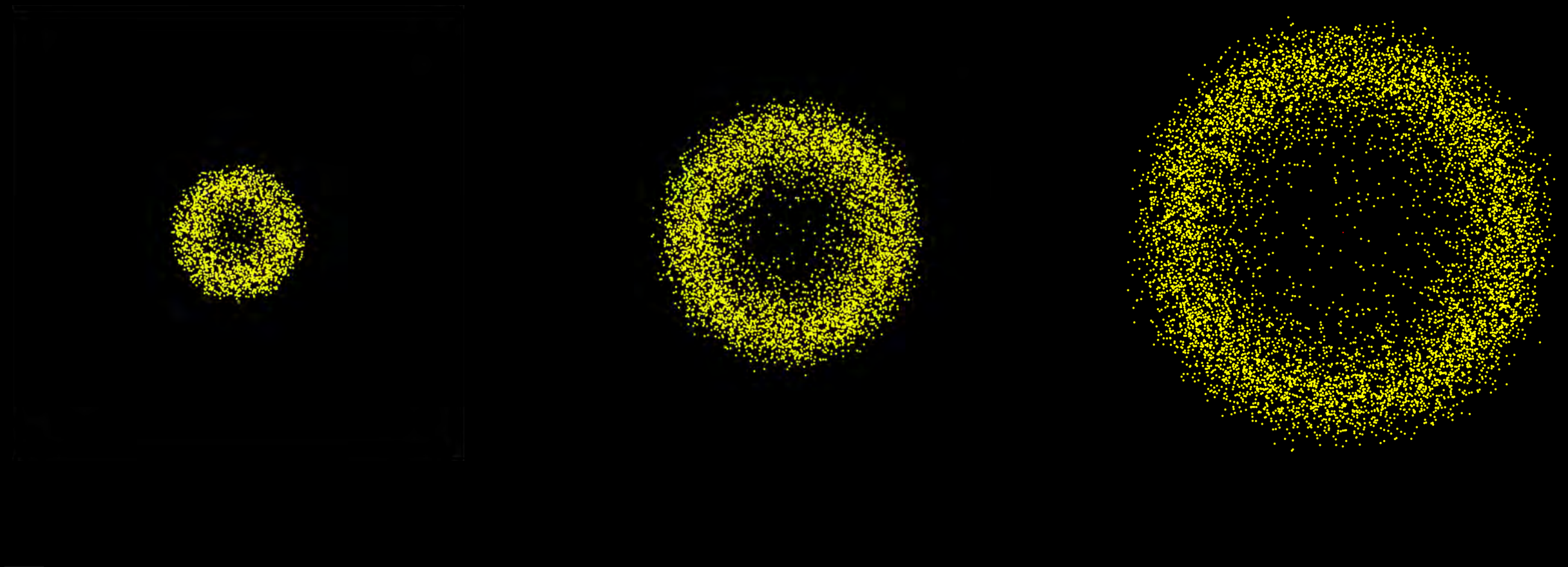


Robustness, sensitivity and pseudospectra around higher order exceptional points



K. G. Makris

Physics Department, University of Crete, Greece

Institute of Electronic Structure and Laser (IESL)-FORTH Heraklion, Greece

Synopsis

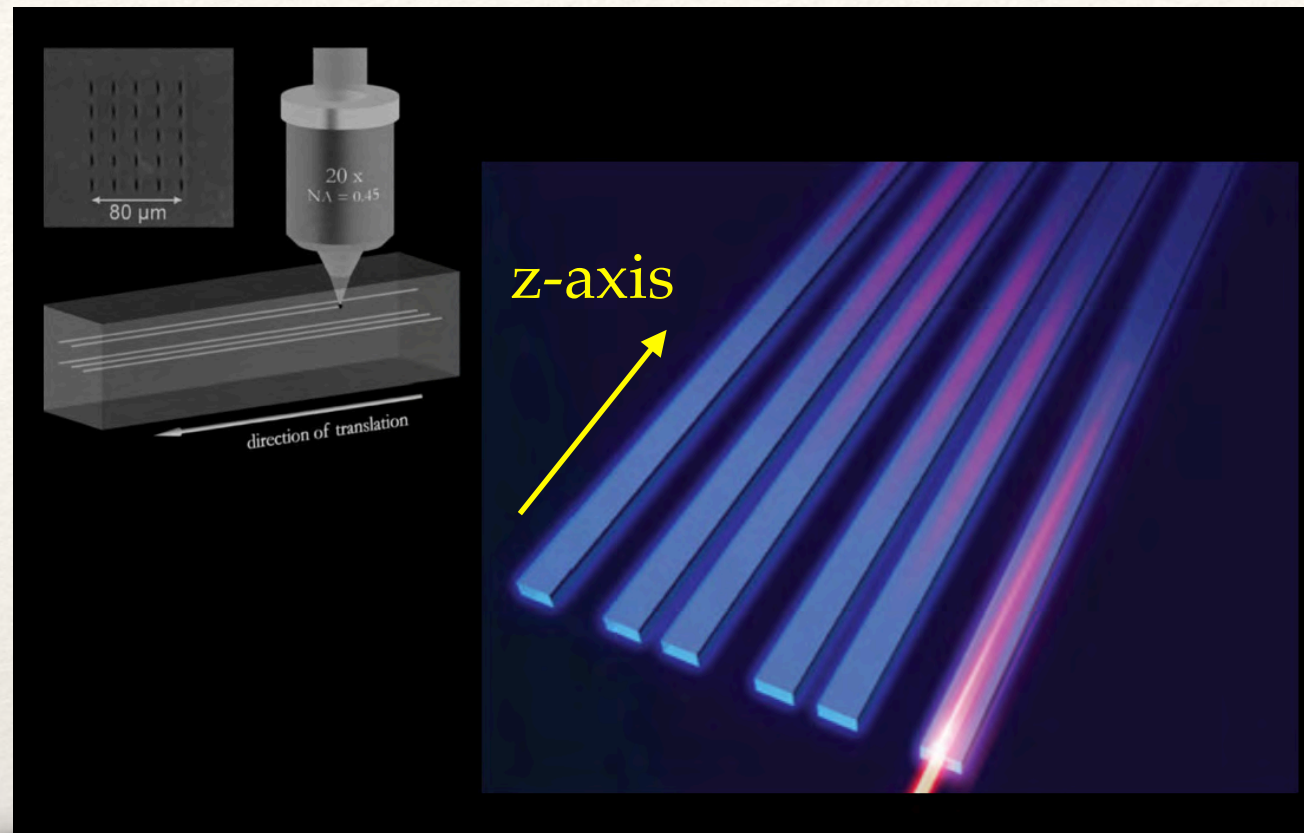
- ❖ Physical Motivation and non-Hermitian Photonics
- ❖ Non-Hermitian Degeneracies: Exceptional points
- ❖ **Part A: Higher-Order Exceptional points in lattices**
- ❖ **Part B: Extreme behaviour around Exceptional Points**
- ❖ Conclusions

Physical Motivation

Non-Hermitian Photonics

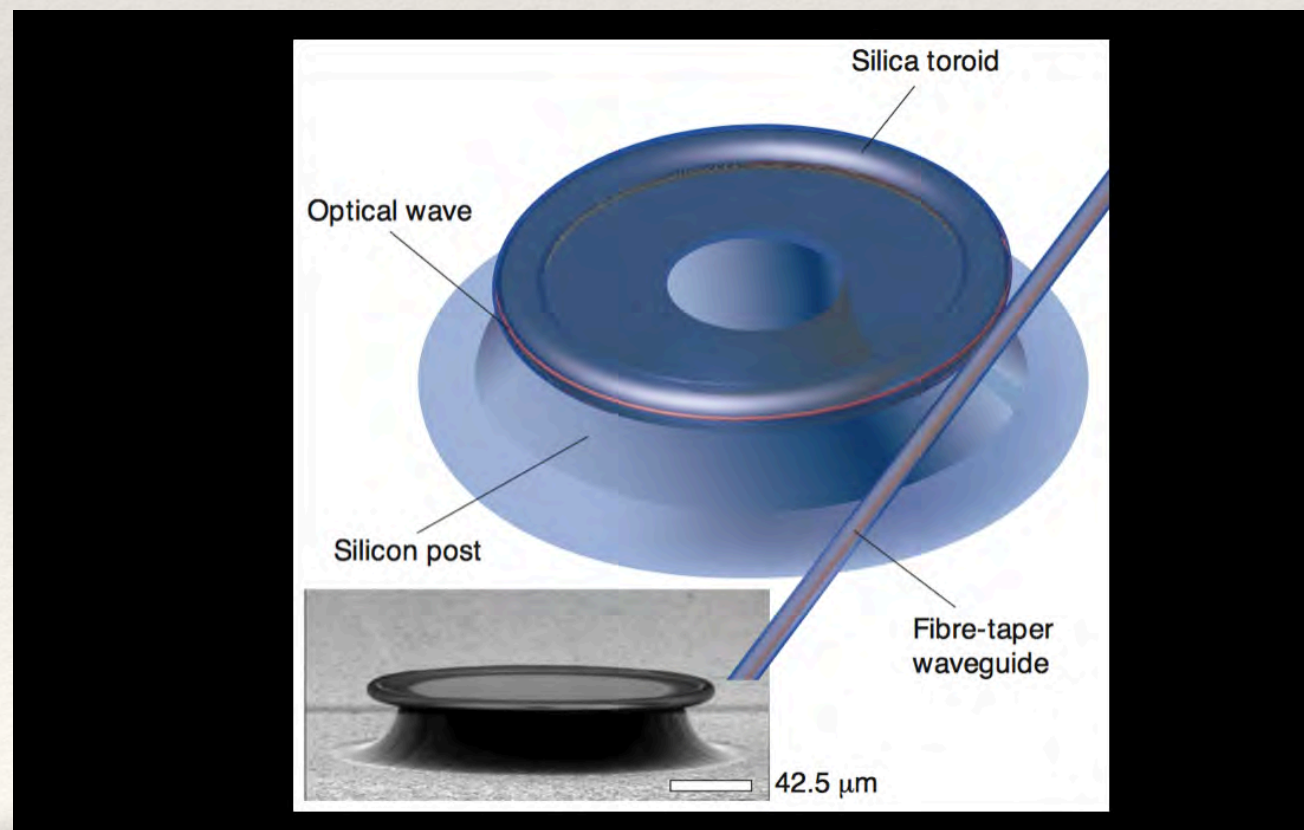
Photons on chip

Guiding light in integrated systems



Waveguides

Real
refractive
index



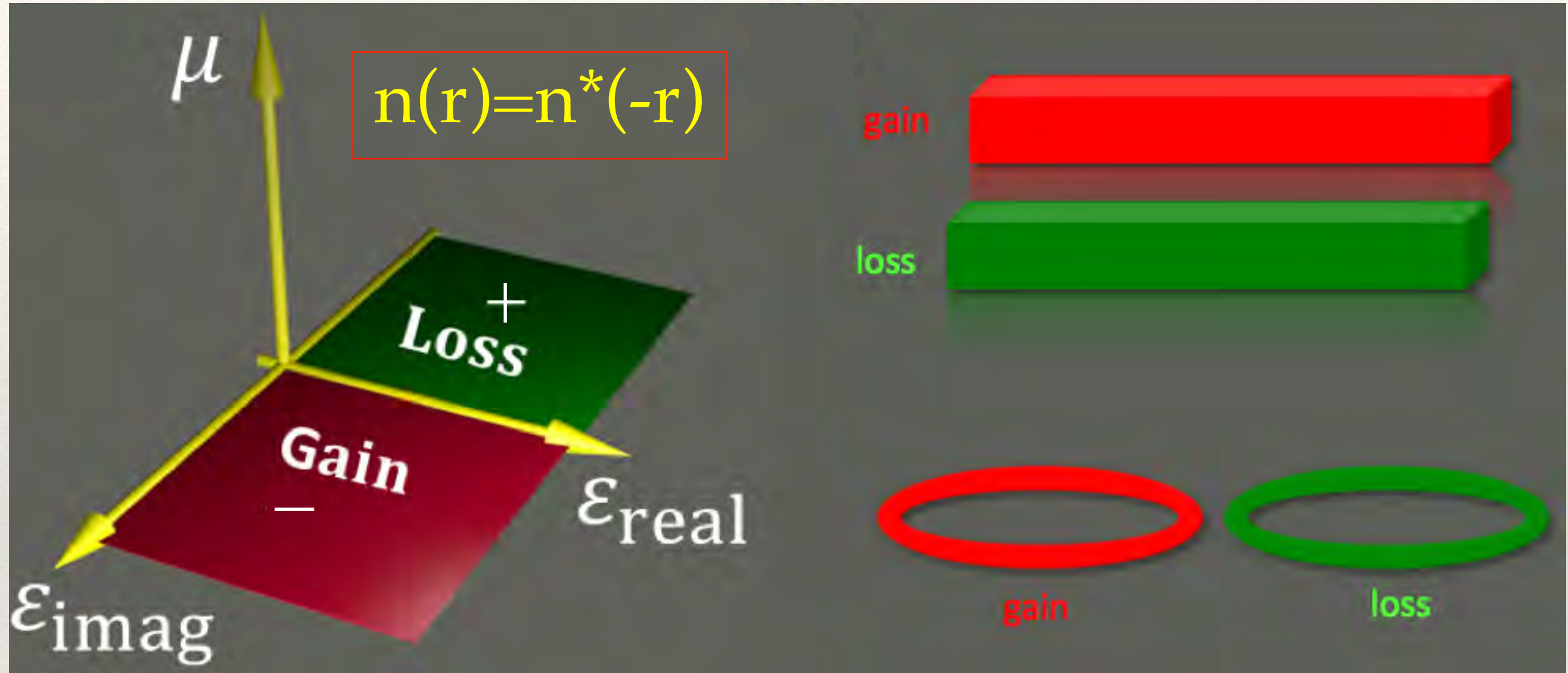
Cavities

Complex Refractive Index

C. M. Bender, and S. Boettcher, Phys. Rev. Lett. 80 5243 (1998)

K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, Phys. Rev. Lett. 100, 103904 (2008)

C. Ruter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev and D. Kip, Nat. Phys. 6, 192 (2010)

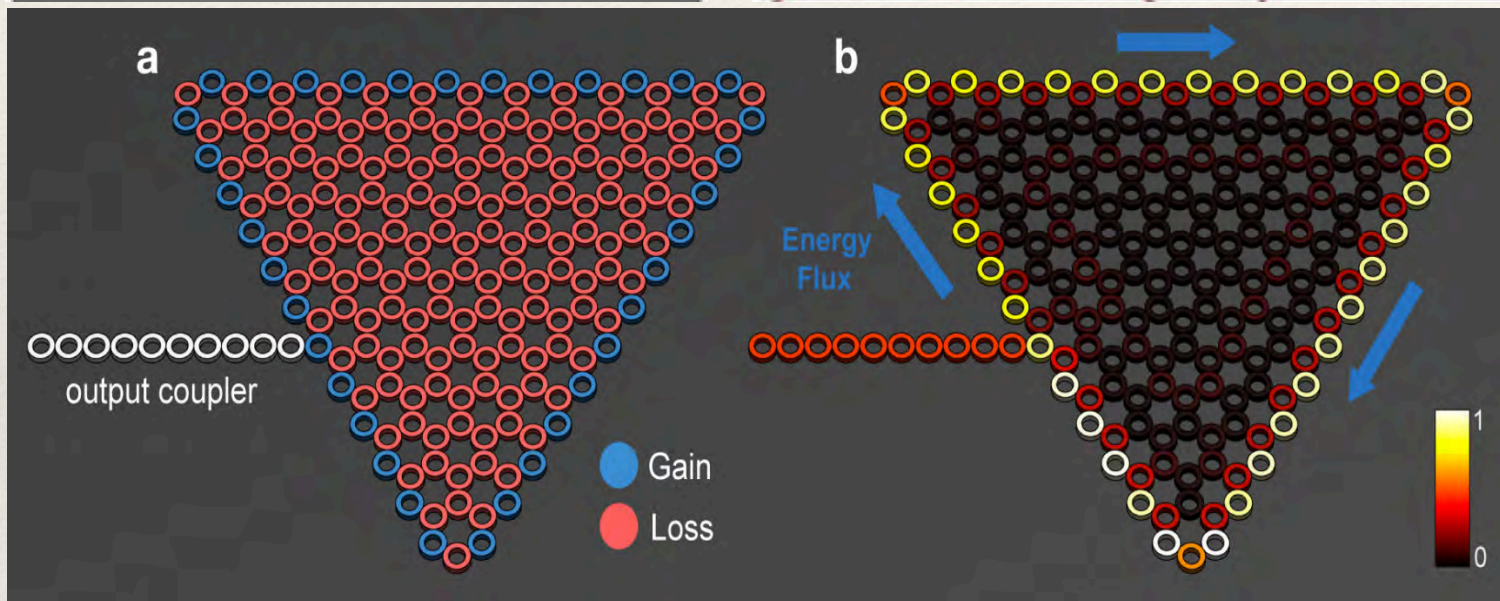
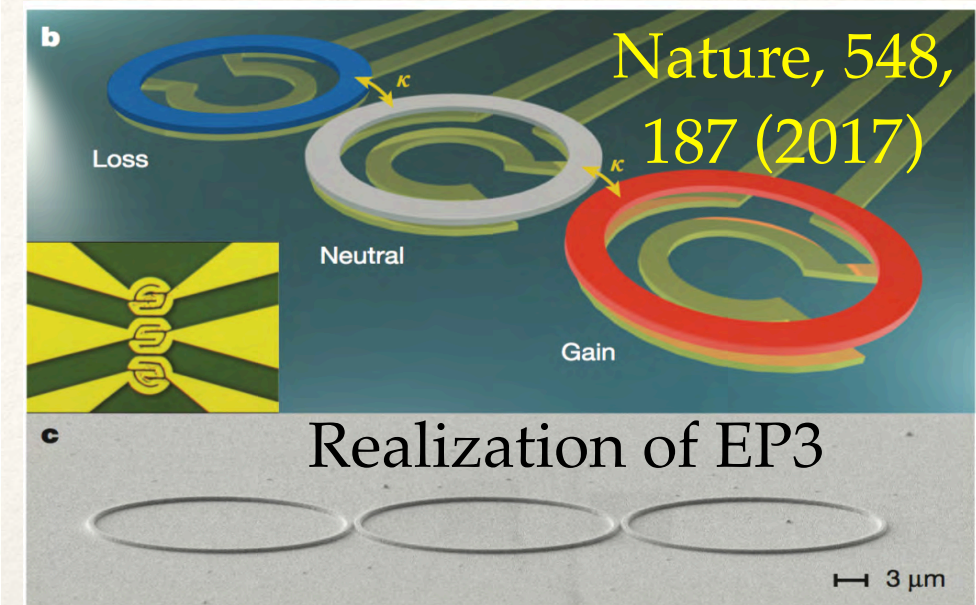
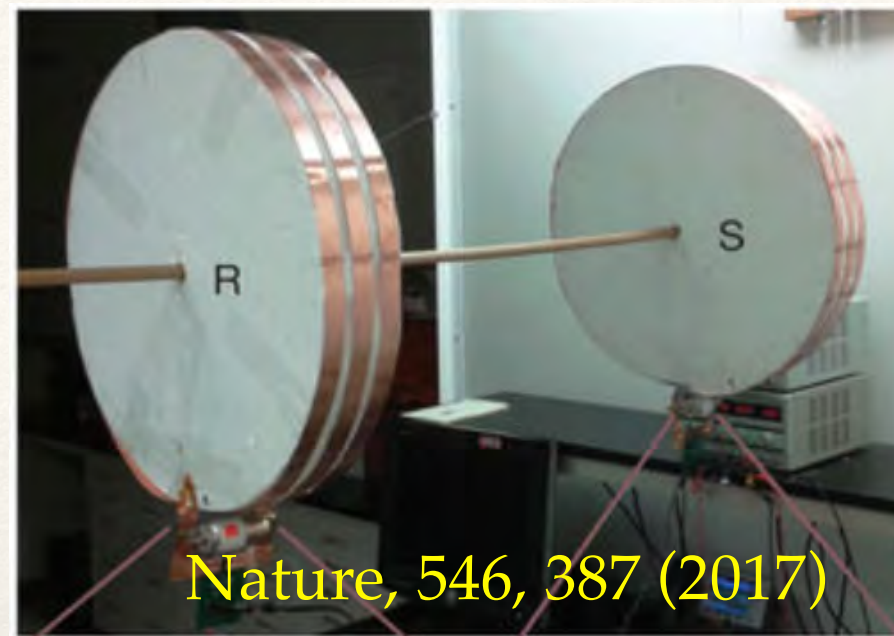
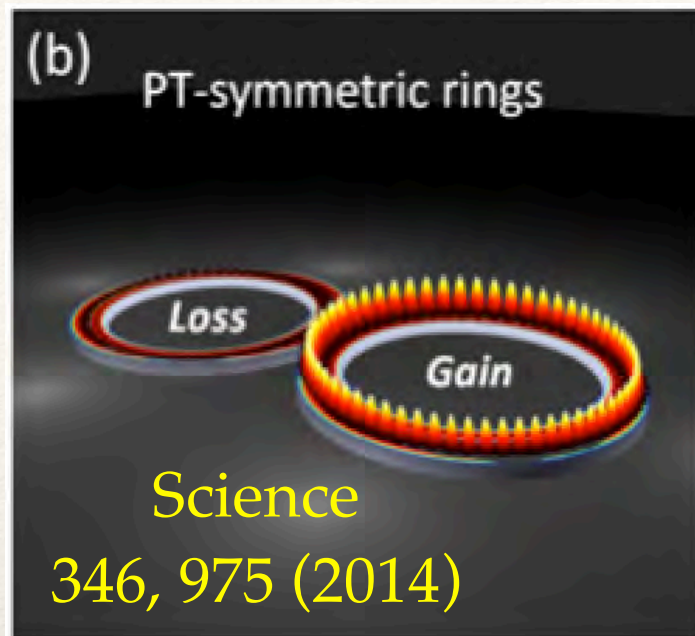


$$\varepsilon = \varepsilon_R + i\varepsilon_I \quad \text{Permittivity}$$

$$n = n_R + in_I \quad \text{Refractive index}$$

$$n_I, \varepsilon_I = \begin{cases} > 0, & \text{loss} \\ < 0, & \text{gain} \end{cases}$$

Recent developments



[Topological insulator laser: theory](#)
Science 359 (6381), eaar4003, (2018).

[Topological insulator laser: Experiments](#)
Science 359 (6381), eaar4005 (2018).

nature
physics

REVIEW ARTICLES

PUBLISHED ONLINE: 5 JANUARY 2017 | DOI: 10.1038/NPHYS4323

Nat. Phys. 14, 11 (2018)

Non-Hermitian physics and PT symmetry

Ramy El-Ganainy¹, Konstantinos G. Makris², Mercedeh Khajavikhan³, Ziad H. Musslimani⁴,
Stefan Rotter⁵ and Demetrios N. Christodoulides^{3*}

Exceptional Points

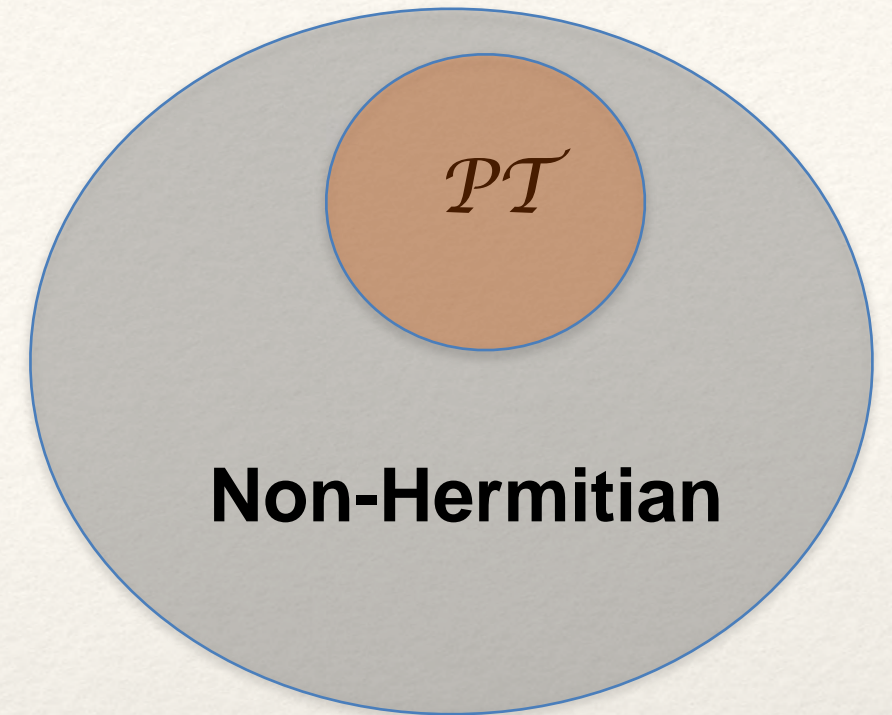
Non-Hermitian Degeneracies

2x2 Non-Hermitian Matrix

- Complex spectrum
- Non-orthogonality
- Exceptional points
- Energy is not conserved
- Ultra-sensitivity

$$M = \begin{pmatrix} \delta - ig_1 & \kappa \\ \kappa & ig_2 \end{pmatrix} \quad M \neq M^\dagger$$

Non-Hermitian matrix



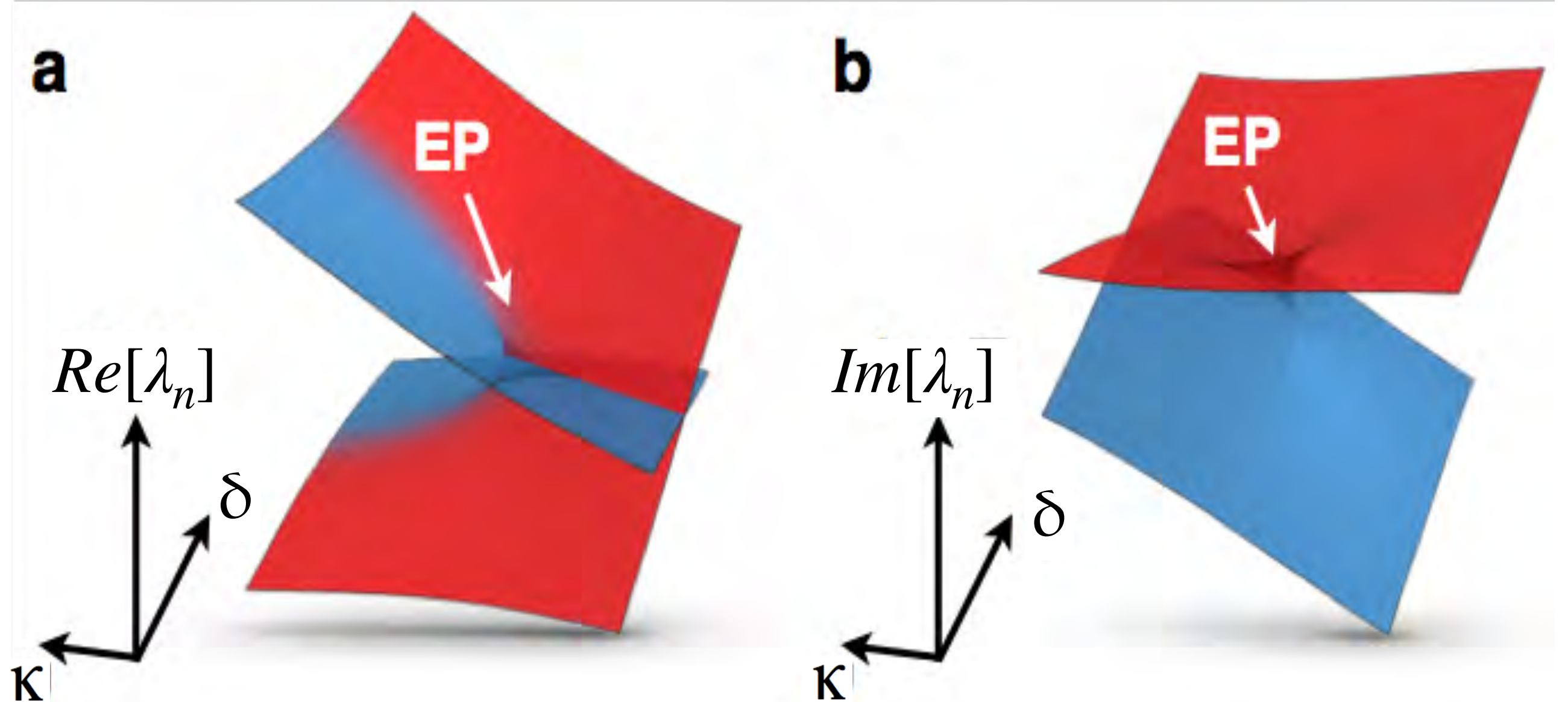
$$M|\phi_n\rangle = \lambda_n|\phi_n\rangle \quad \langle \tilde{\phi}_m | \phi_n \rangle = \delta_{n,m}$$

Bi-orthogonality Condition

$$\lambda_{1,2} = \frac{1}{2} \left\{ \delta - i(g_1 - g_2) \pm \sqrt{\delta^2 + 4\kappa^2 - (g_1 + g_2)^2 - 2i\delta(g_1 + g_2)} \right\}$$

A non-Hermitian matrix has complex eigenvalues in general

Exceptional Points



$$\delta = 0, \quad \kappa = \frac{|g_1 + g_2|}{2} \Rightarrow \lambda_1 = \lambda_2 = -i \frac{g_1 - g_2}{2}$$

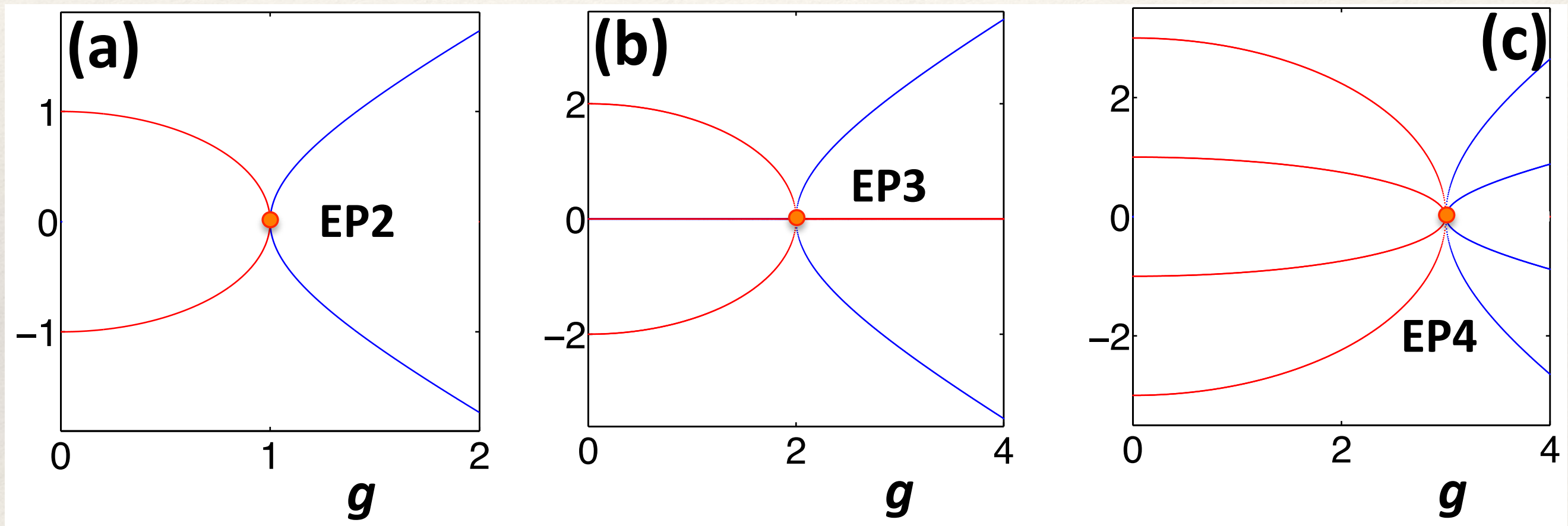
Eigenvalues and eigenvectors coalesce: Exceptional Point (EP)

Higher order EPs

$$M_2 = i \begin{pmatrix} ig & \kappa \\ \kappa & -ig \end{pmatrix}$$

$$M_3 = i \begin{pmatrix} -2ig & \sqrt{2}\kappa & 0 \\ \sqrt{2}\kappa & 0 & \sqrt{2}\kappa \\ 0 & \sqrt{2}\kappa & 2ig \end{pmatrix}$$

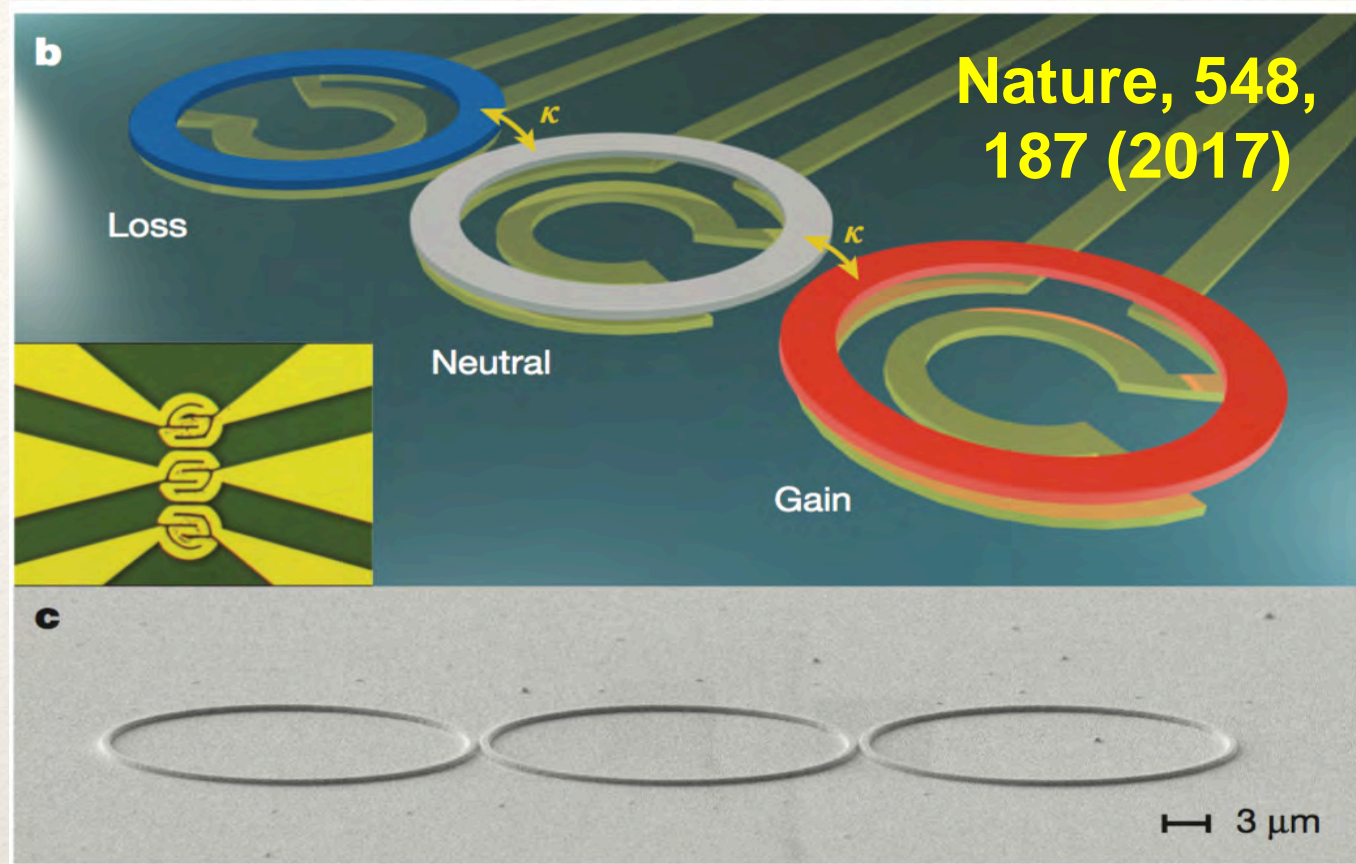
$$M_4 = i \begin{pmatrix} -2ig & \sqrt{3}\kappa & 0 & 0 \\ \sqrt{3}\kappa & -ig & \sqrt{3}\kappa & 0 \\ 0 & \sqrt{3}\kappa & ig & \sqrt{3}\kappa \\ 0 & 0 & \sqrt{3}\kappa & 2ig \end{pmatrix}$$



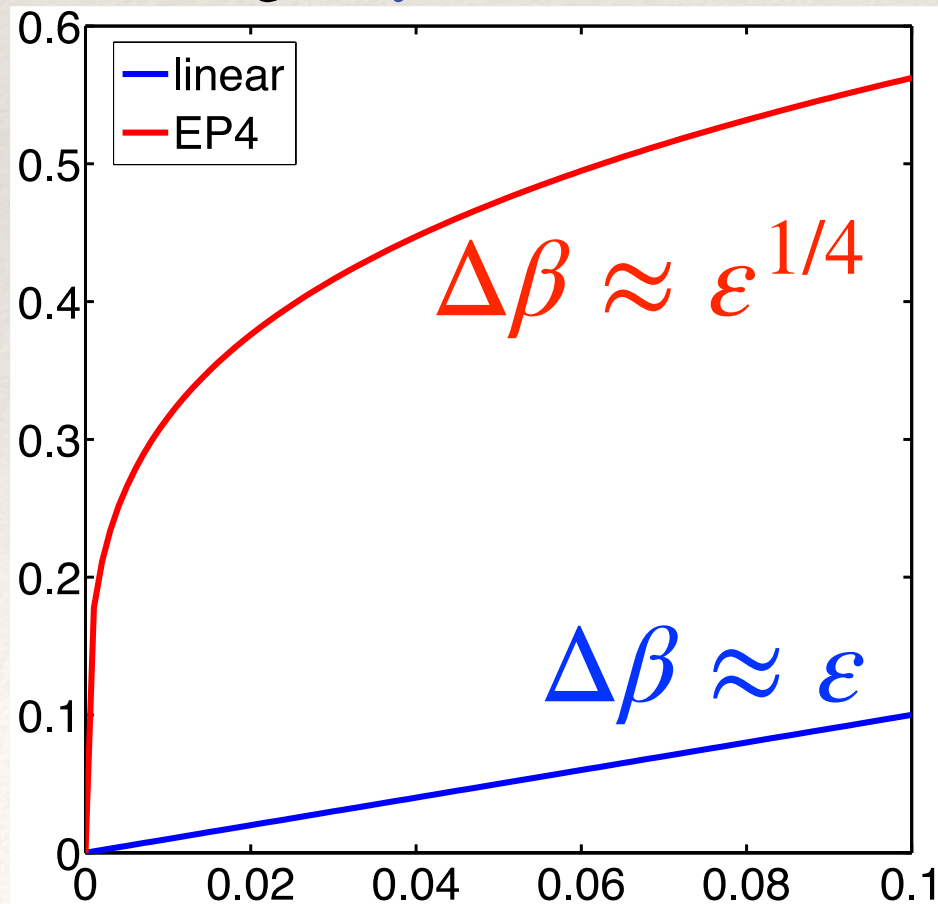
E. M. Graefe, U. Günther, H. J. Korsch, and A. E. Niederle, [J. Phys. A **41**, 255206 \(2008\)](#).

D. Gilles and E.M.Graefe, [J. Phys. A **45**, 025303 \(2012\)](#). W. D. Heiss and G. Wunner, [J. Phys. A **49**, 495303 \(2016\)](#).

Amplification and Sensitivity close to EPNs

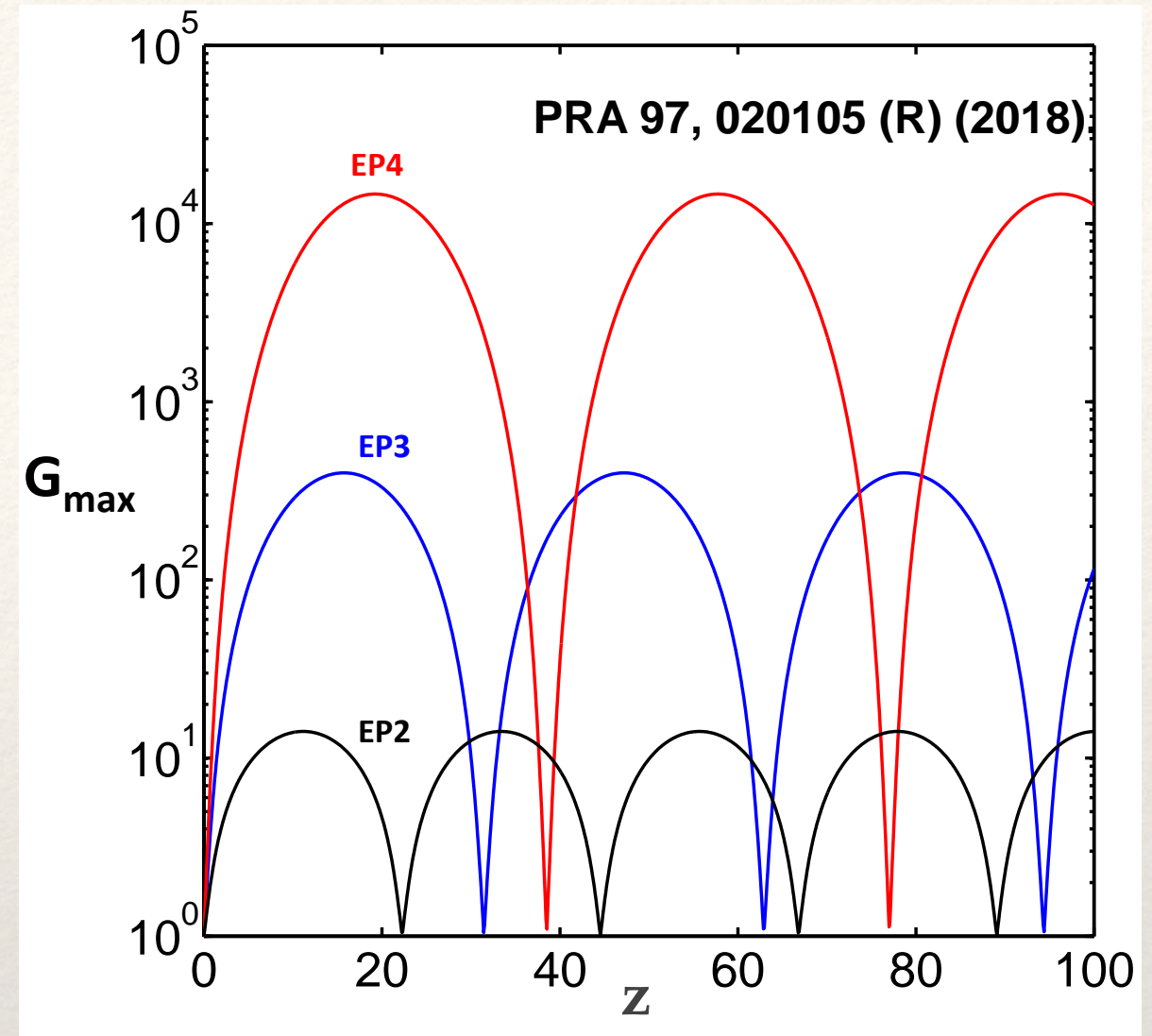


J. Wiersig, *Phys. Rev. Lett.* **112**, 203901 (2014)



$$\Delta\beta \approx \varepsilon^{1/N}$$

**Sensitivity
at EPNs**



$$\tilde{g} \equiv g/\kappa$$

$$G_{\max} = \max_z \|e^{zH}\| = \left(\frac{1 + \tilde{g}}{1 - \tilde{g}}\right)^{N-1}$$

Maximal amplification at EPNs

Can spectrum alone determine dynamics?

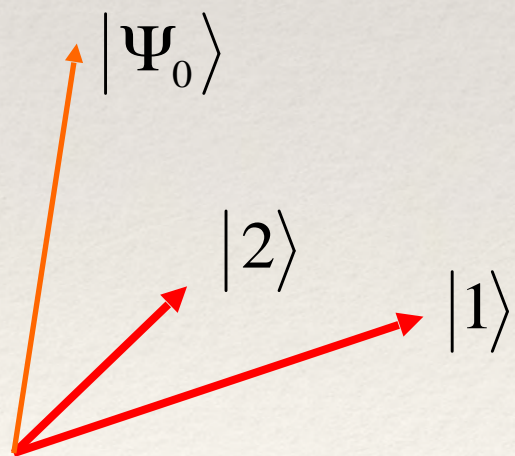
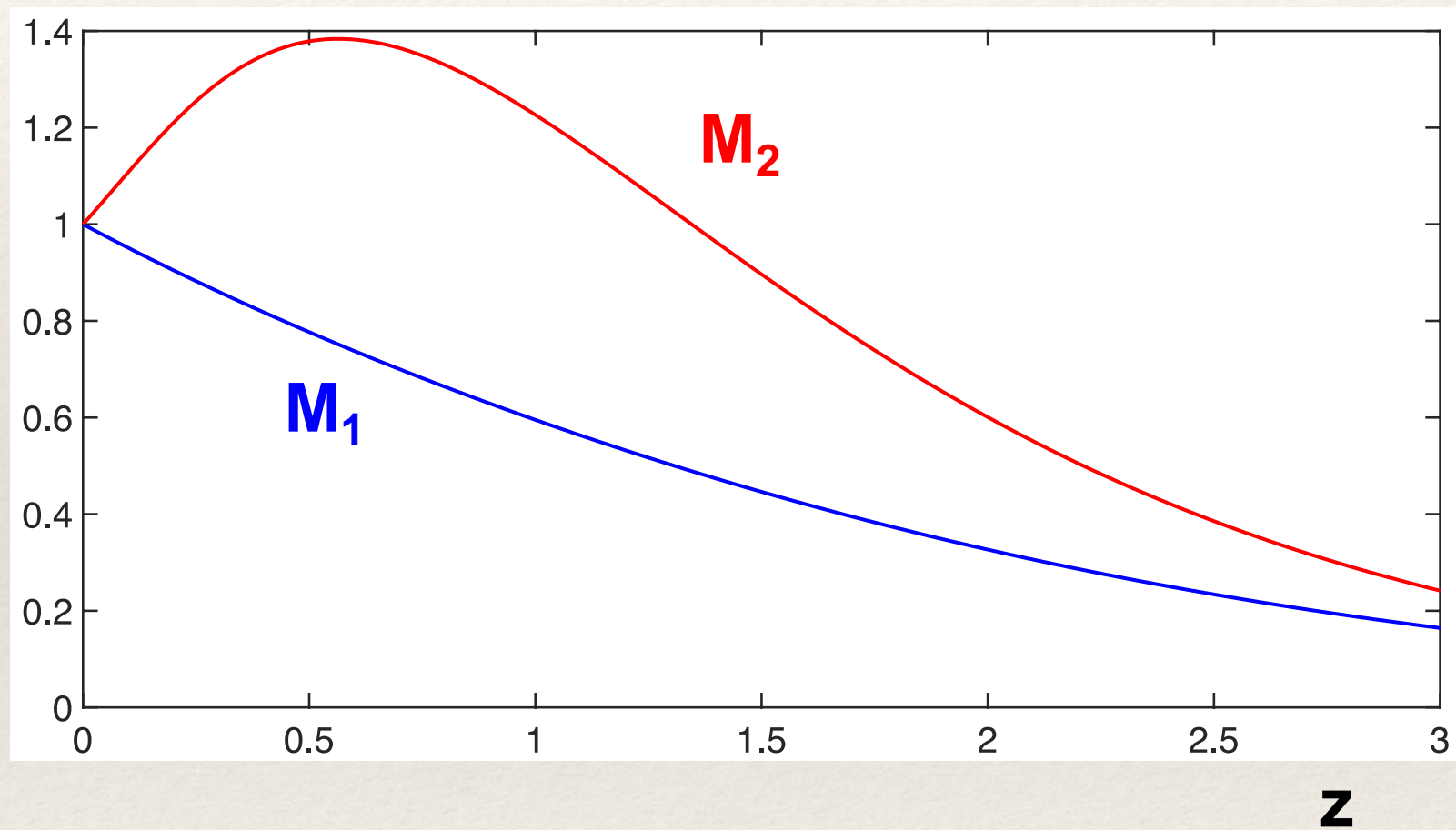
$$M_1 = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$$

$$\lambda_{1,2} = -1, -1 < 0$$

$$M_2 = \begin{pmatrix} -1 & 5 \\ 0 & -2 \end{pmatrix}$$

$$\lambda_{1,2} = -1, -2 < 0$$

$$\|e^{zM}\|$$



$$|\Psi_0\rangle = |1\rangle \exp(i\beta_1 z) + |2\rangle \exp(i\beta_2 z)$$

$$\langle \Psi_0 | \Psi_0 \rangle = \langle 1 | 1 \rangle + \langle 2 | 2 \rangle + \langle 2 | 1 \rangle \exp(i\Delta\beta z) + cc$$

Pseudospectrum

$$\|x\| \quad \text{Norm of a vector} \qquad \|M\| \equiv \max_x \frac{\|Mx\|}{\|x\|} \quad \text{Norm of a matrix}$$

$$\sigma(H) \equiv \text{Eigenvalue spectrum}$$

Pseudospectrum

$$\sigma_\varepsilon(H) \equiv \bigcup_{i=1, \|E_i\| < \varepsilon}^M \sigma(H + E_i)$$

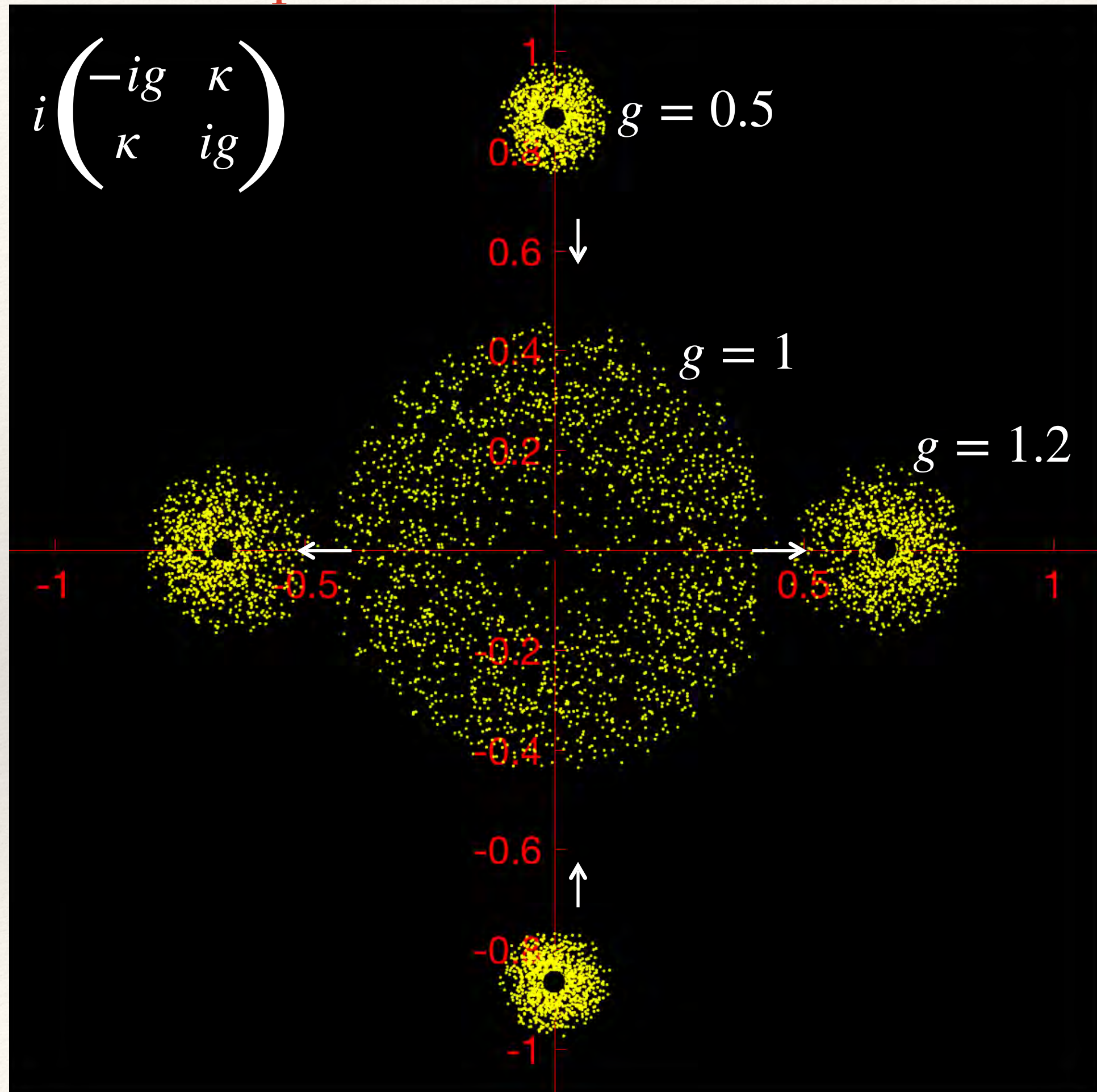
Pseudospectral radius

$$\rho_\varepsilon \equiv \max_{\mathbf{z} \in \sigma_\varepsilon(H)} |\mathbf{z}|$$

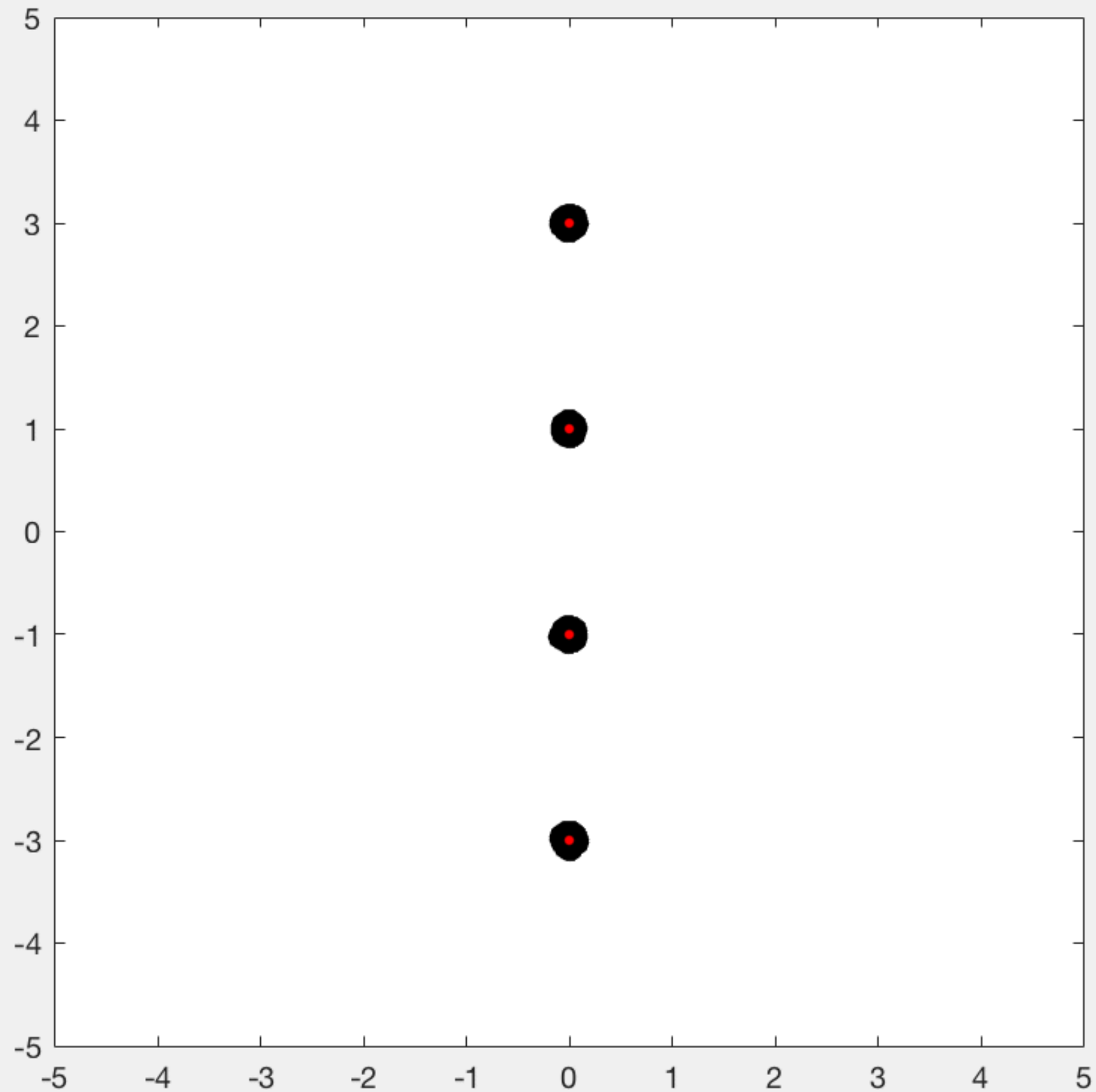
$$\sigma_\varepsilon(H) \rightarrow \underbrace{\begin{pmatrix} \delta - ig_1 & \kappa \\ \kappa & ig_2 \end{pmatrix}}_H + \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}$$

Non-Hermitian
random matrix

Pseudospectrum of a PT-matrix with EP2



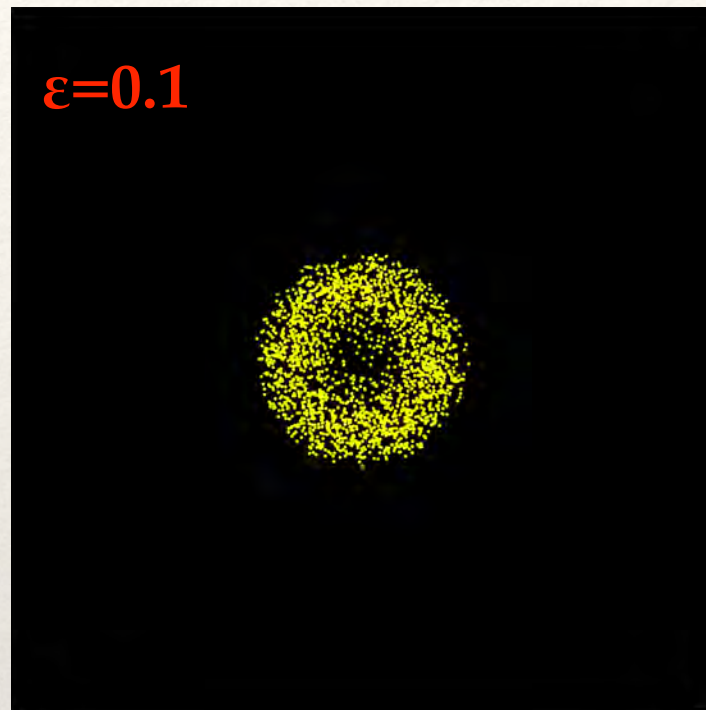
Pseudospectrum of a PT-matrix with EP4



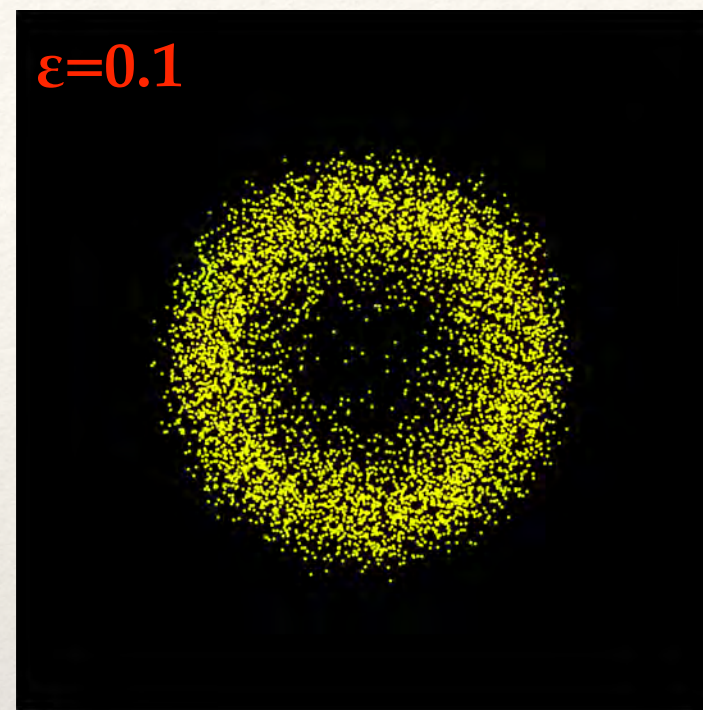
Extreme Sensitivity close to EPNs

Perturbation theory: $\Delta\omega^{EP2} = 2\kappa^{1/2}\epsilon^{1/2}$, $\Delta\omega^{EP3} = 1.5\kappa^{2/3}\epsilon^{1/3}$

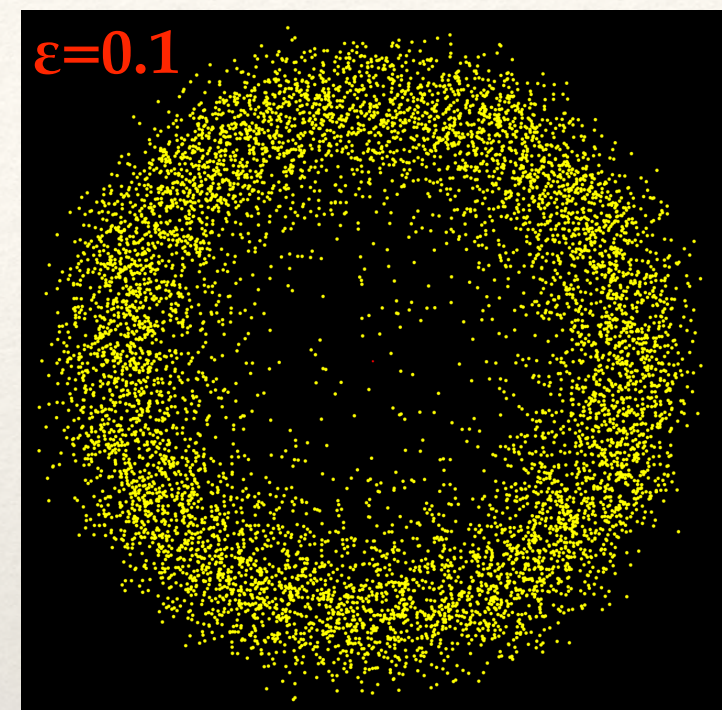
J. Wiersig, Physical Review A 93, 033809 (2016), W. Chen, et al., Nature 548, 192 (2017) H. Hodaei, et al, Nature, 548, 187 (2017).



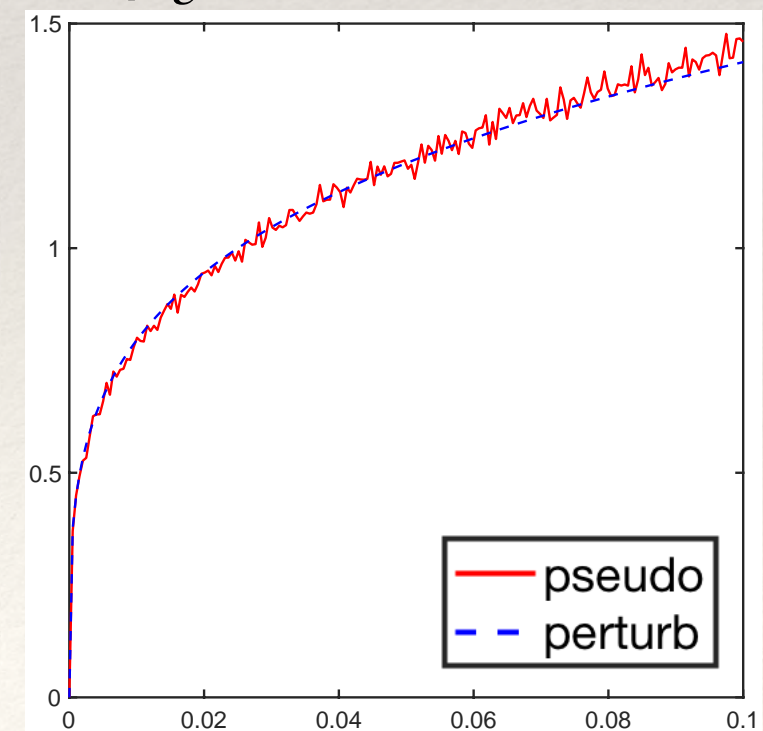
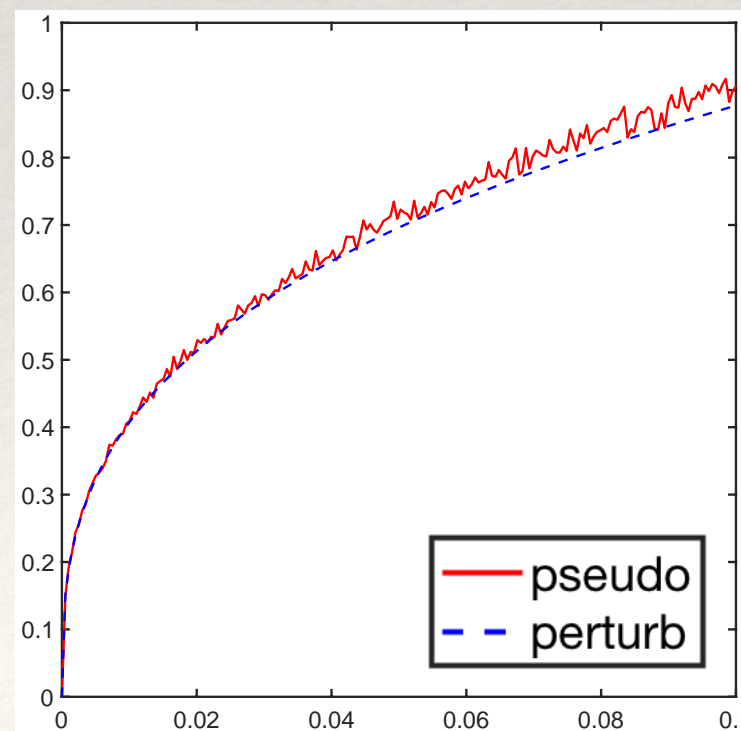
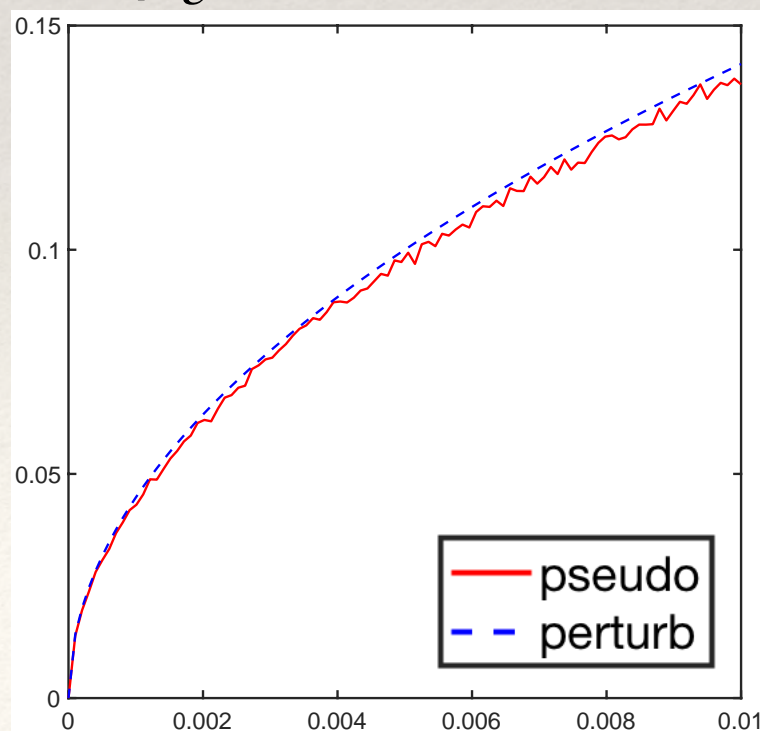
$$\rho_\epsilon \approx \epsilon^{1/2} \approx 0.3$$



$$\rho_\epsilon \approx \epsilon^{1/3} \approx 0.46$$

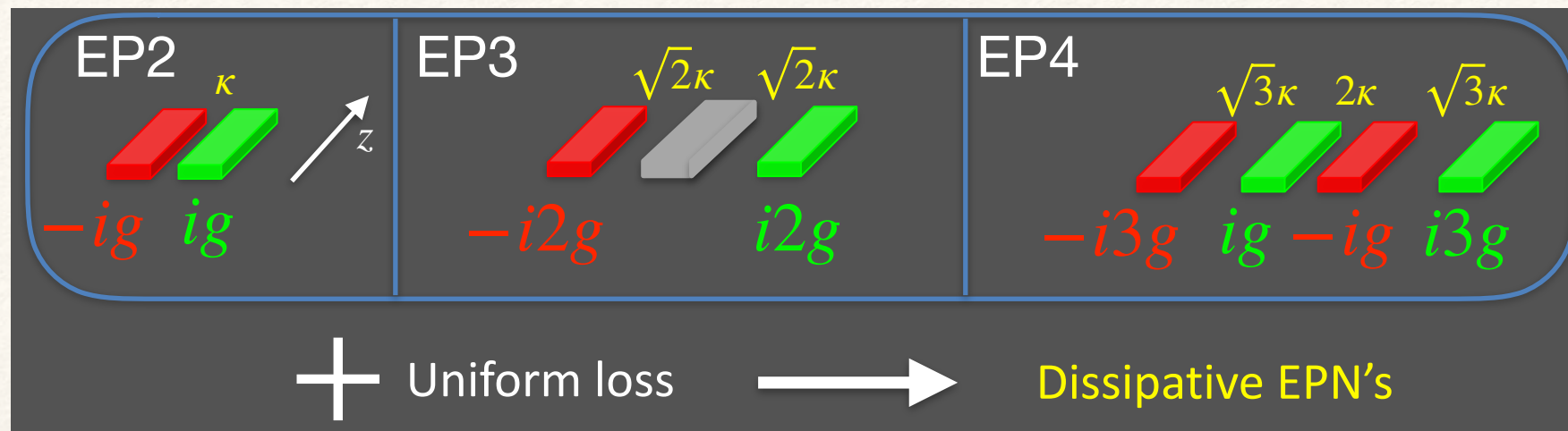


$$\rho_\epsilon \approx \epsilon^{1/4} \approx 0.56$$



ρ_ϵ

Dissipative Higher order EPs



PHYSICAL REVIEW E **104**, 054218 (2021)

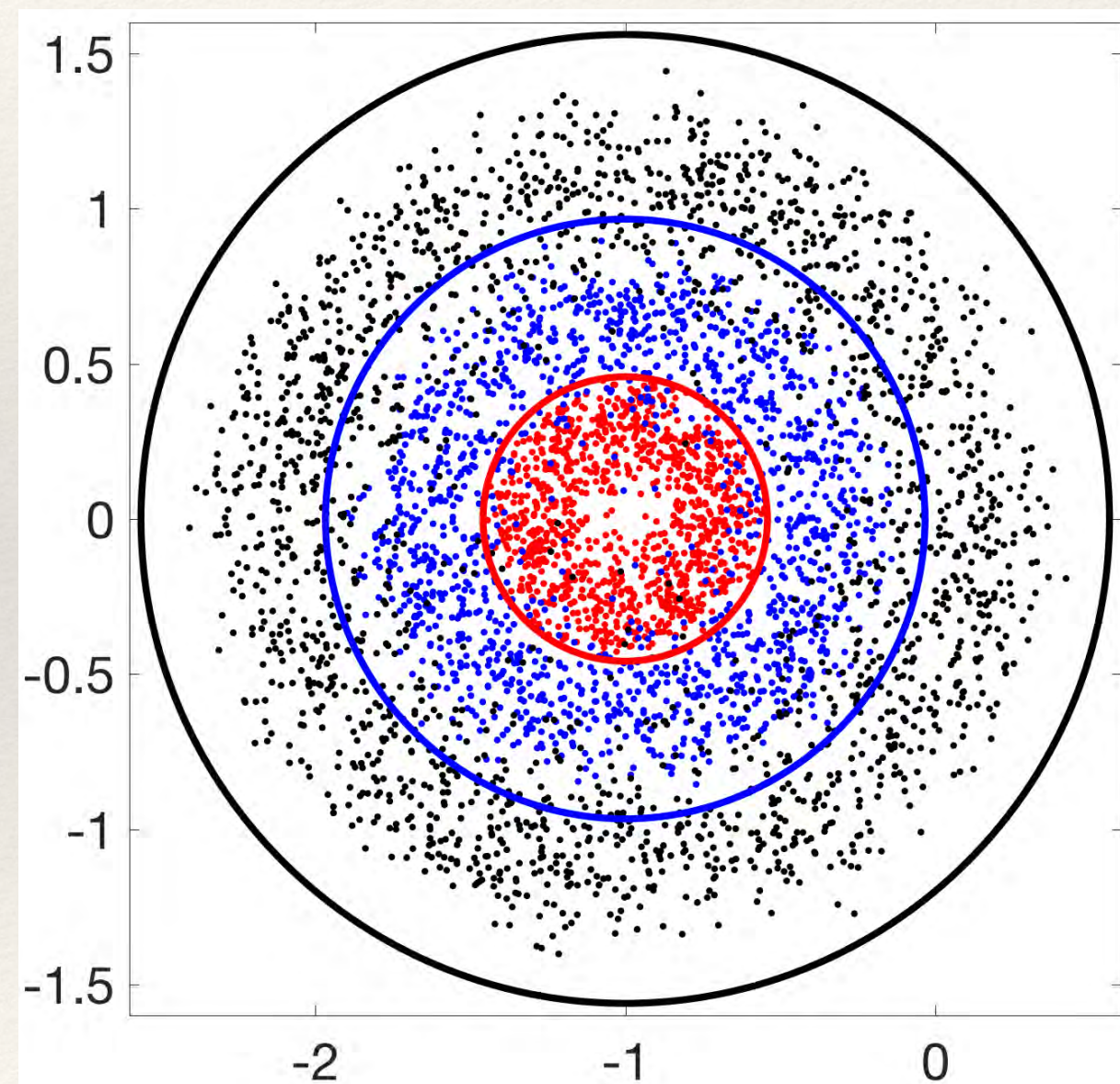
Transient growth and dissipative exceptional points

K. G. Makris

*ITCP-Physics Department, University of Crete, 71003 Heraklion, Greece
and Institute of Electronic Structure and Laser, FORTH, 71110 Heraklion, Greece*

Non-perturbative approach

Pseudospectra on the Complex Plane

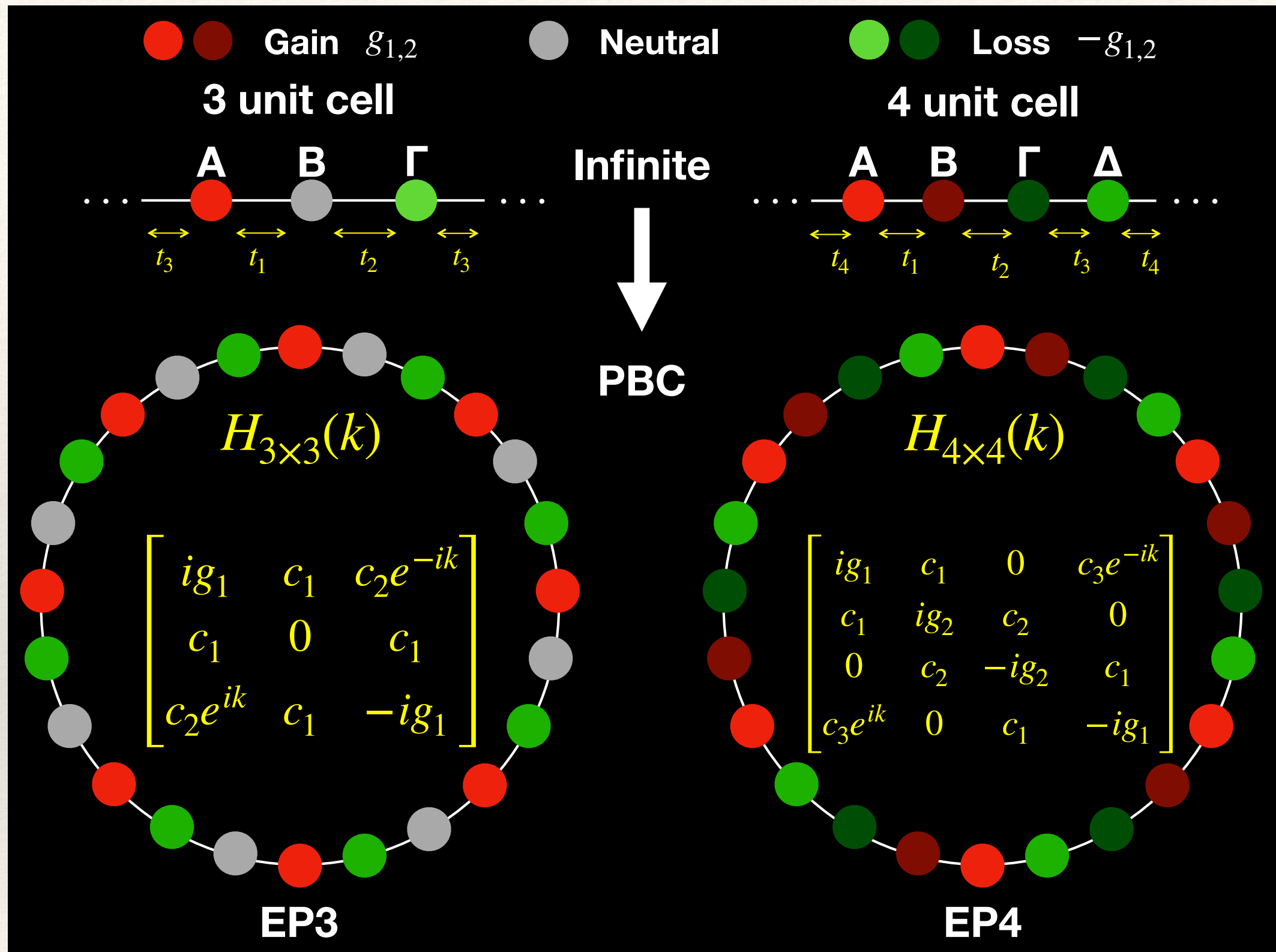


Part A:

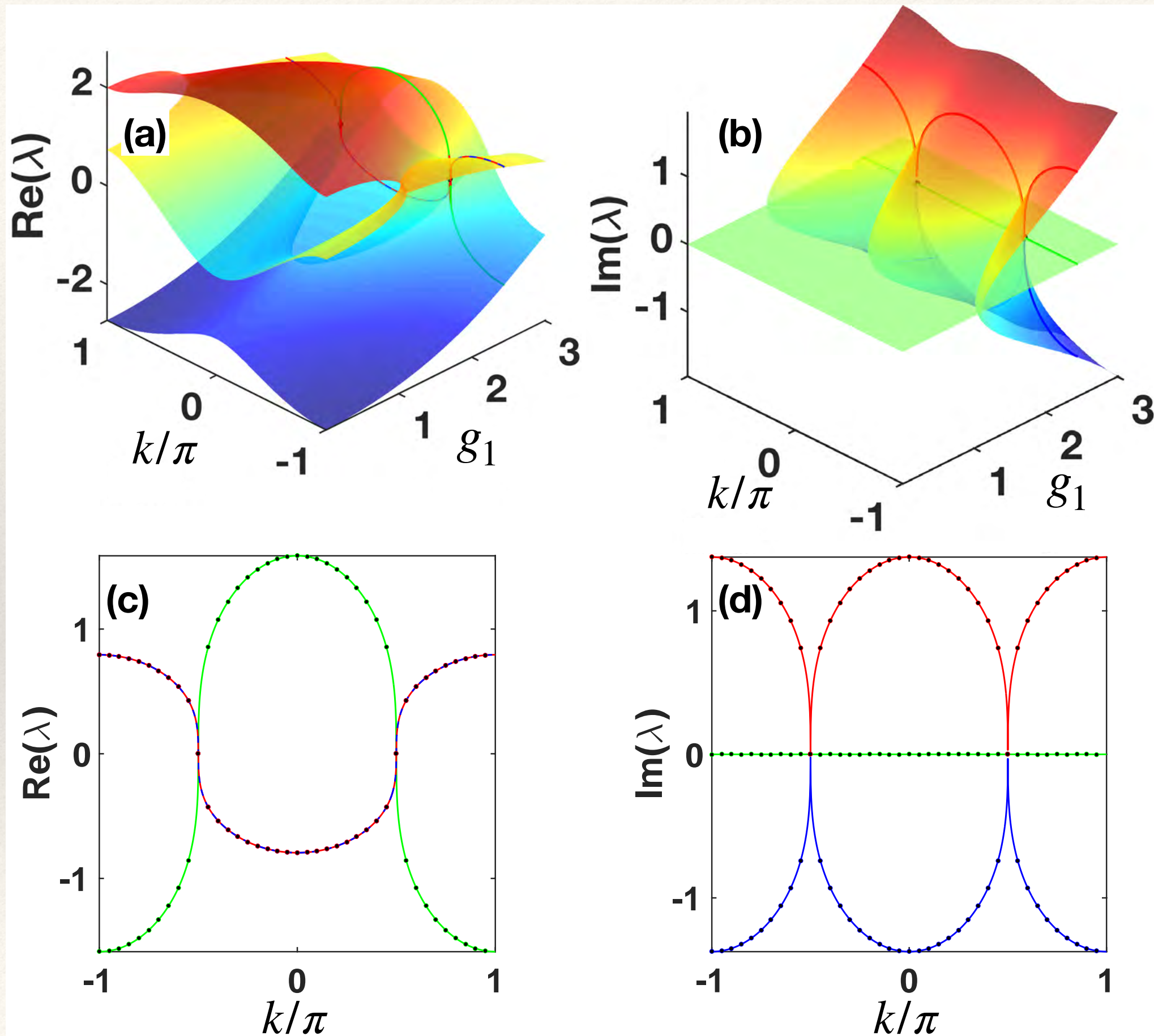
Higher order Exceptional points in lattices

Infinite Optical lattices

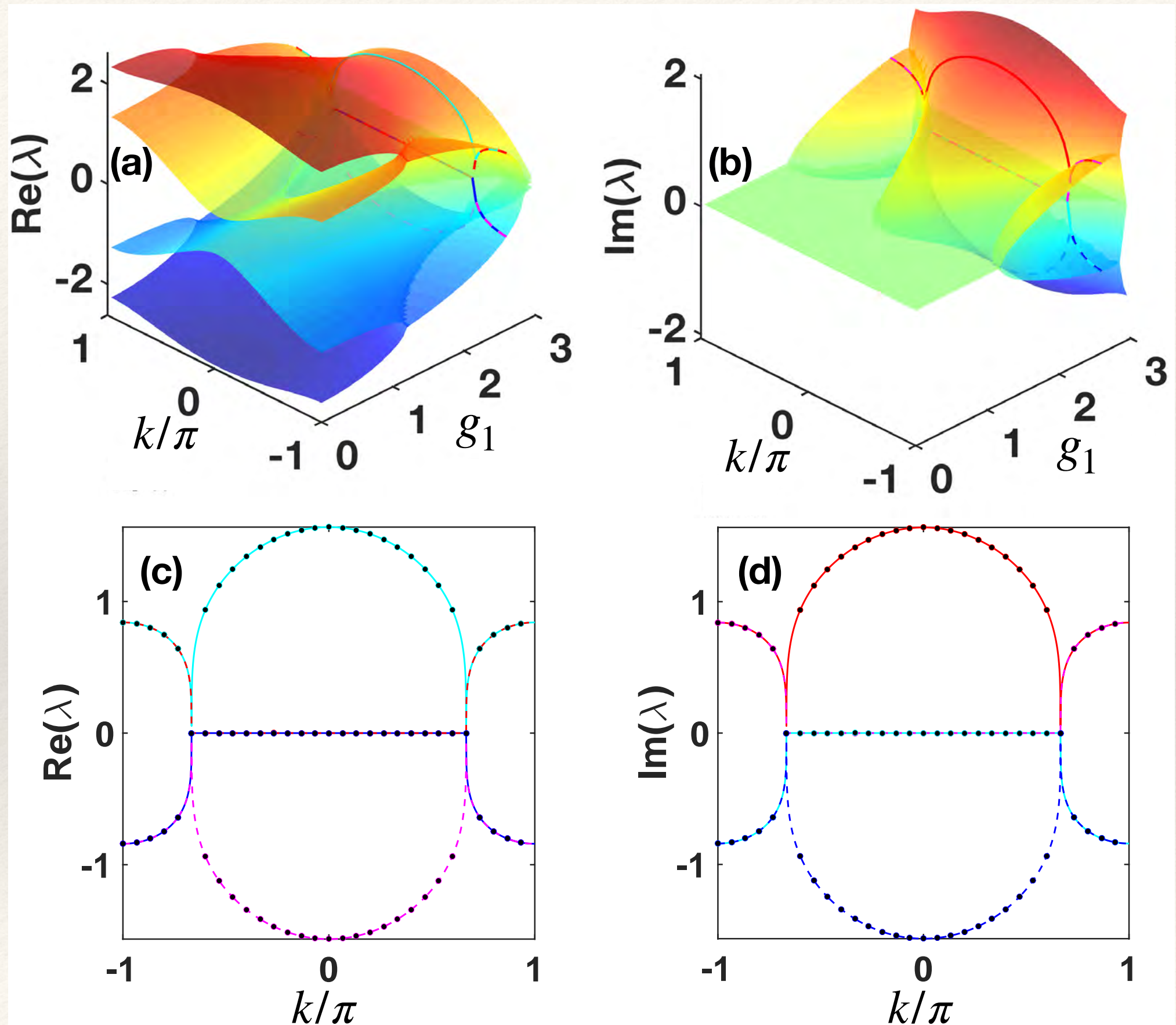
HEPs in infinite optical lattices



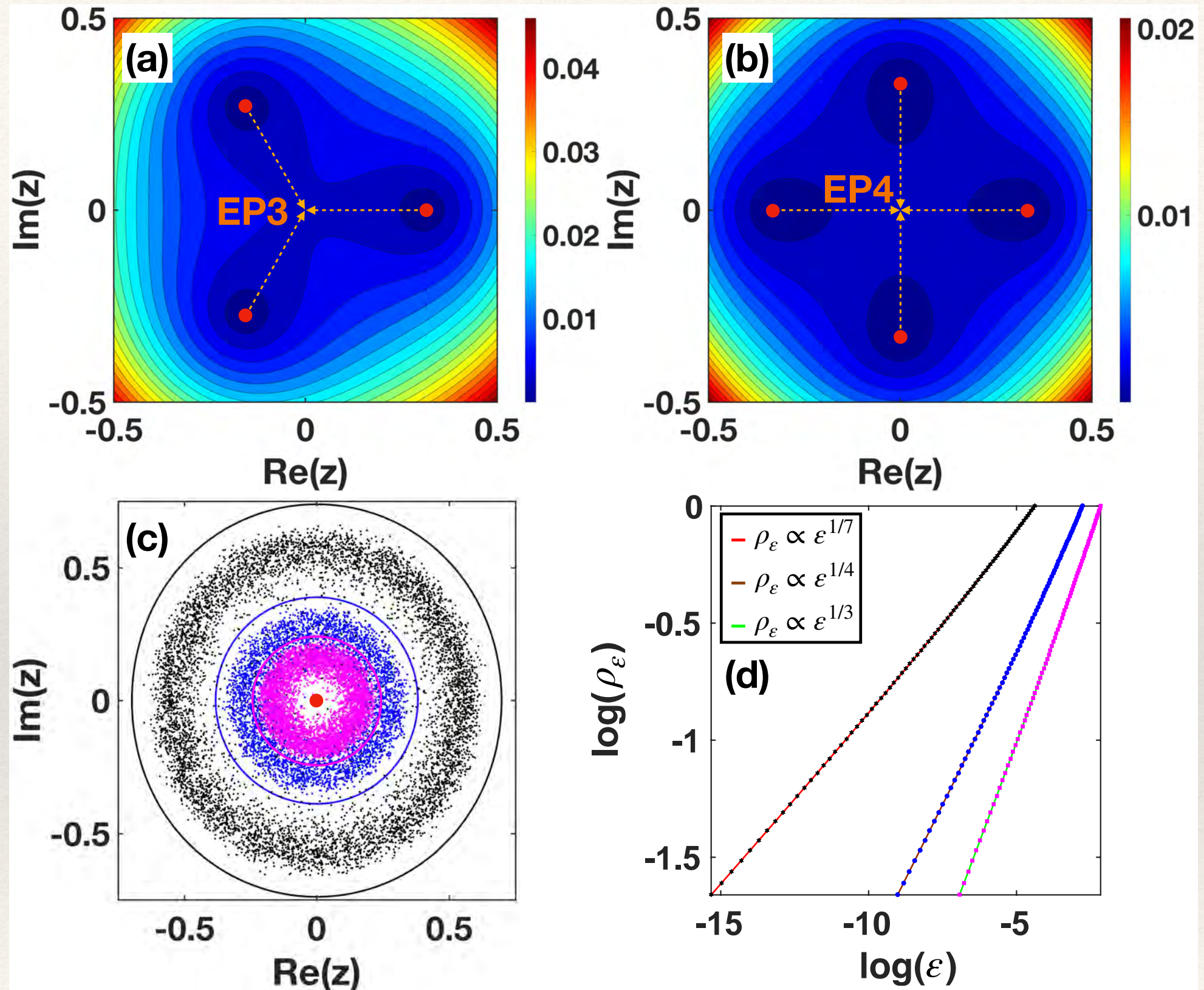
Trimerized lattice - EP3



Tetramerized lattice - EP4



Sensitivity around HEP's



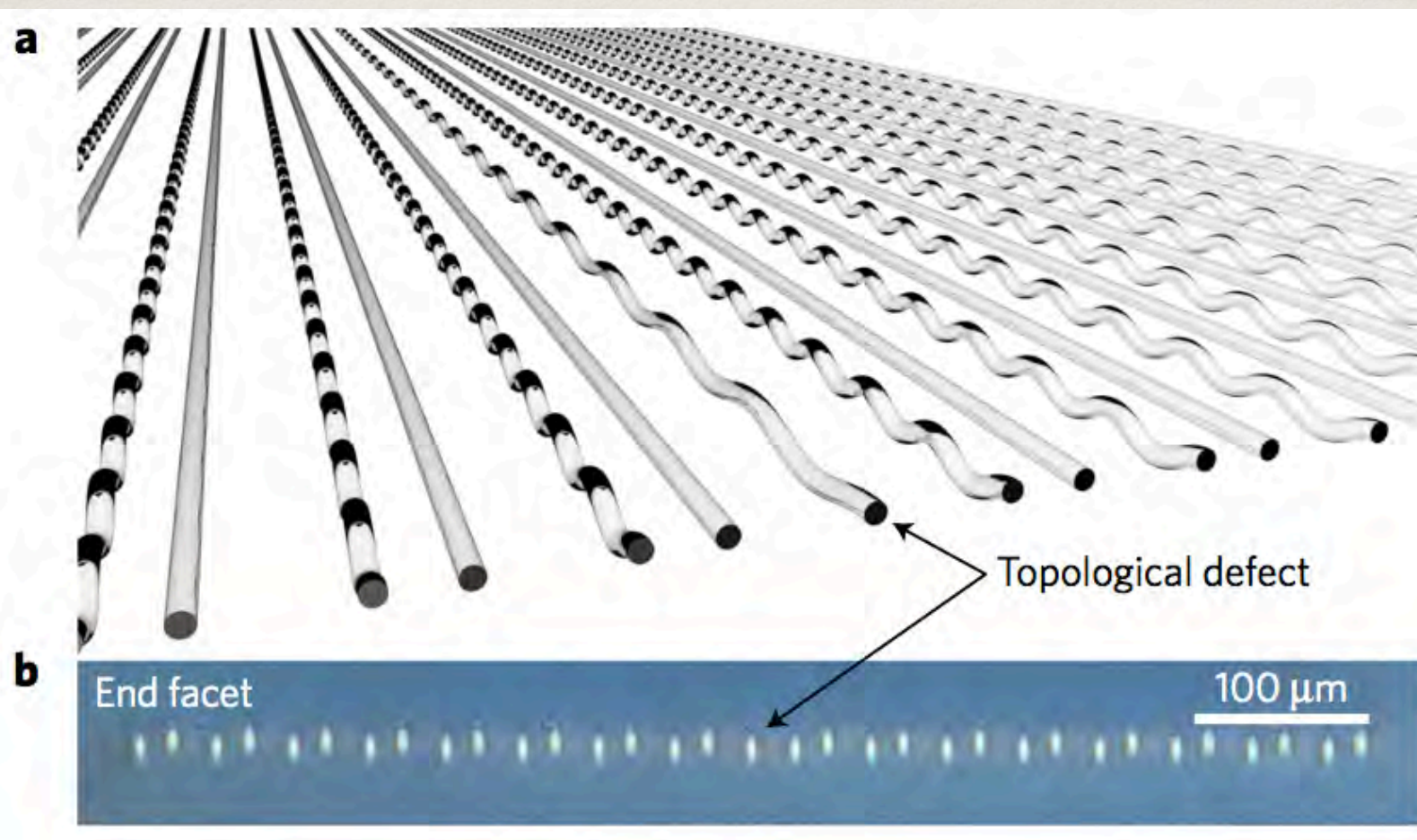
Part B:

Extreme behaviour around HEPs

Sensitivity vs Robustness

Topologically protected bound states in photonic parity-time-symmetric crystals

S. Weimann^{1†}, M. Kremer^{1†}, Y. Plotnik², Y. Lumer², S. Nolte¹, K. G. Makris^{3,4}, M. Segev²,
M. C. Rechtsman⁵ and A. Szameit^{1*}



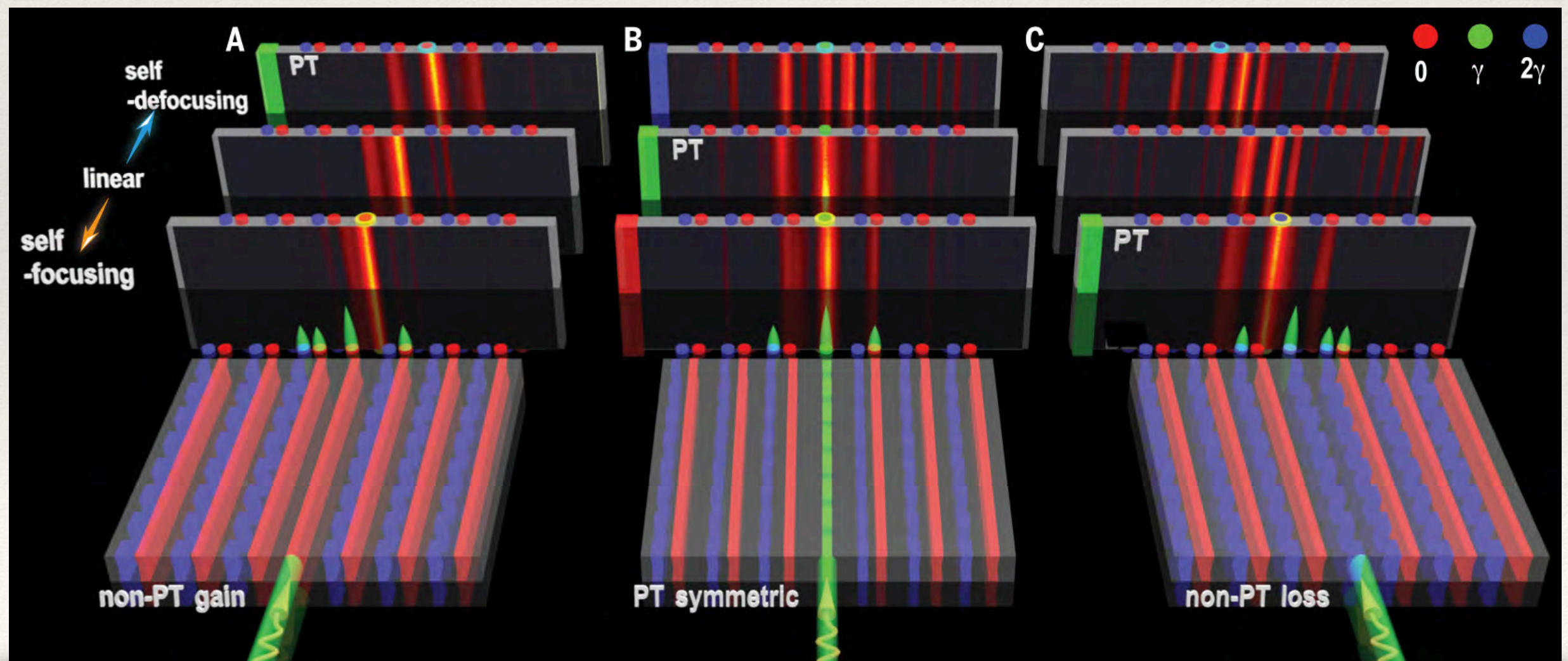
Nonlinearly induced complexity and topology

TOPOLOGICAL OPTICS

Nonlinear tuning of PT symmetry and non-Hermitian topological states

Science 372, 72 (2021)

**Shiqi Xia^{1*}, Dimitrios Kaltsas^{2*}, Daohong Song^{1*}, Ioannis Komis², Jingjun Xu¹, Alexander Szameit³,
Hrvoje Buljan^{1,4†}, Konstantinos G. Makris^{2,5†}, Zhigang Chen^{1,6†}**



Topological Protection against Fluctuations

Robustness versus sensitivity in non-Hermitian topological lattices probed by pseudospectra

I. Komis,^{1,2} D. Kaltsas,¹ S. Xia,³ H. Buljan,^{4,3} Z. Chen,^{3,5} and K. G. Makris^{1,2}

¹*ITCP-Physics Department, University of Crete, Heraklion, 71003, Greece*

²*Institute of Electronic Structure and Laser (IESL) – FORTH, Heraklion, 71110, Greece*

³*The MOE Key Laboratory of Weak-Light Nonlinear Photonics,*

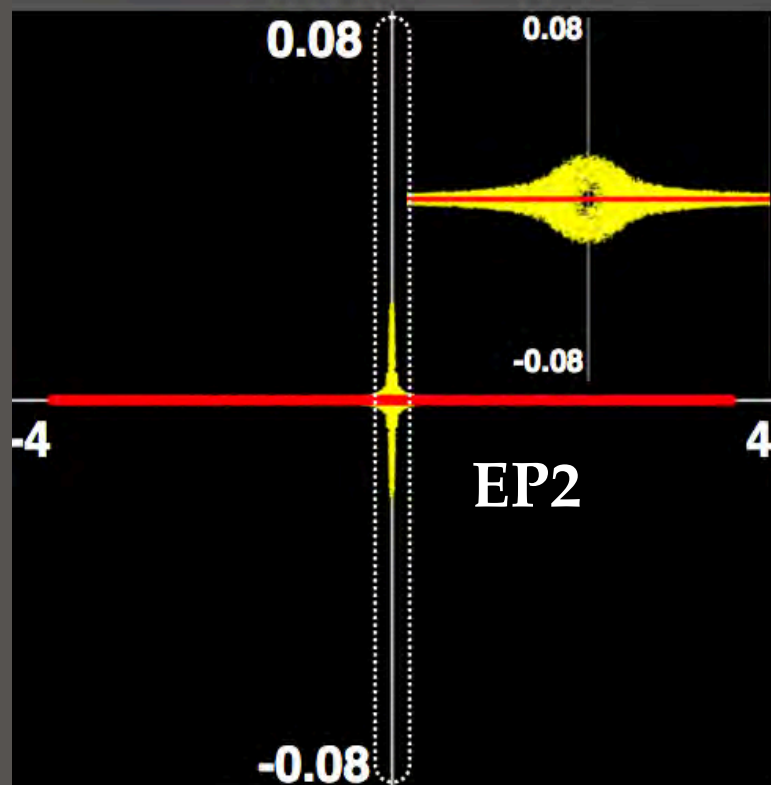
TEDA Applied Physics Institute and School of Physics, Nankai University, Tianjin 300457, China

⁴*Department of Physics, Faculty of Science, University of Zagreb, Bijenička c. 32, 10000 Zagreb, Croatia*

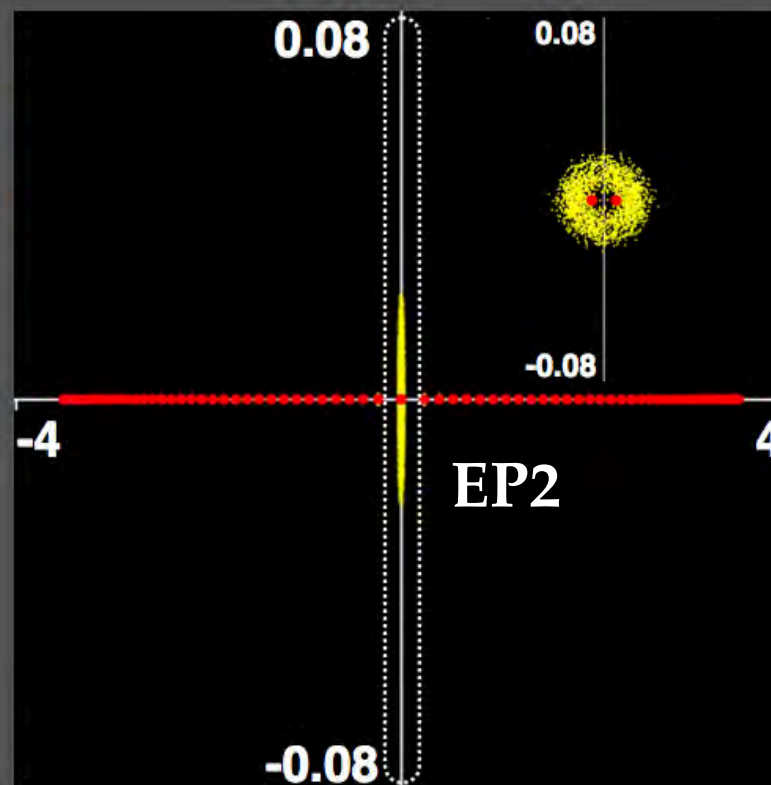
⁵*Department of Physics and Astronomy, San Francisco State University, California 94132, USA*

(Dated: March 8, 2022)

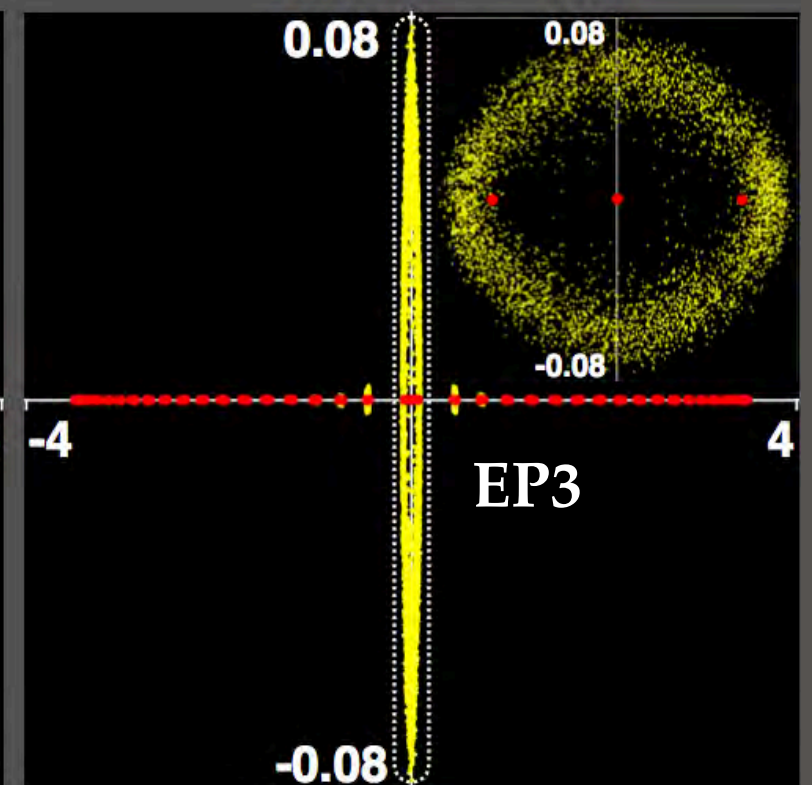
1 Infinite SSH



2 Finite SSH



3 Interface of two SSH

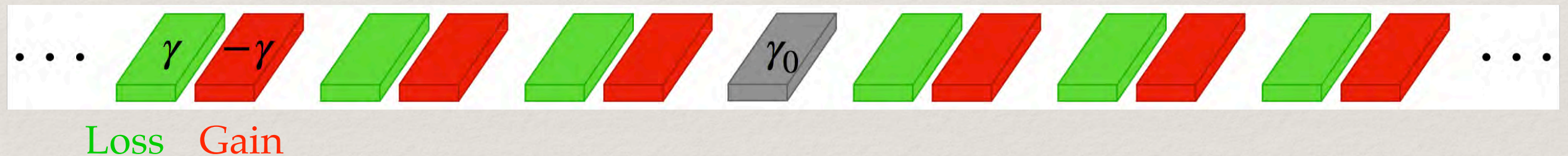


Structured Perturbations

$$\sigma_{\varepsilon}^{str}(H) \equiv \bigcup_{j=1, E\text{-structured}, ||E_j|| < \varepsilon}^s \sigma(H + E_j)$$

Structured Pseudospectrum

NH- SSH interface lattice

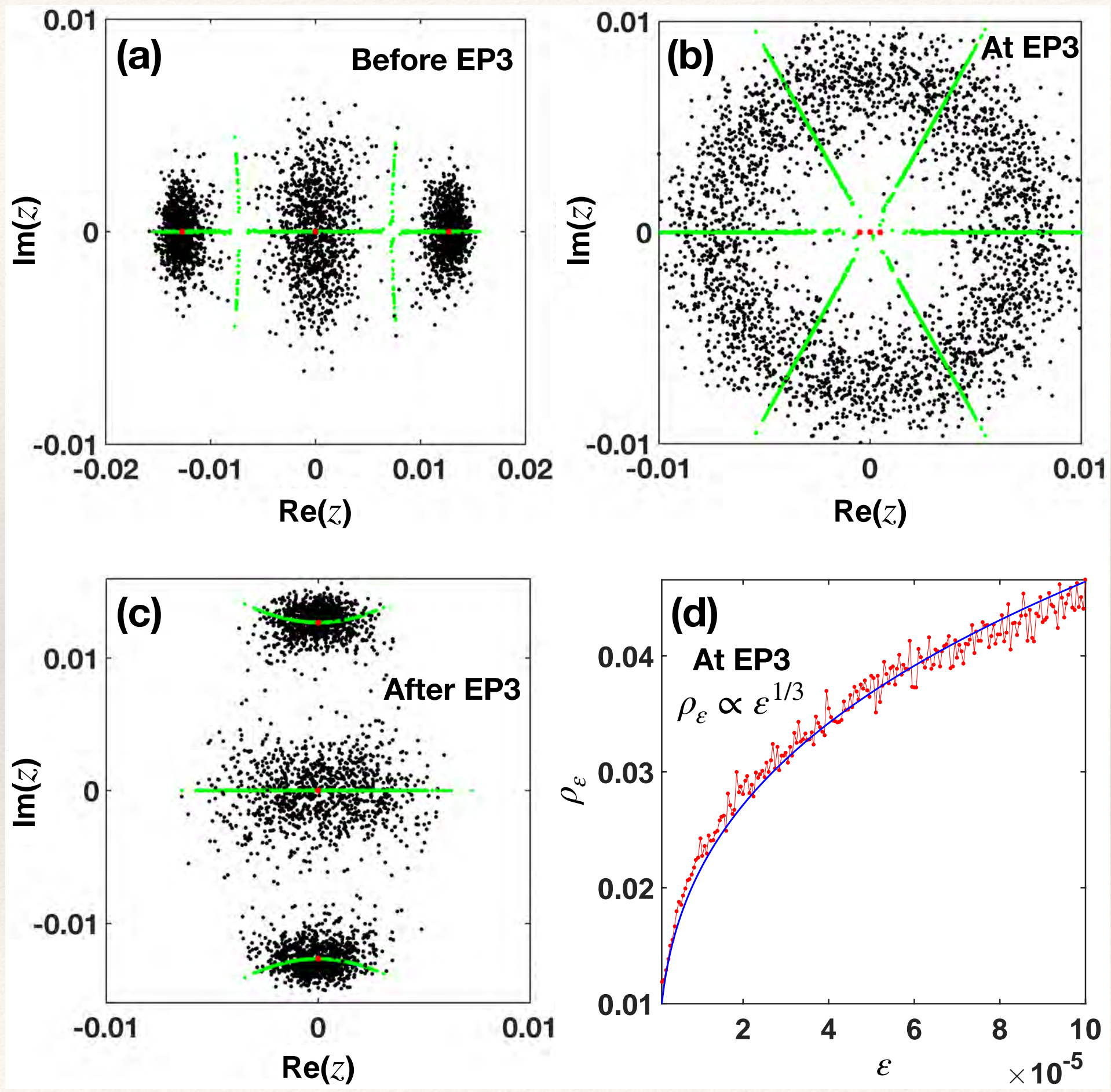


Physical Perturbations

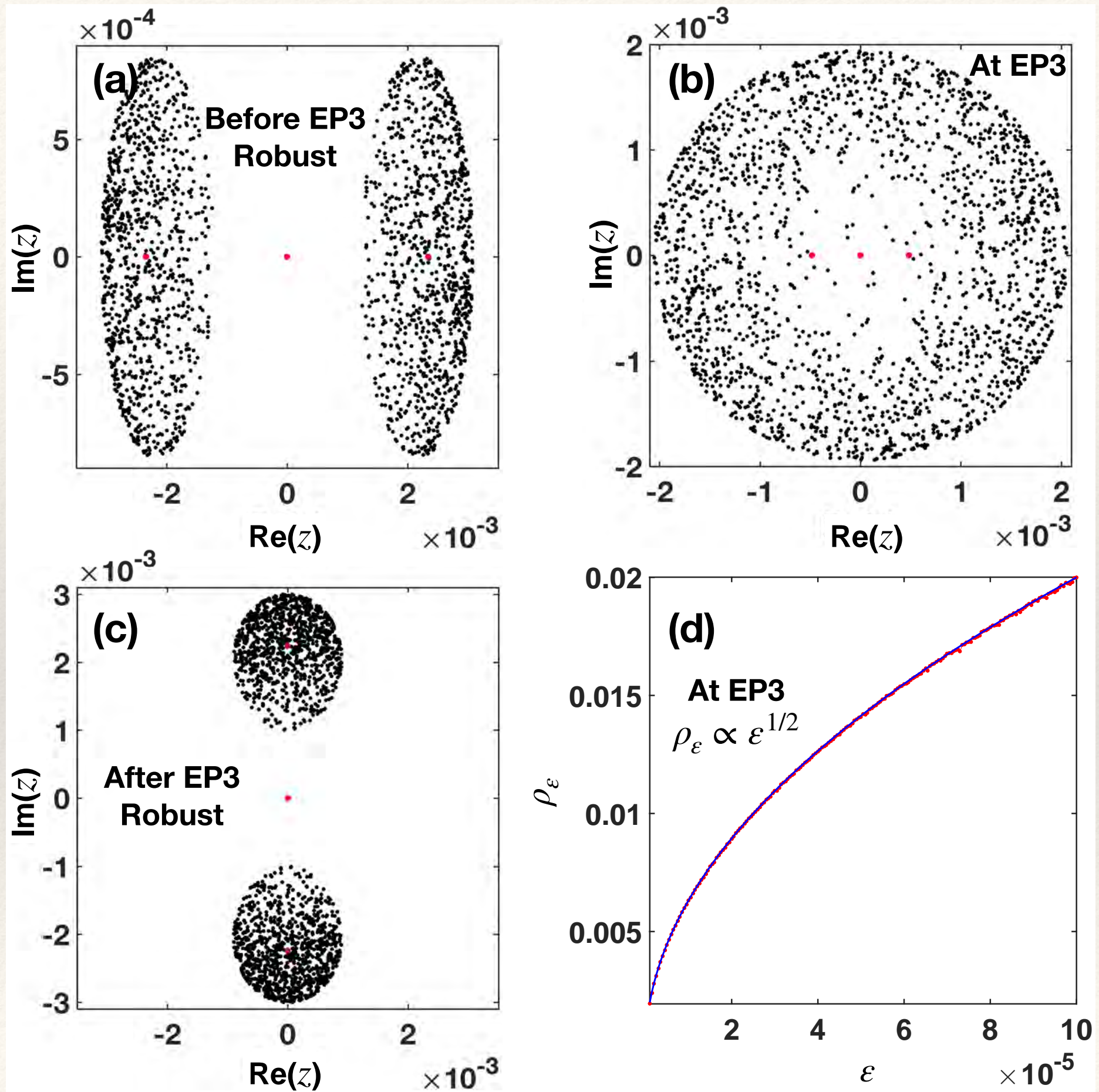
Diagonal perturbations: Change of the potential strength and the gain/loss amplitude

Off-Diagonal perturbations: Change of the coupling coefficient between waveguides

Diagonal Perturbations



Off-Diagonal Perturbations



Acknowledgements

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students



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D. Kaltsas



International Collaborations

Experiment

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China



A. Szameit

Rostock University,
Germany



M. Segev

Solid State
Institute, Israel



M. Rechtsman
Penn State, USA



E. Rivet, H. Lissek, R. Fleury
School of Engineering,
Switzerland



Theory



University of
Zagreb

H. Buljan

Department of Physics, Croatia

D. N. Christodoulides

CREOL, USA



S. Rotter

TU-Wien, Austria



Z. H. Musslimani

Department of
Mathematics, FSU, USA



Epilogue

Exceptional Point in Optics

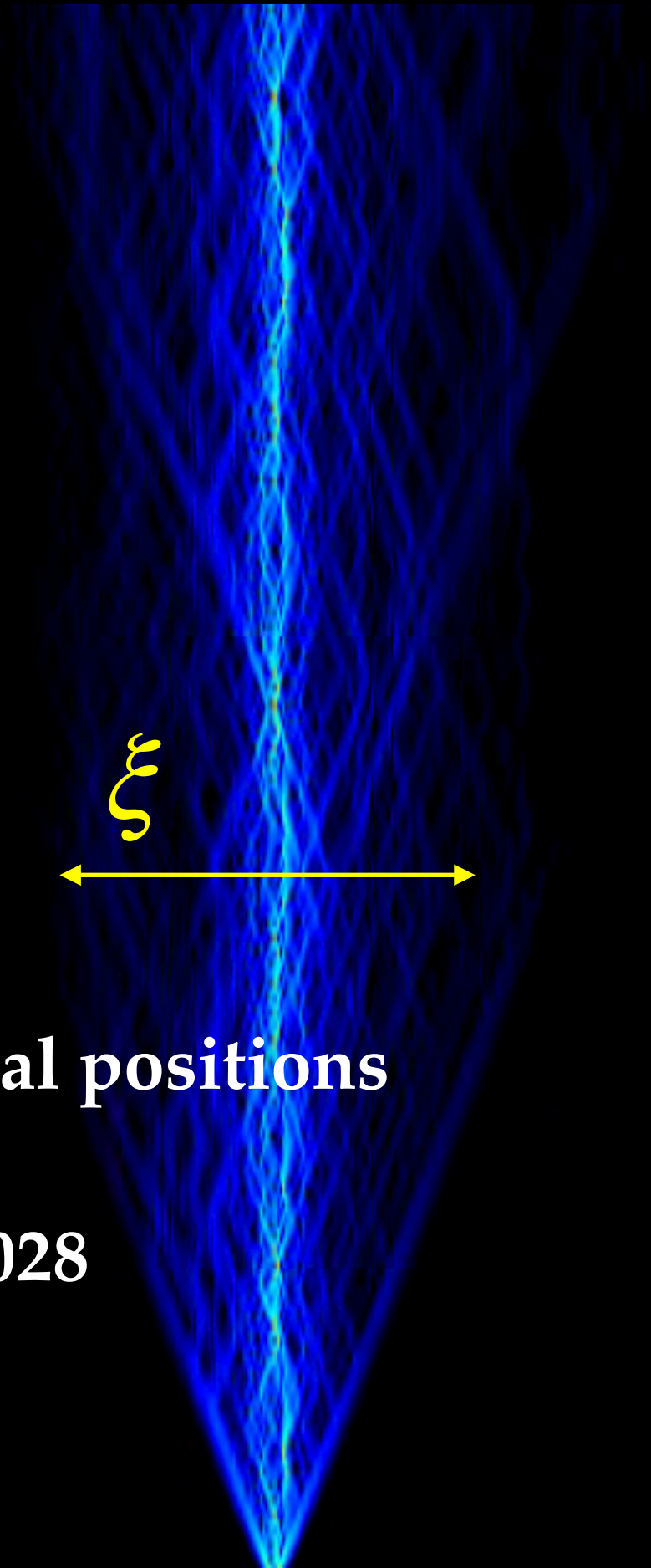
Higher order Exceptional Points in lattices

Sensitivity and Pseudospectra

Hermitian
Localization



No Transport



Open Postdoctoral positions

2023 - 2028

Thank you



Beyond Anderson?