

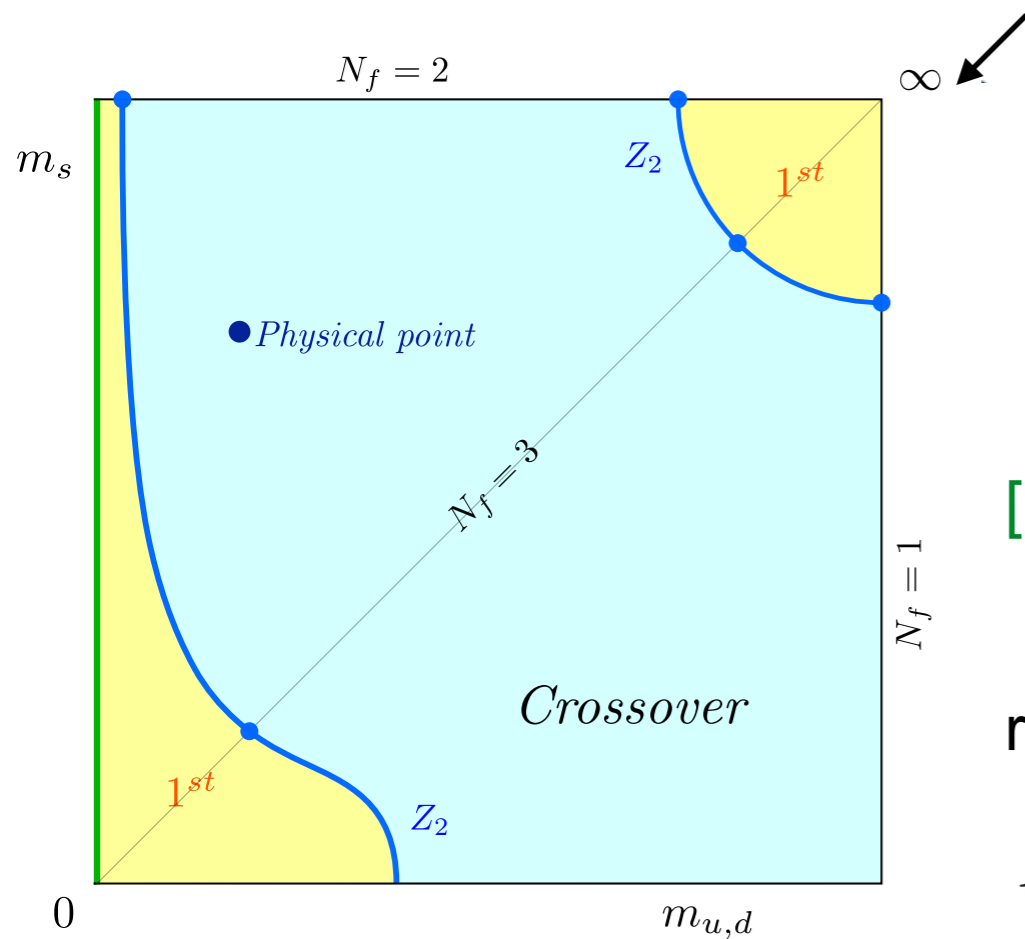
# Surprises on the way to the QCD phase diagram

Owe Philipsen

- Chiral phase transition in massless limit constrains the QCD phase diagram
- ~40 years of “common wisdom” + inconclusive lattice results:  
Resolution with a surprise
- Emergent chiral spin symmetry:  
Additional evidence from screening masses+spectral functions
- A modified phase diagram

# The nature of the QCD thermal transition, $\mu_B = 0$

deconfinement p.t.:  
breaking of global  $Z(3)$  symmetry

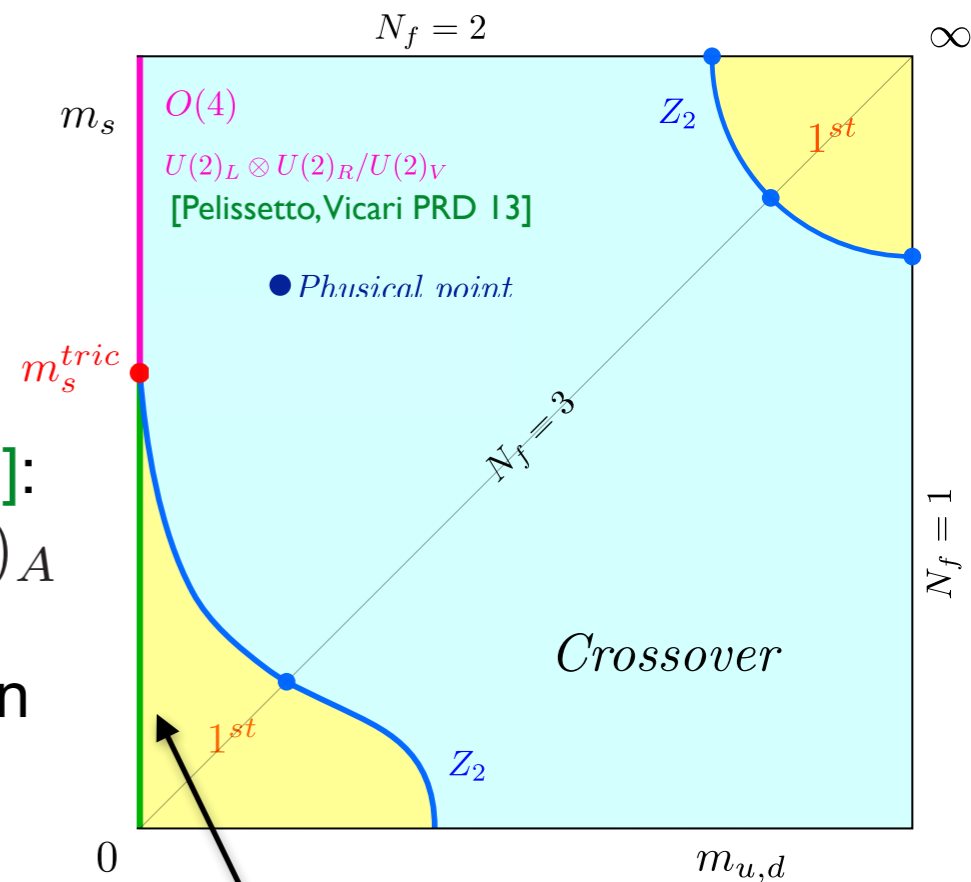


[Pisarski, Wilczek, PRD 84]:  
 $N_f = 2$  depends on  $U(1)_A$

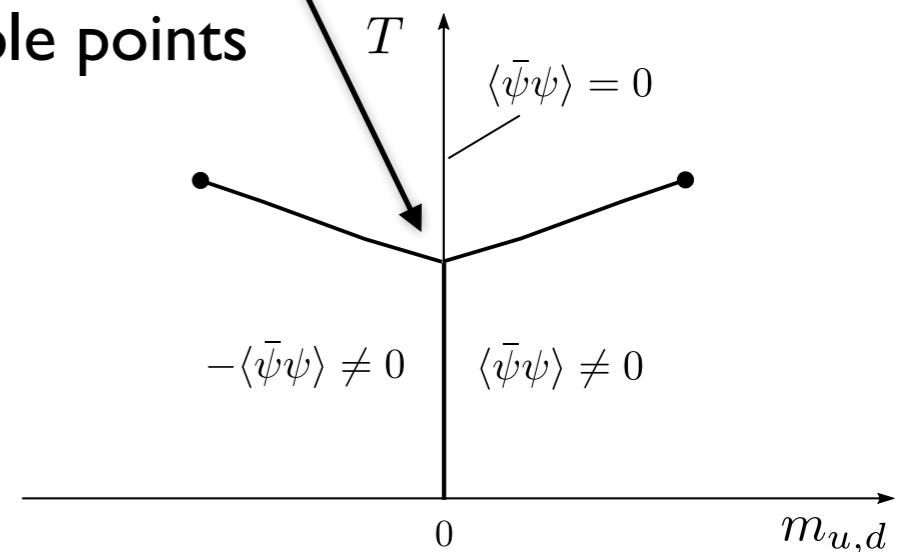
restored

broken

$N_f \geq 3$  1st order



triple points



chiral p.t.

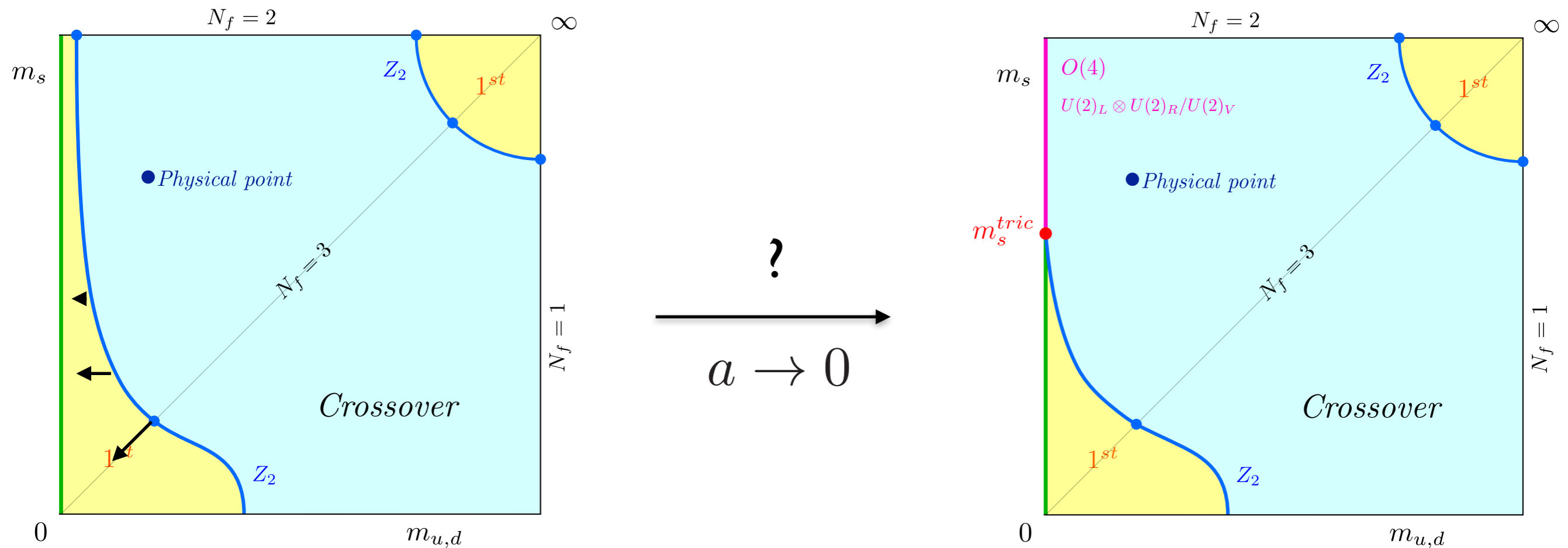
restoration of global symmetry in flavour space

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

↑  
anomalous

# The nature of the QCD chiral transition

...is elusive, massless limit **not simulable!**

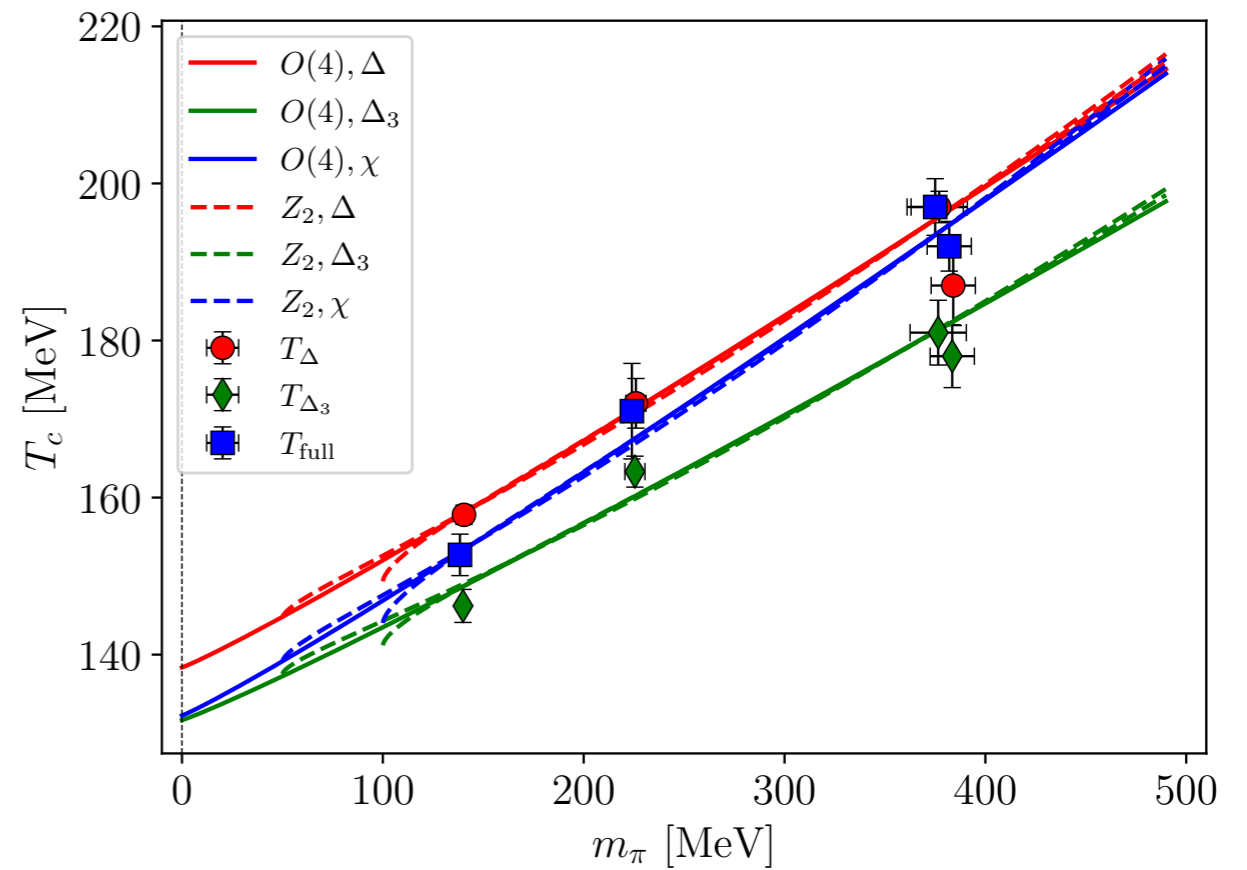
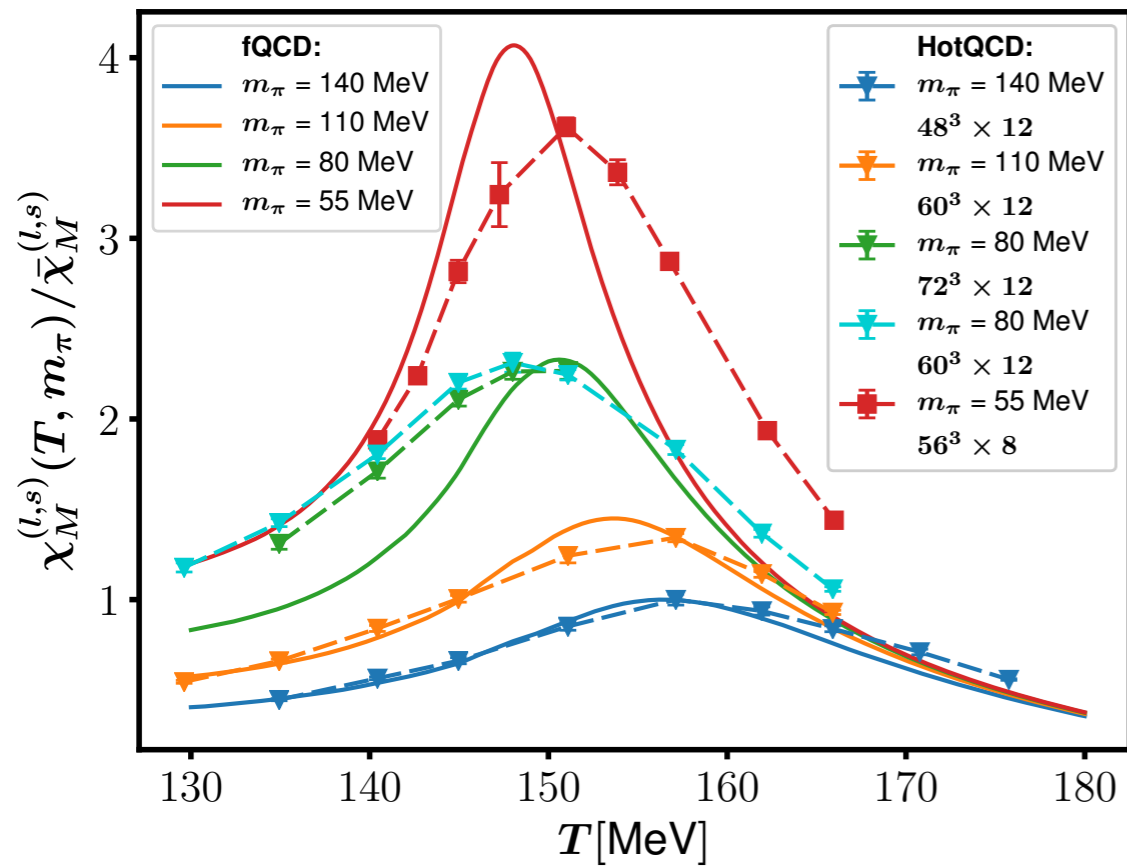


- Coarse lattices or unimproved actions: 1st order for  $N_f = 2, 3$
- 1st order region shrinks rapidly as  $a \rightarrow 0$
- Improved staggered actions: no 1st order region so far, even for  $N_f = 3$   $m_{PS} > 45\text{MeV}$

Details and reference list: [\[O.P., Symmetry 13, 21\]](#)

# From the physical point to the chiral limit

arXiv:2012.06231



[HotQCD, PRL 19] HISQ (staggered)

[Kotov, Lombardo, Trunin, PLB 21] Wilson twisted mass

$$T_c^0 = 132_{-6}^{+3} \text{ MeV}$$

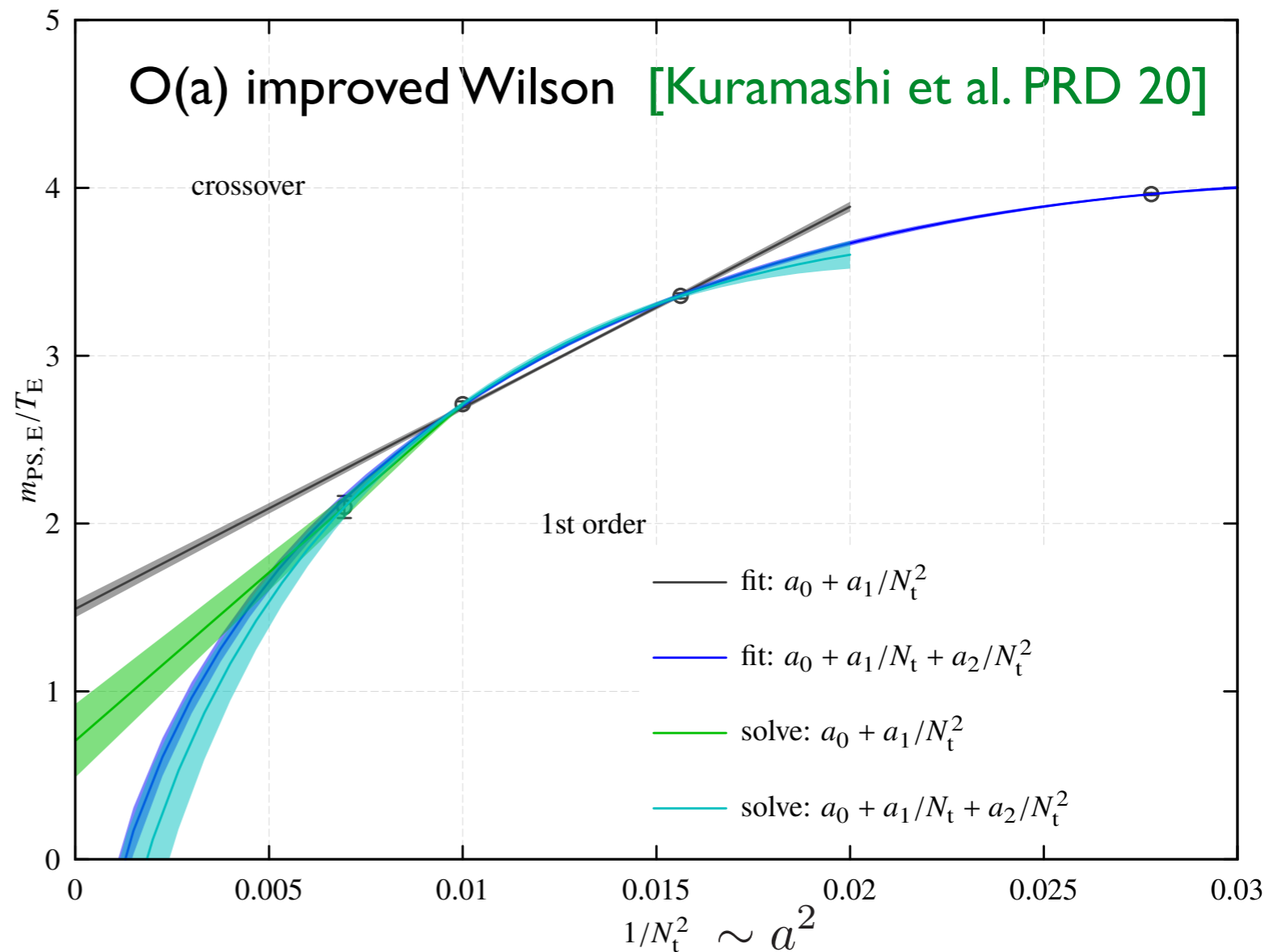
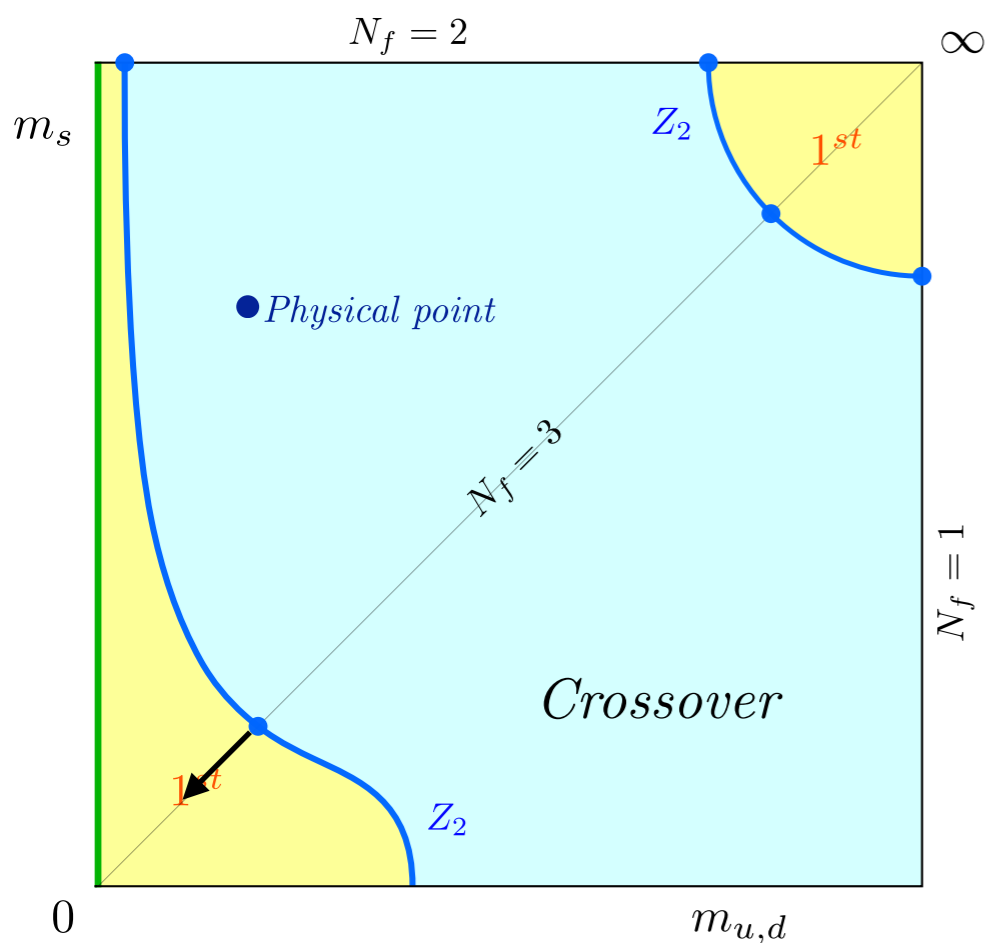
$$T_{pc}(m_l) = T_c^0 + K m_l^{1/\beta\delta}$$

$$T_c^0 = 134_{-4}^{+6} \text{ MeV}$$

- Keep strange quark mass fixed, crossover gets stronger as chiral limit approached
- Cannot distinguish between  $Z(2)$  vs.  $O(4)$  exponents, need exponential accuracy!
- Determination of chiral critical temperature possible, but not the order of the transition
- Comparison with fRG:  $T_c^0 \approx 142 \text{ MeV}$ , "most likely  $O(4)$ " [Braun et al., PRD 20,21]

# The nature of the QCD chiral transition, $N_f=3$

...has enormously large cut-off effects!

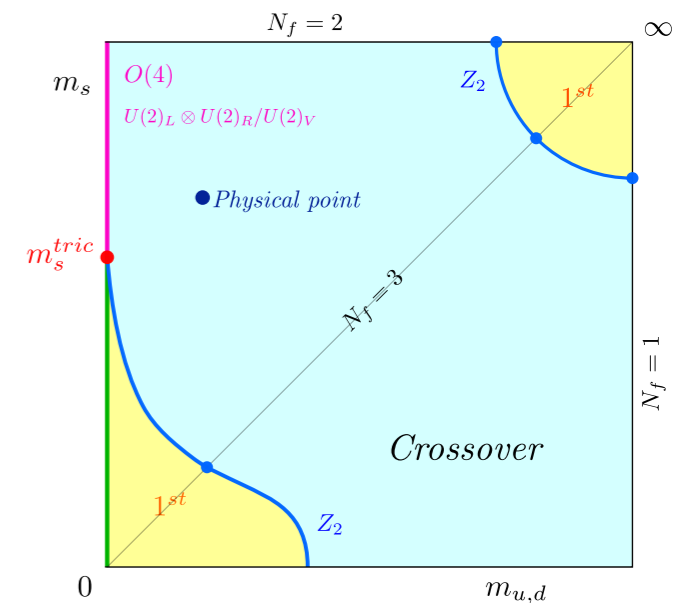
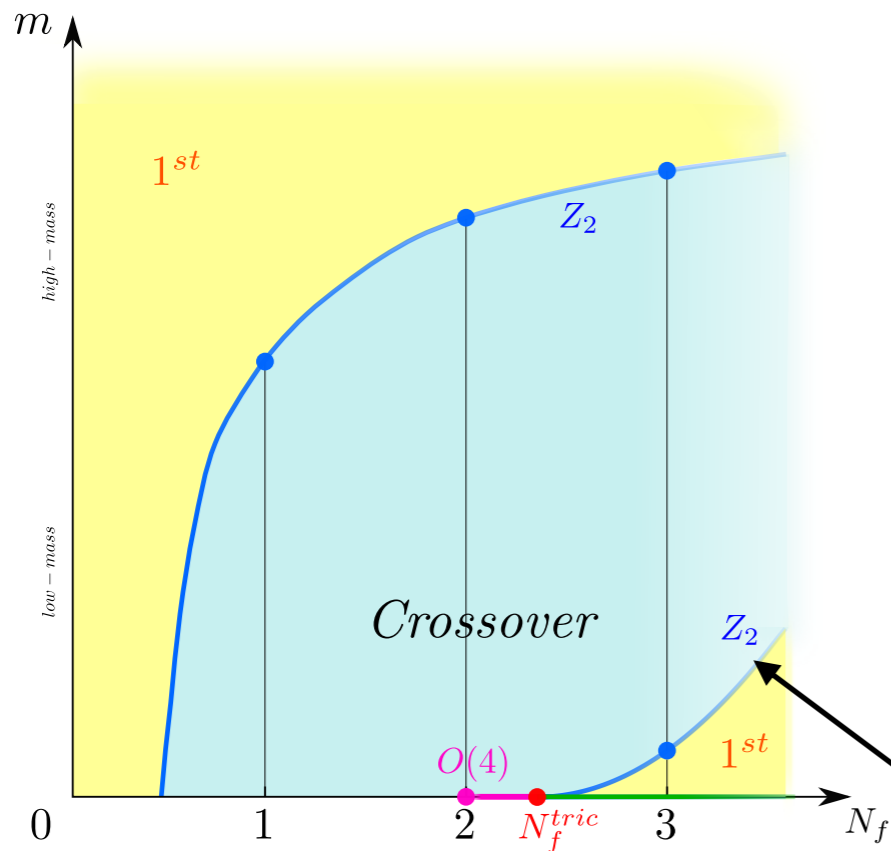


O(a)-improved Wilson:

1st order region shrinks for  $a \rightarrow 0$

$$m_\pi^c \leq 110 \text{ MeV} \quad N_\tau = 4, 6, 8, 10, 12$$

# Different view point: mass degenerate quarks

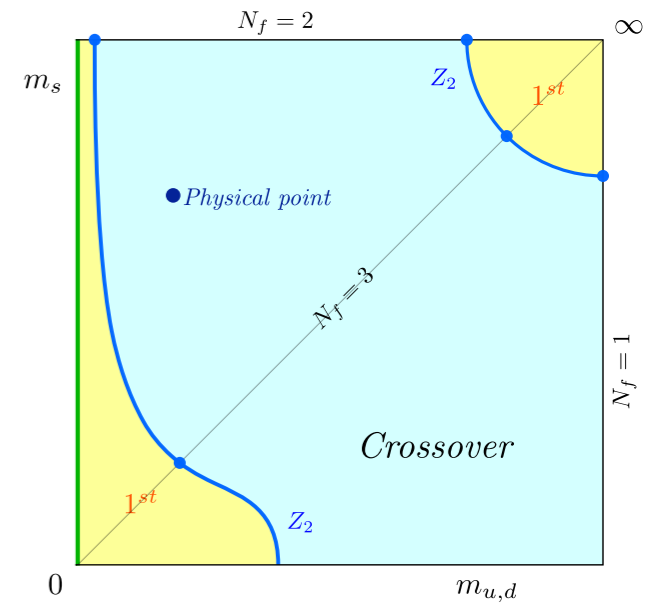
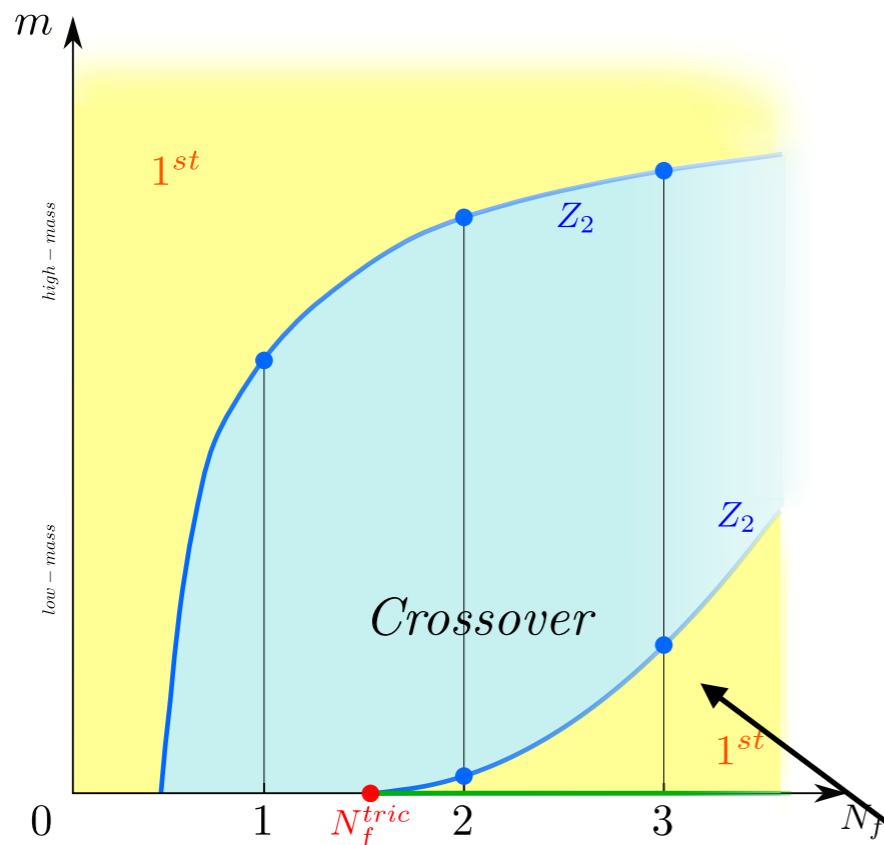


$$Z(N_f, g, m) = \int \mathcal{D}A_\mu (\det M[A_\mu, m])^{N_f} e^{-\mathcal{S}_{\text{YM}}[A_\mu]}$$

$$N_f^c(am) = N_f^{\text{tric}} + \mathcal{B}_1 \cdot (am)^{2/5} + \mathcal{O}((am)^{4/5})$$

- Consider analytic continuation to continuous  $N_f$
- Tricritical point **guaranteed** to exist if there is 1st order at any  $N_f$
- Known exponents for critical line entering tric. point!
- Continuation to  $a \neq 0$ :  $Z(2)$  surface ends in tricritical line

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# Methodology to determine order of transition

Finite size scaling of generalised cumulants

$$B_n = \frac{\langle (\bar{\psi}\psi - \langle \bar{\psi}\psi \rangle)^n \rangle}{\langle (\bar{\psi}\psi - \langle \bar{\psi}\psi \rangle)^2 \rangle^{n/2}}$$

Standard staggered fermions, bare parameters:

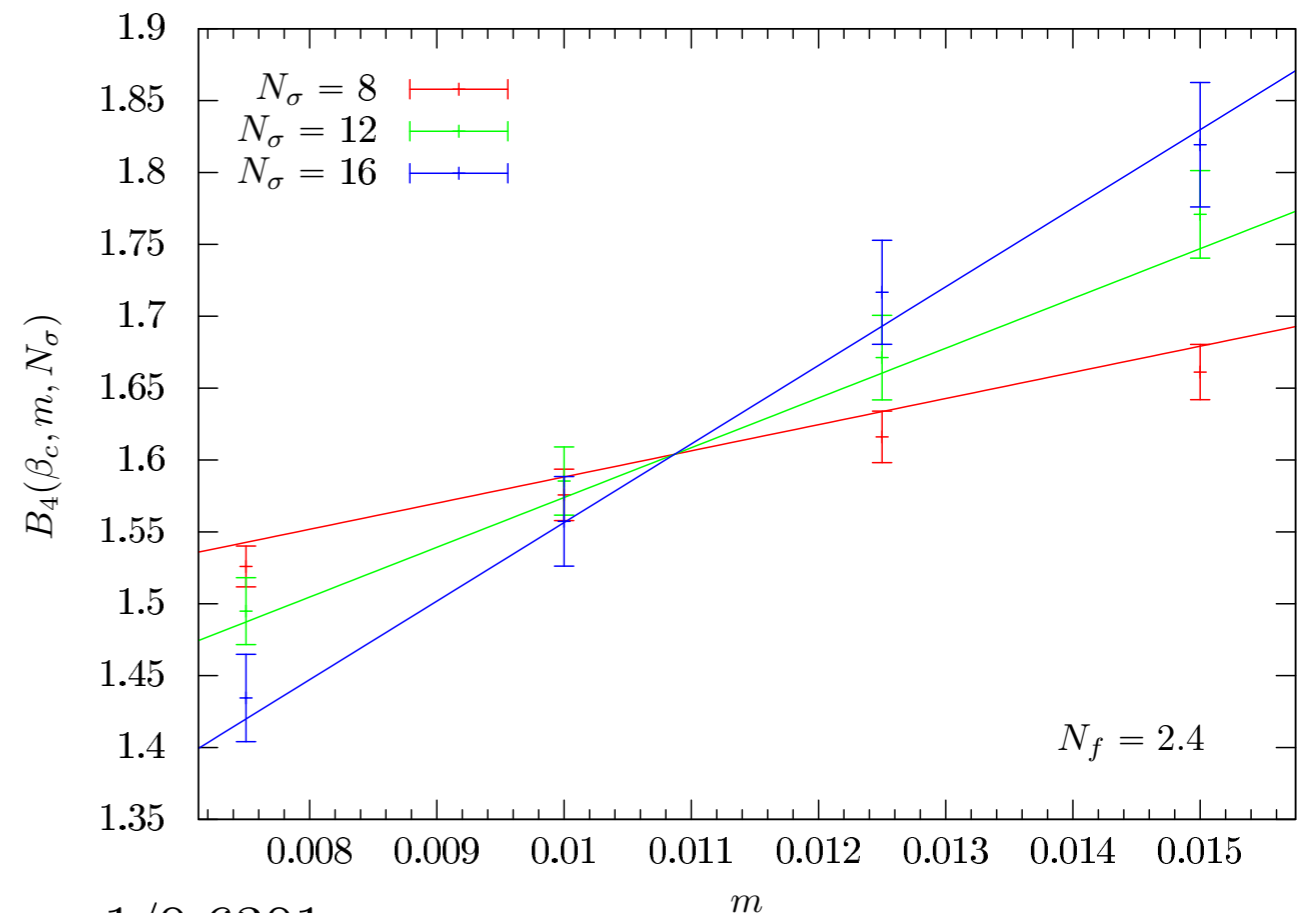
$$\beta, am, N_f, N_\tau$$

(Pseudo-critical) phase boundary:  $B_3 = 0$

3d manifold

Second-order 3d Ising:

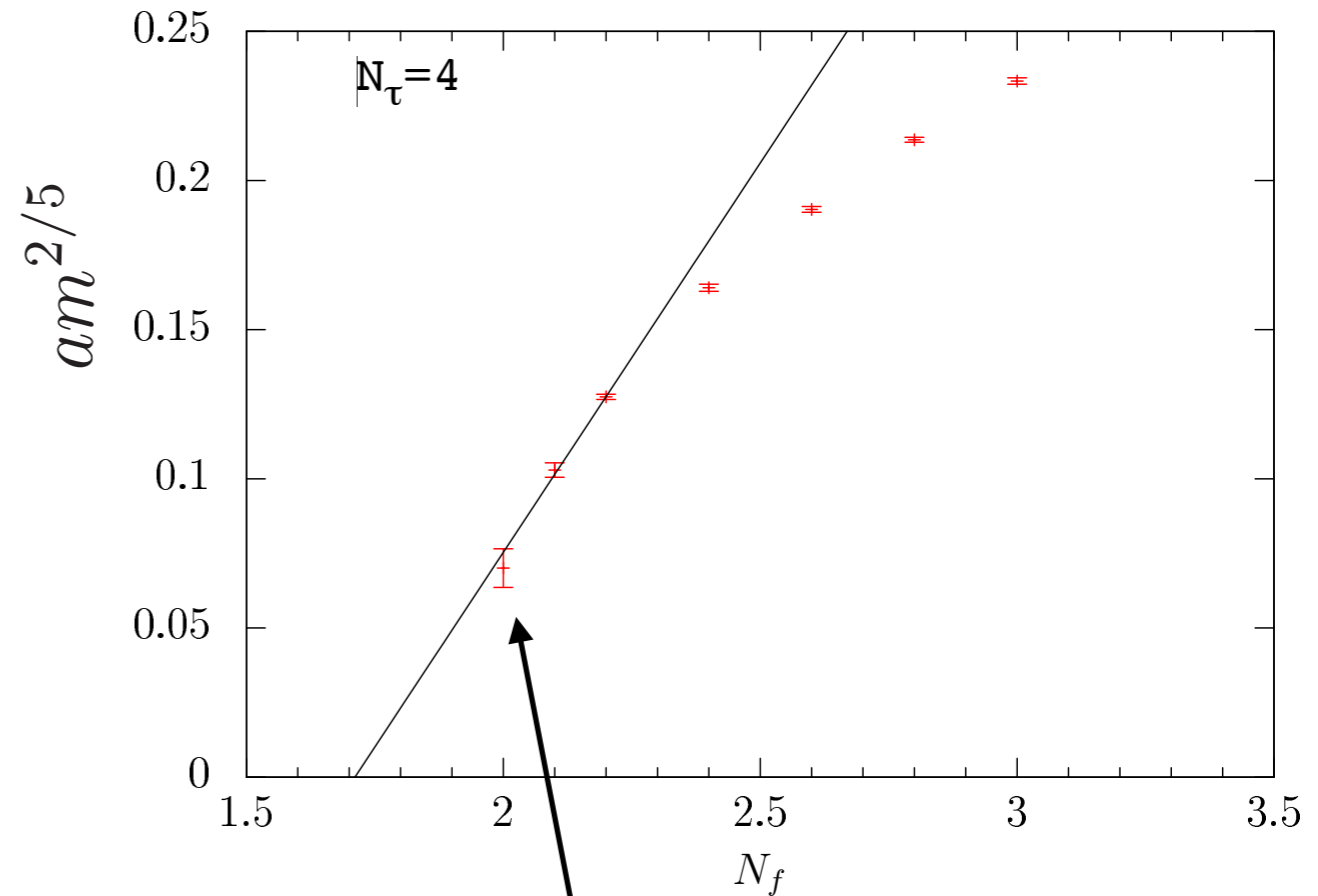
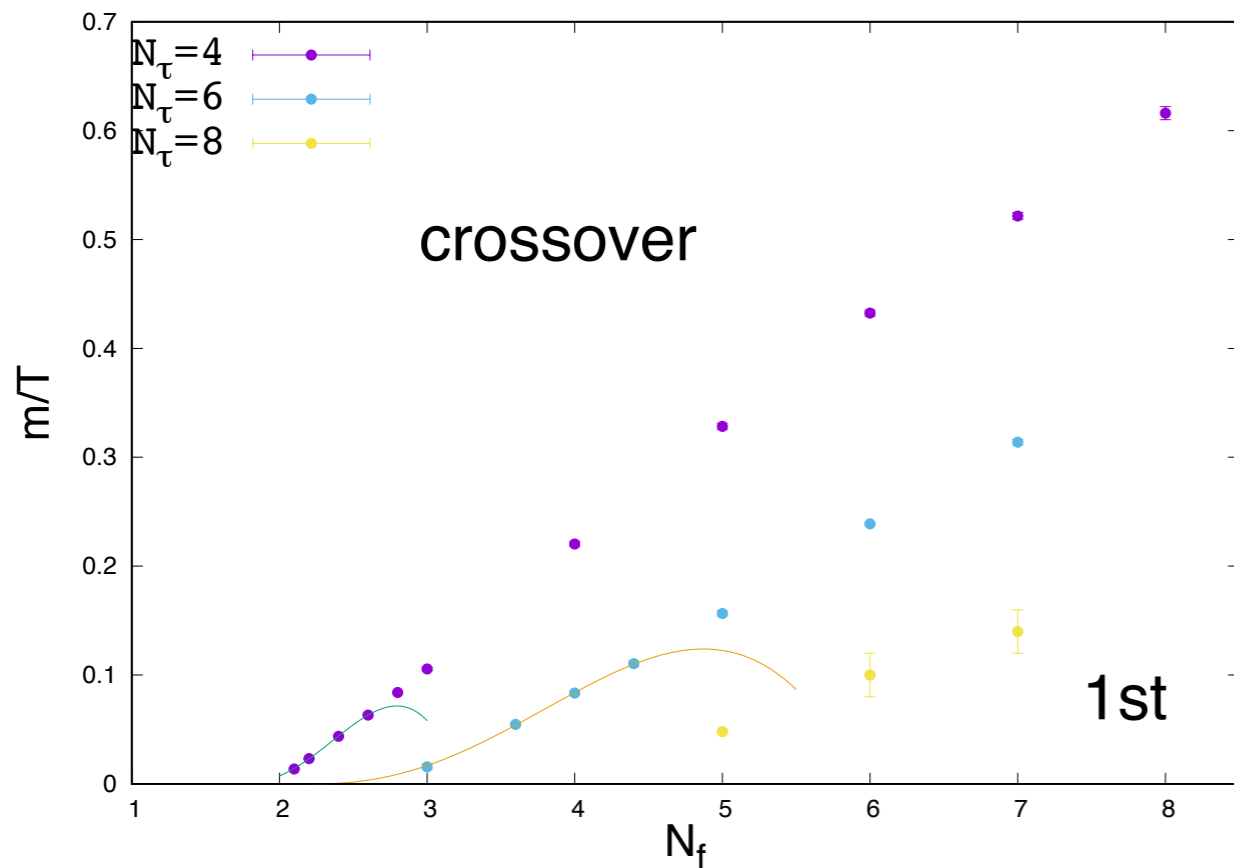
2d chiral critical surface separates 1st order from crossover



$$B_4(\beta_c, am, N_\sigma) \approx 1.604 + c(am - am_c) N_\sigma^{1/0.6301}$$

# Bare parameter space of unimproved staggered LQCD

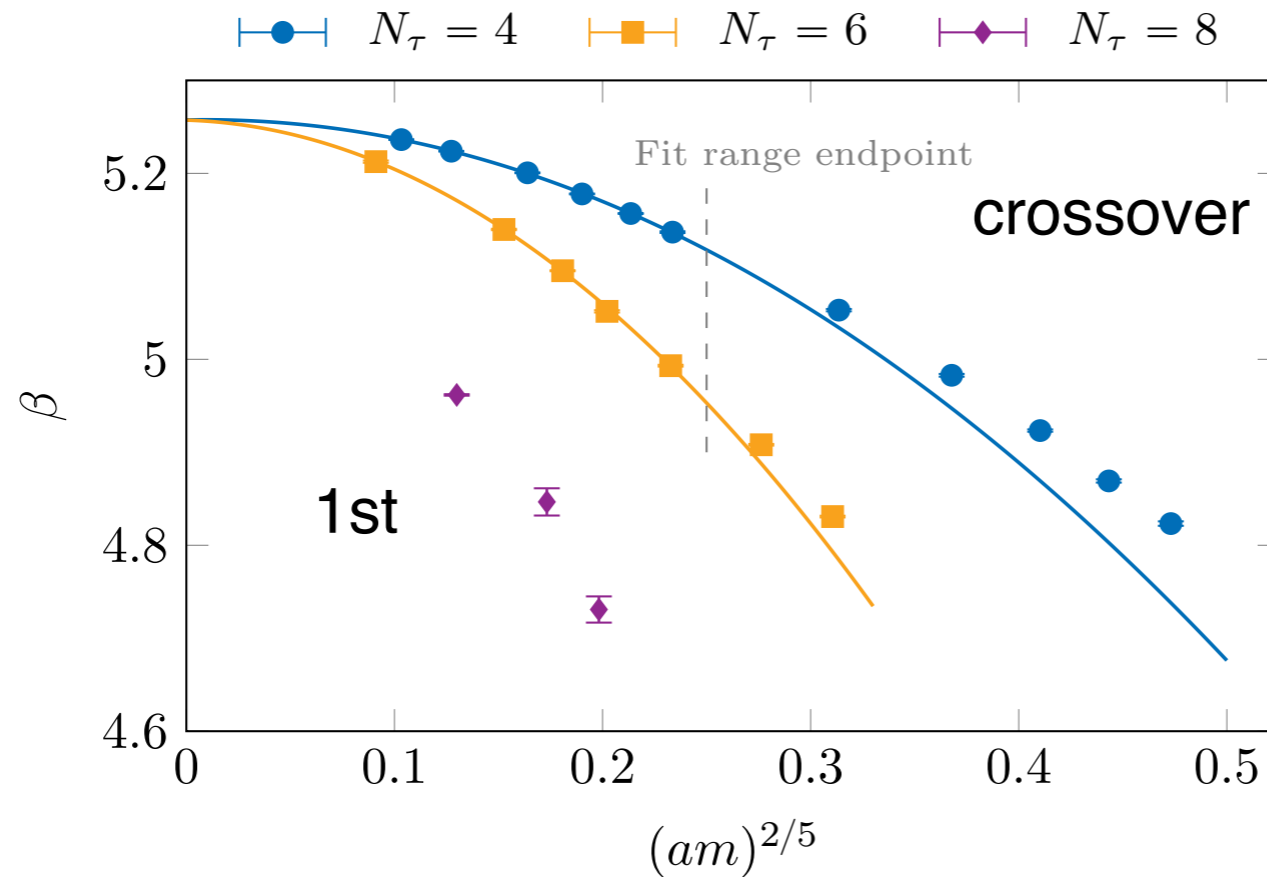
[Cuteri, O.P., Sciarra 21] ~120 M Monte Carlo trajectories with light fermions, aspect ratios 3,4,5



- Tricritical scaling observed in different variable pairings
- Consistent with tric. scaling from finite imaginary  $\mu$  [Bonati et al. PRD 14]
- Old question:  $m_c/T = 0$  or  $\neq 0$  ? Answered for  $N_f = 2$
- New question: will  $N_f^{\text{tric}}$  slide beyond  $N_f = 3$  ?

# Bare parameter space of unimproved staggered LQCD

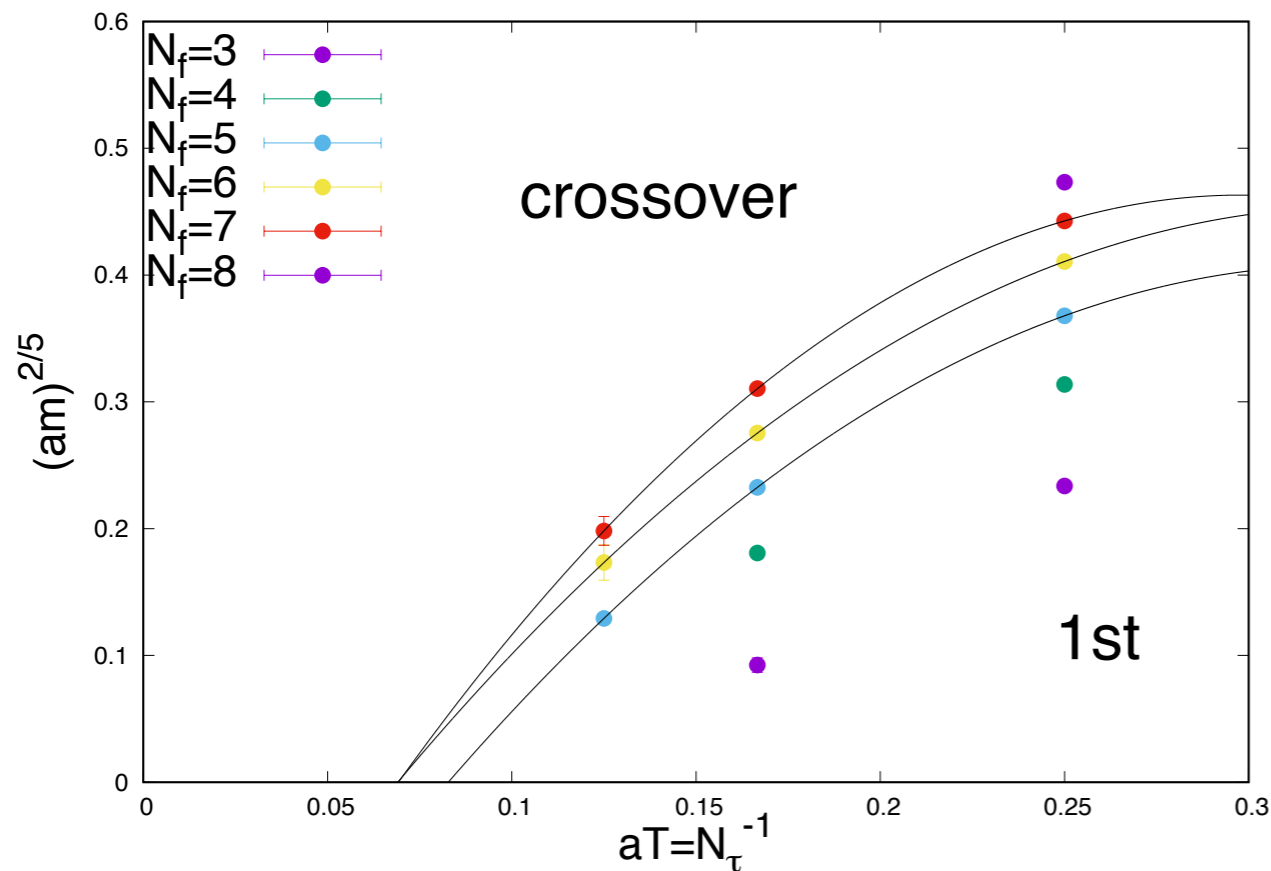
[Cuteri, O.P., Sciarra JHEP 21]



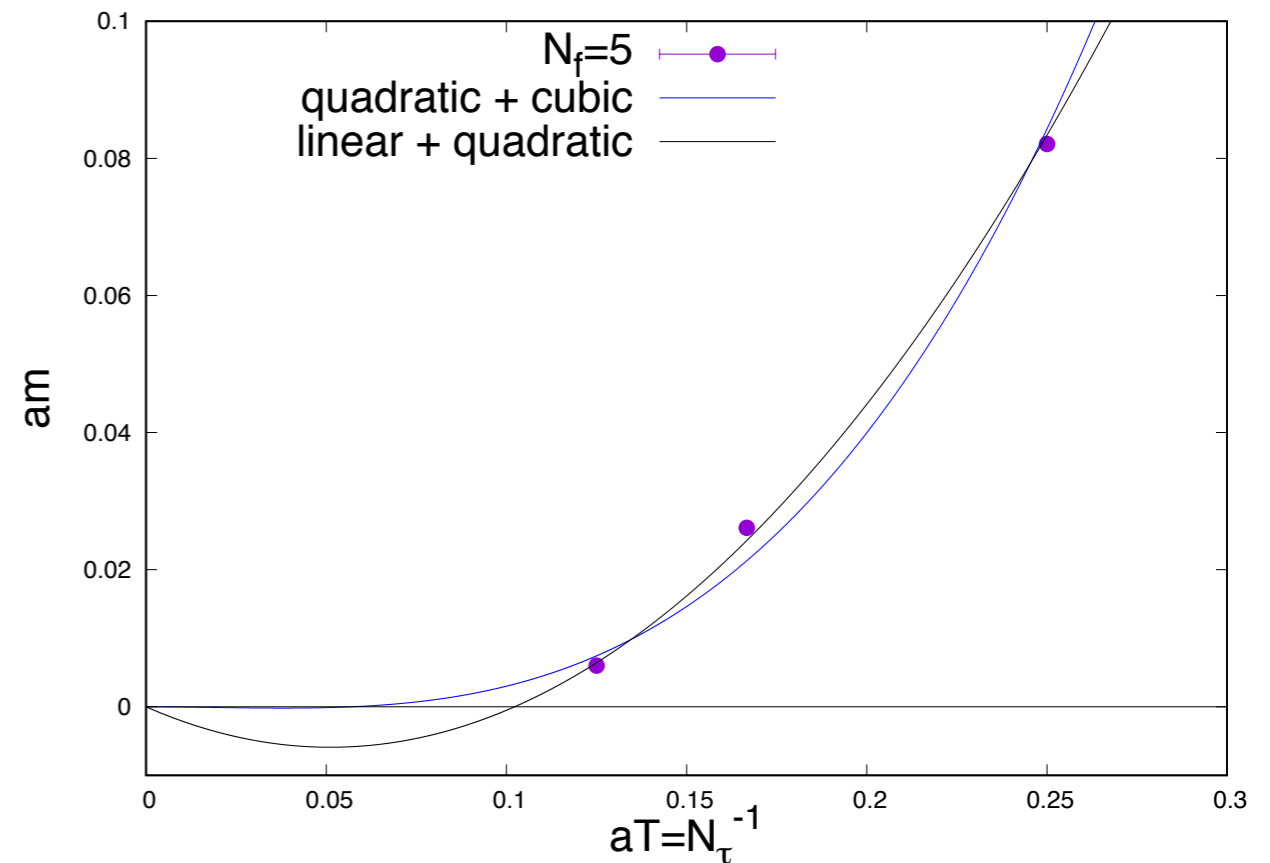
- Tricritical scaling observed in lattice bare parameter space
- Fixed  $N_\tau$ , tricritical extrapolation always possible

# Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra JHEP 21]



1st order scenario does not fit!

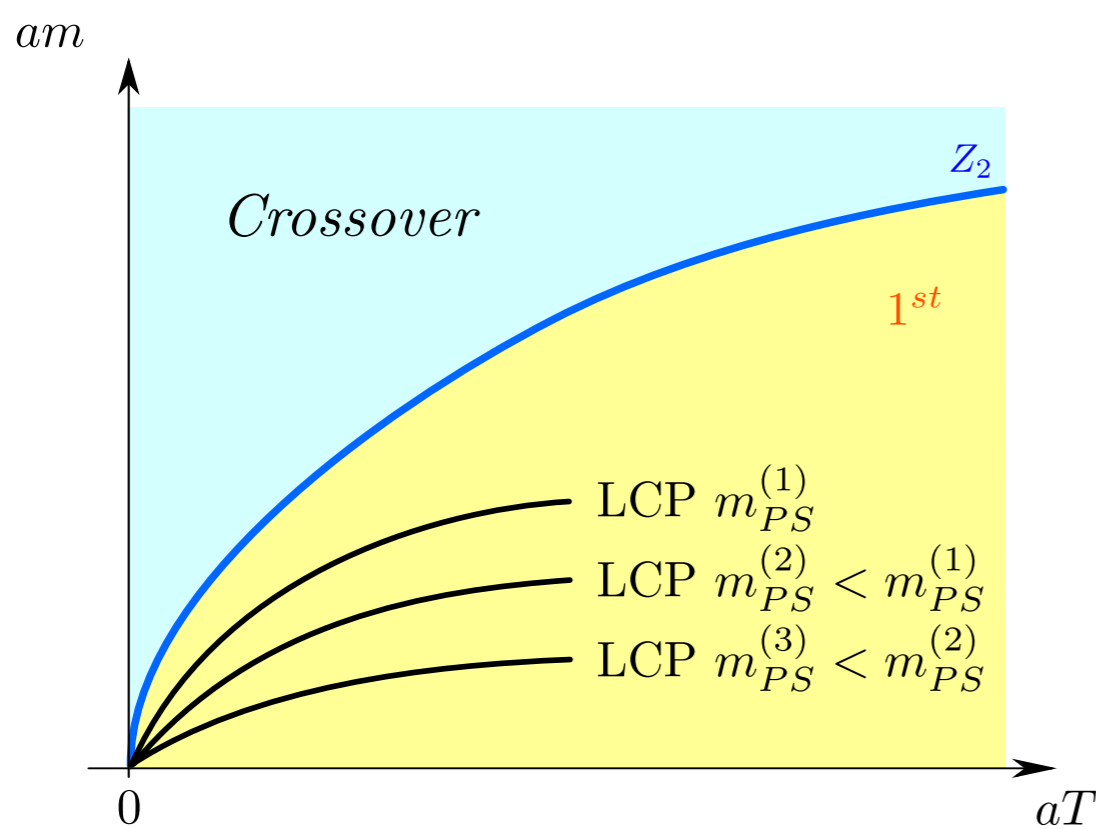


- Tricritical scaling observed also in plane of mass vs. lattice spacing
- Allows extrapolation to lattice chiral limit, tricritical points  $N_\tau^{\text{tric}}(N_f)$
- 1st order scenario:  $m_c(a) = m_c(0) + c_1(aT) + c_2(aT)^2 + \dots$

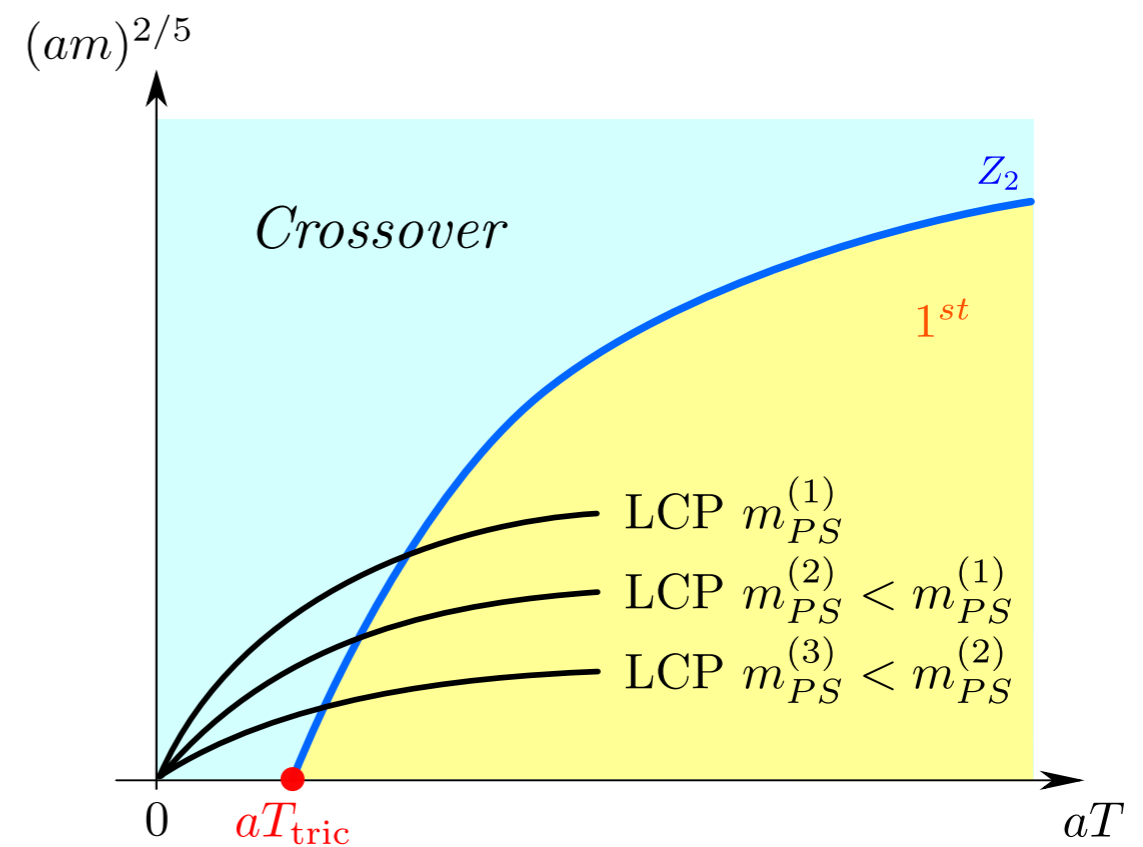
Incompatible with data!  $\chi_{\text{dof}}^2 > 10$

# Implications for the continuum

- Finite  $N_{\tau}^{\text{tric}}(N_f)$  implies that 1st order transition is not connected to continuum
- Approaching continuum first, then chiral limit:  
**Continuum chiral phase transition second-order!**



1st order scenario



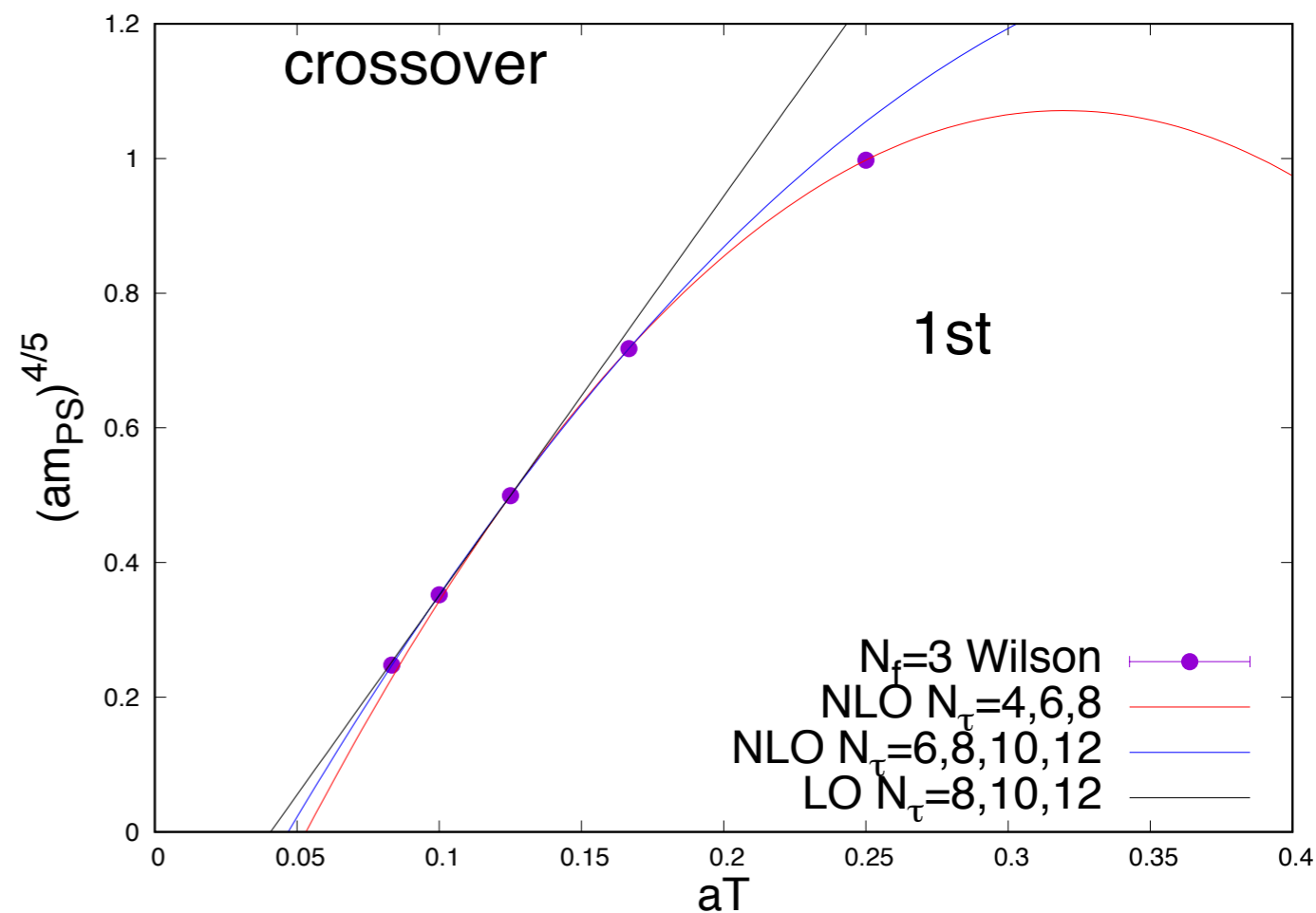
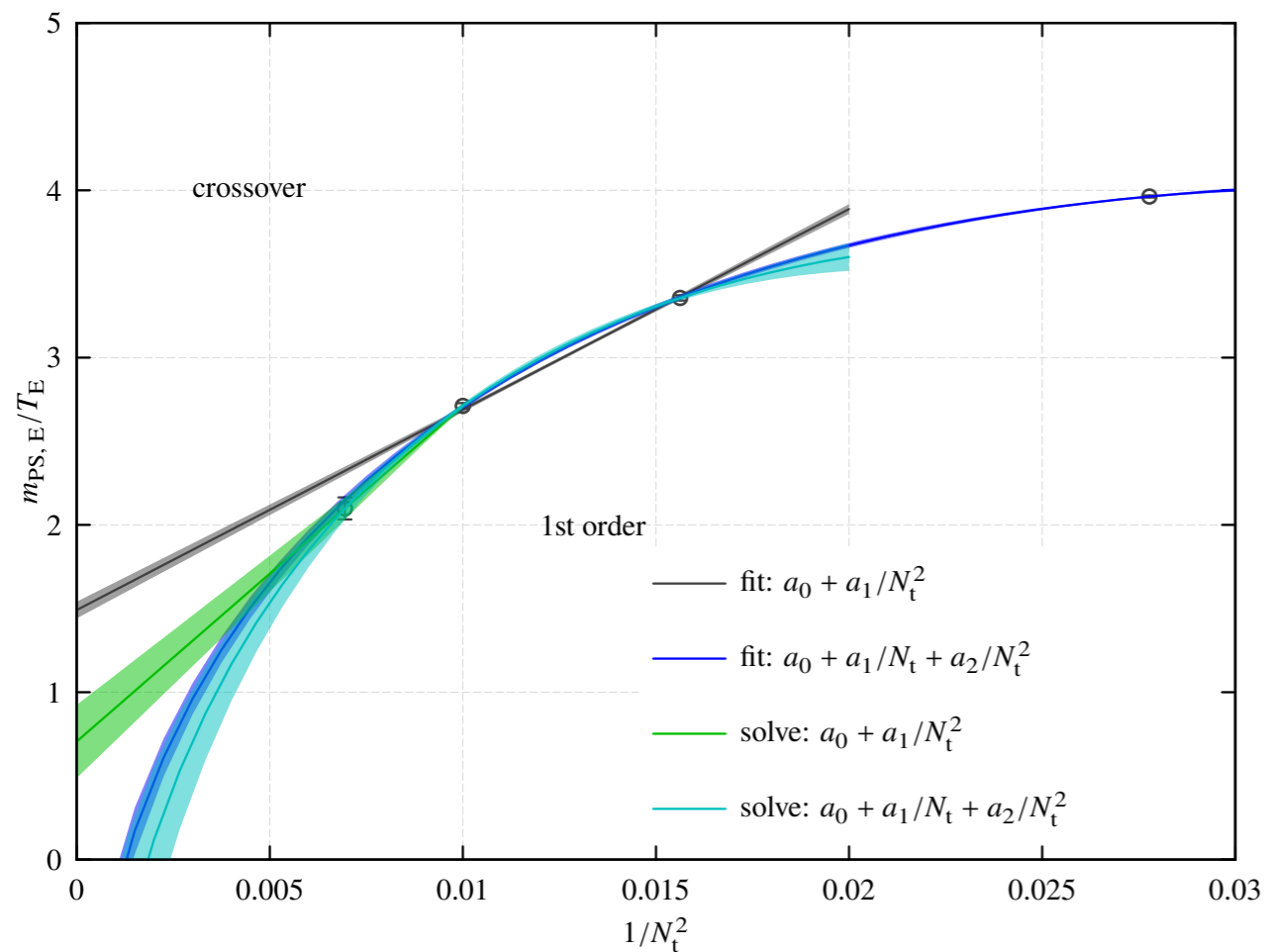
2nd order scenario

# Nf=3 O(a)-improved Wilson fermions

[Kuramashi et al. PRD 20]

$$m_\pi^c \leq 110 \text{ MeV} \quad N_\tau = 4, 6, 8, 10, 12$$

Re-analysis using:  $am_{PS}^2 \propto am_q$

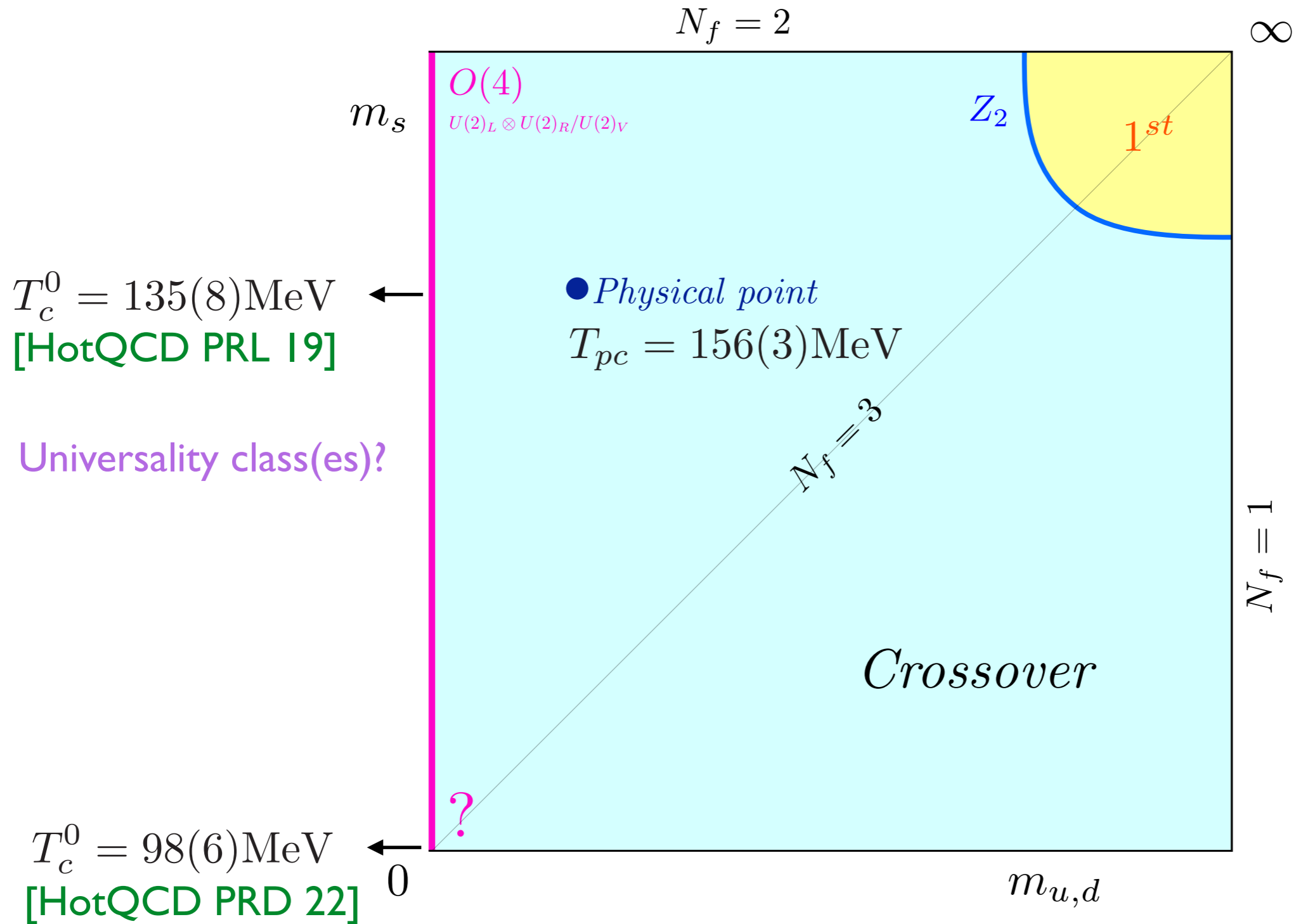


Tricritical scaling! [Cuteri, O.P., Sciarra, JHEP 21]

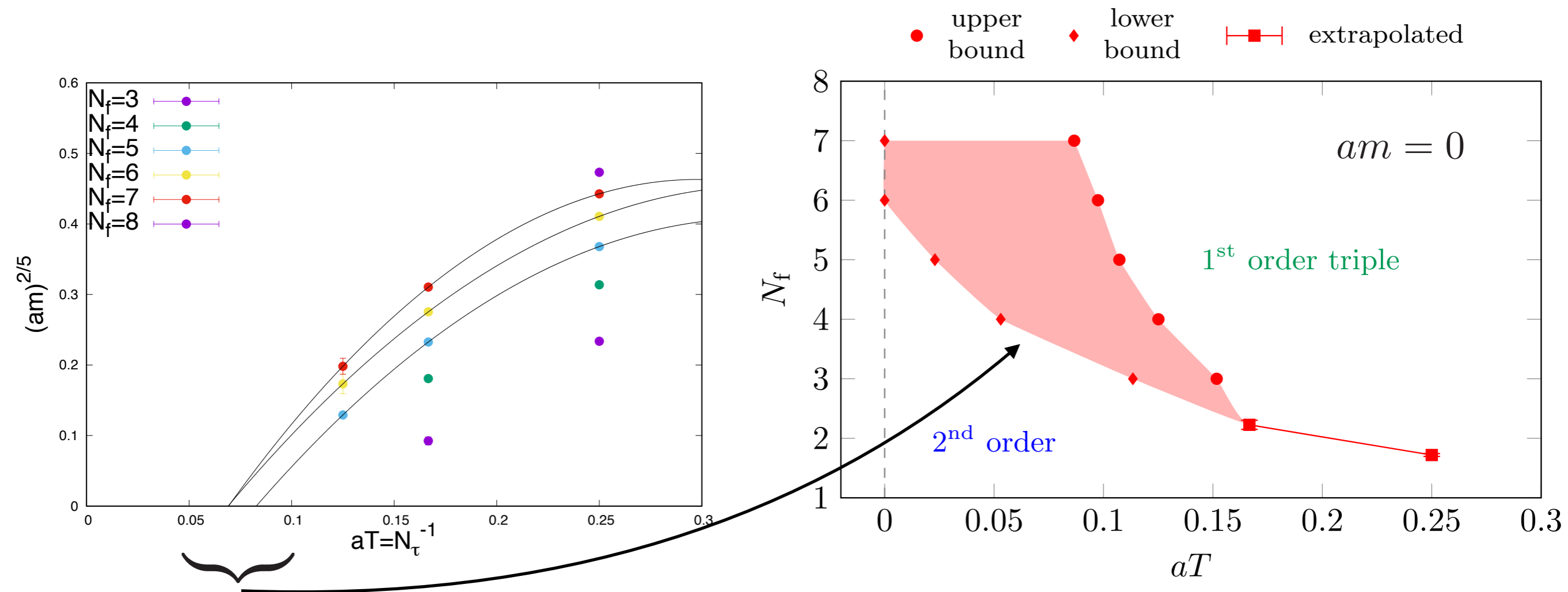
Nf=3 consistent with staggered, 2nd order in chiral continuum limit!

# The Columbia plot in the continuum

[Cuteri, O.P., Sciarra JHEP 21]



# Staggered: tricritical points as function of $N_f$

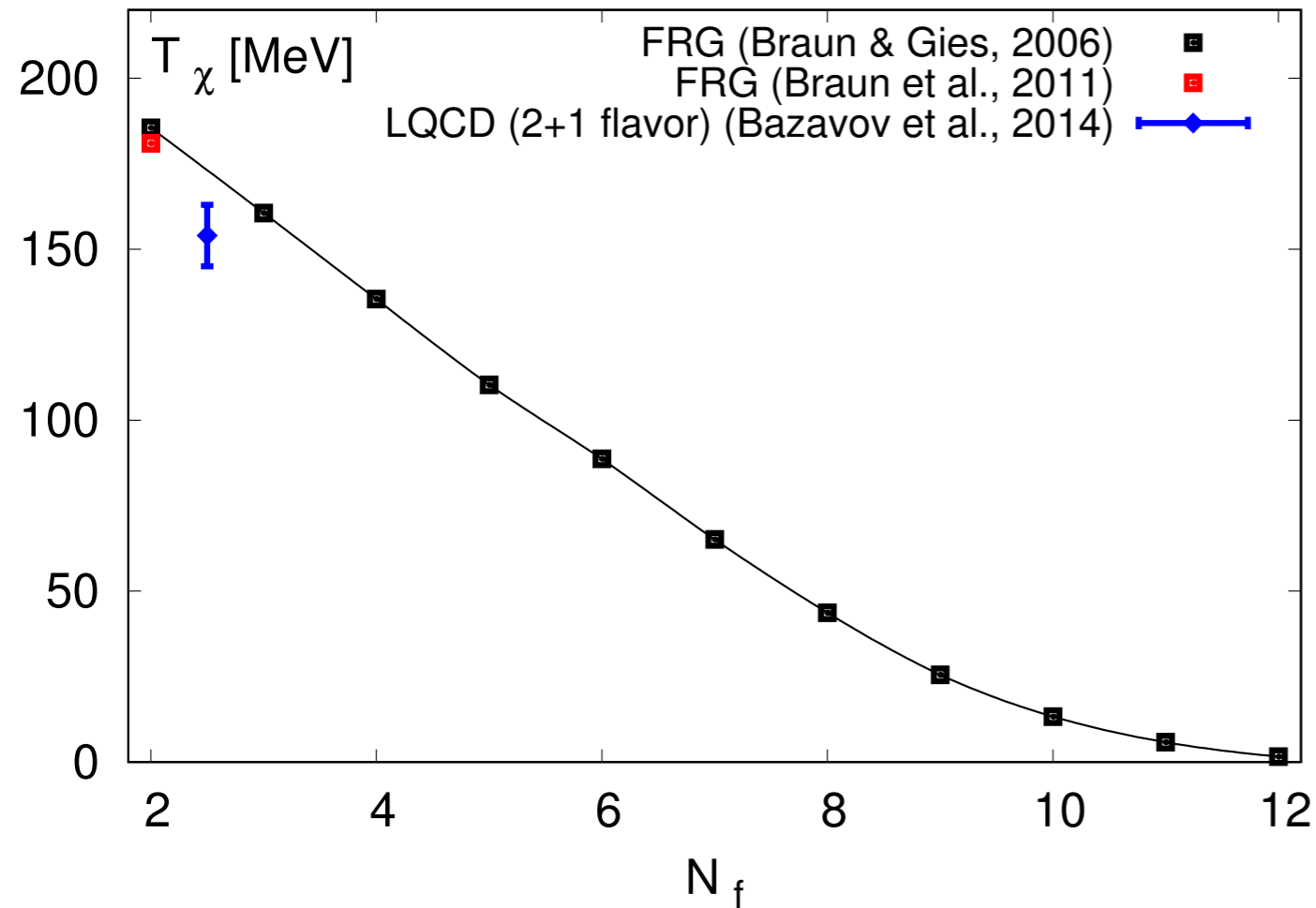


●  $N_\tau^{\text{tric}}(N_f)$  increasing function

● Tricritical line in the plane of the lattice chiral limit, separates 1st from 2nd

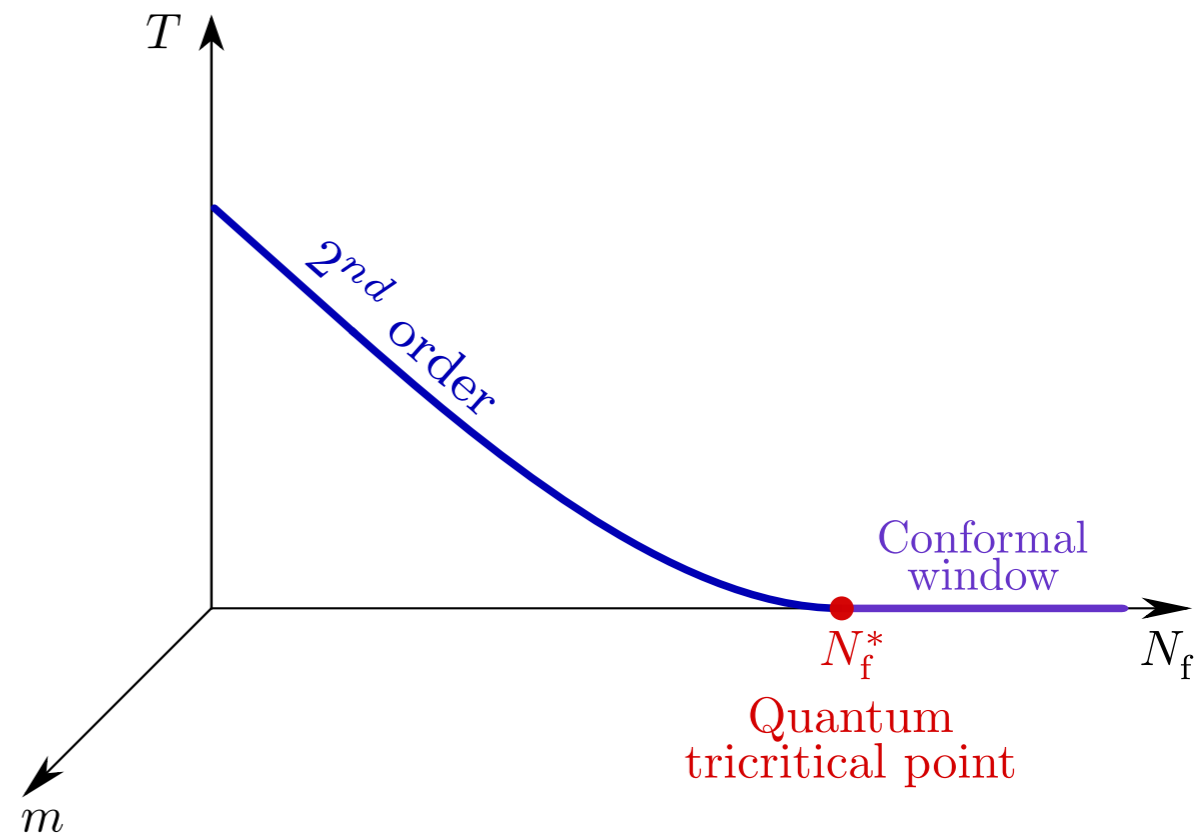
# The chiral phase transition for different $N_f$

Temperature dependence:



For lattice, see [\[Miura, Lombardo, NPB 13\]](#)

Order of the transition:



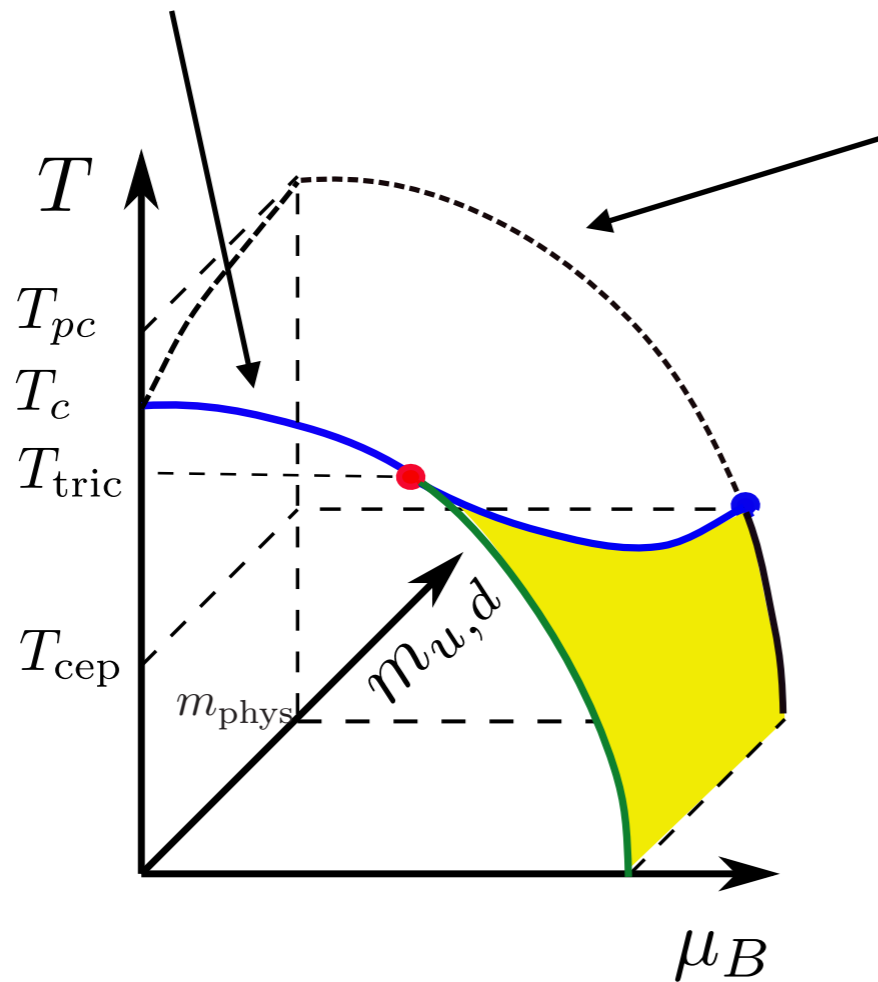
[\[Cuteri, O.P., Sciarra, JHEP 21\]](#)

The chiral phase transition in the massless limit is likely second-order for all  $N_f$

# From the chiral limit to the physical point

The “standard scenario”: [Halasz et al., PRD 98; Hatta, Ikeda, PRD 03...]

Importance of the chiral limit!



$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 - \kappa_2 \left( \frac{\mu_B}{T_{pc}(0)} \right)^2 + \dots$$

$\kappa_2$	Action
0.0158(13)	imag. $\mu$ , stout-smearred staggered
0.0135(20)	imag. $\mu$ , stout-smearred staggered
0.0145(25)	Taylor, stout-smearred staggered
0.016(5)	Taylor, HISQ

[Bellwied et al, PLB 15]  
 [Bonati et al, NPA 19]  
 [Bonati et al, PRD 18]  
 [HotQCD, PLB 19]

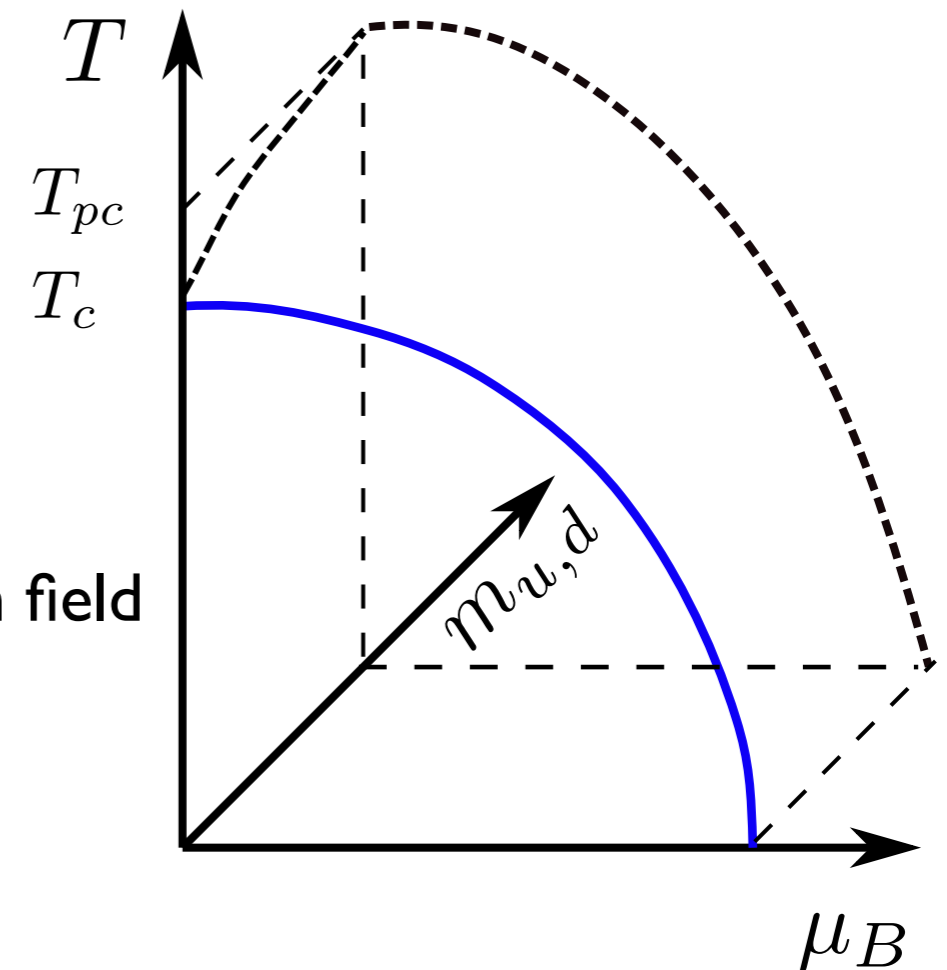
$$T_{pc} > T_c > T_{tric} > T_{cep}$$



$$\mu_B^{cep} > 3.1 T_{pc}(0) \approx 485 \text{ MeV}$$

# But what if..... ??

- ....the chiral limit is all second order?
- Then for physical masses: all crossover!
- So far consistent with **all** available lattice results
- Predicted by nucleon-meson models, beyond mean field  
[Brandes, Kaiser, Weise, EPJA 21]
- **Need to rule out one or the other scenario!**



# Three temperature regimes of QCD

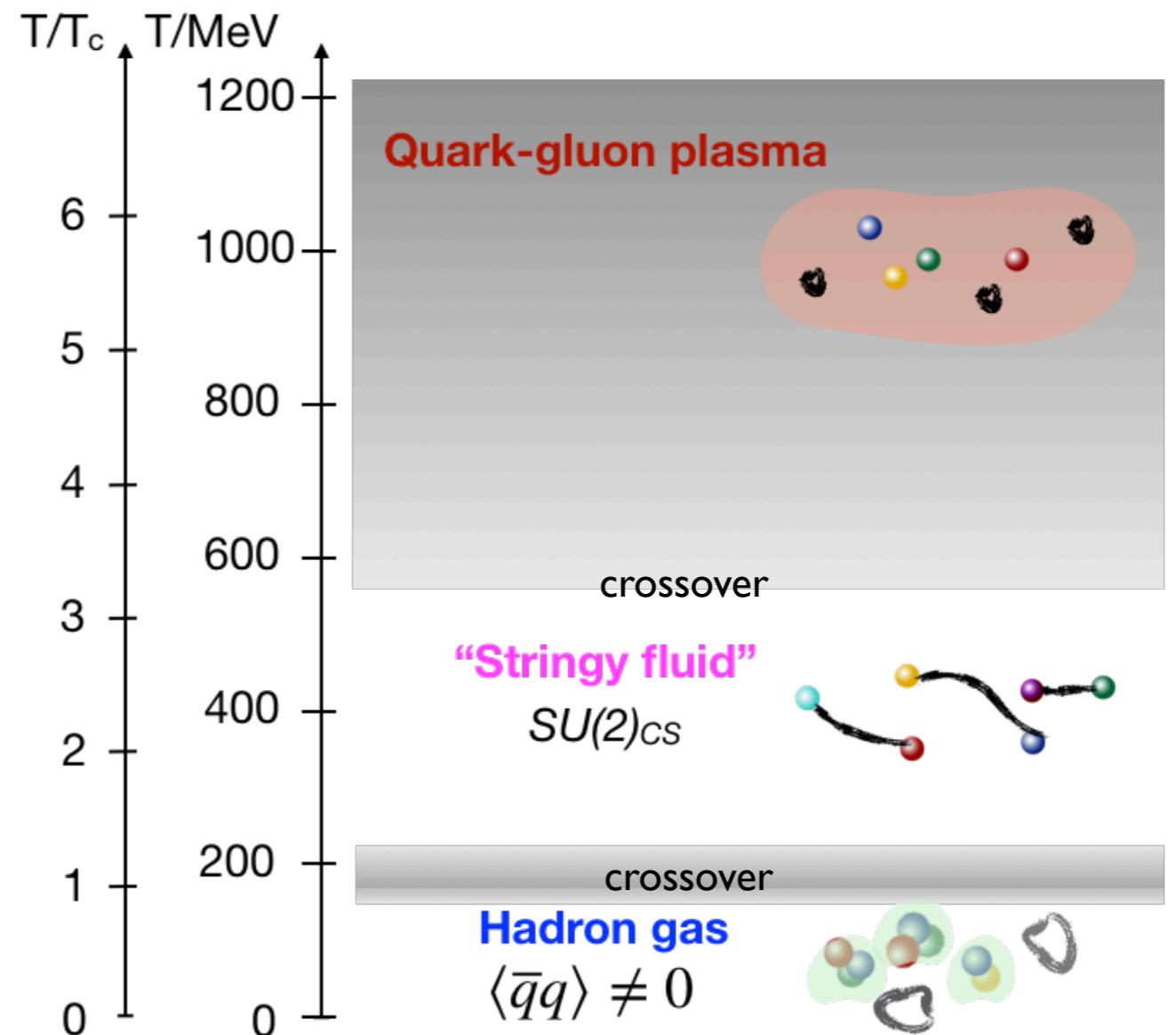
Symmetries (verified):

Degrees of freedom (to be verified):

Chiral symmetry (approximate)

Chiral spin symmetry (approximate)

Chiral symmetry broken



Rohrhofer et al., Phys. Rev. D 100 (2019)

# Check well-studied observables: screening masses

$$C_{\Gamma}^s(z) = \sum_{x,y,\tau} C_{\Gamma}(\tau, \mathbf{x}) \xrightarrow{z \rightarrow \infty} \text{const. } e^{-m_{scr} z}$$

Directly related to the partition function and equation of state

by transfer matrices:

$$T = e^{-aH}, T_z = e^{-aH_z}$$

$$\begin{aligned} e^{pV/T} = Z &= \text{Tr}(e^{-aH N_{\tau}}) \\ &= \text{Tr}(e^{-aH_z N_z}) = \sum_{n_z} e^{-E_{n_z} N_z} \end{aligned}$$

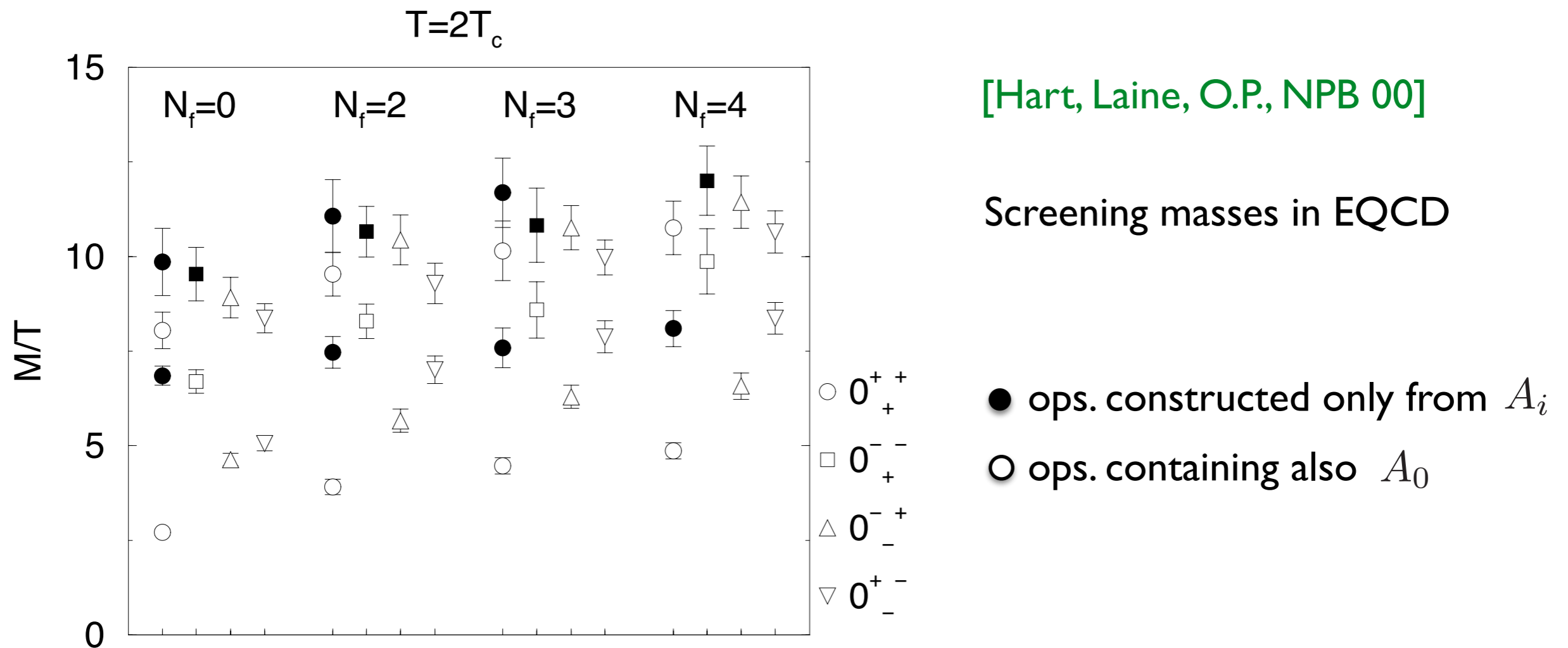
Screening masses: eigenvalues of  $H_z$

For  $T=0$  equivalent to eigenvalues of  $H$ , for  $T \neq 0$  “finite size effect”

# Colour-electric vs. colour magnetic fields

Scales at finite T:

Matsubara $\sim \pi T$ ,	hard modes, fermions	QCD
Debye/electric $\sim gT$ ,	$A_0$	EQCD
magnetic $\sim g^2 T$ ,	$A_i$	MQCD



Colour-electric fields dynamically dominant, perturbative ordering reversed!

**No quark hadron duality;** expected for soft scales of EQCD at low T

# Meson screening masses at high temperatures

[Dalla Brida et al., JHEP 22]

Nf=3, T=1 GeV -160 GeV

Highly non-trivial technically:  
shifted b.c. + step-scaling techniques  
(Alpha-Collaboration)

$$\frac{m_{PS}}{2\pi T} = 1 + p_2 \hat{g}^2(T) + p_3 \hat{g}^3(T) + p_4 \hat{g}^4(T)$$

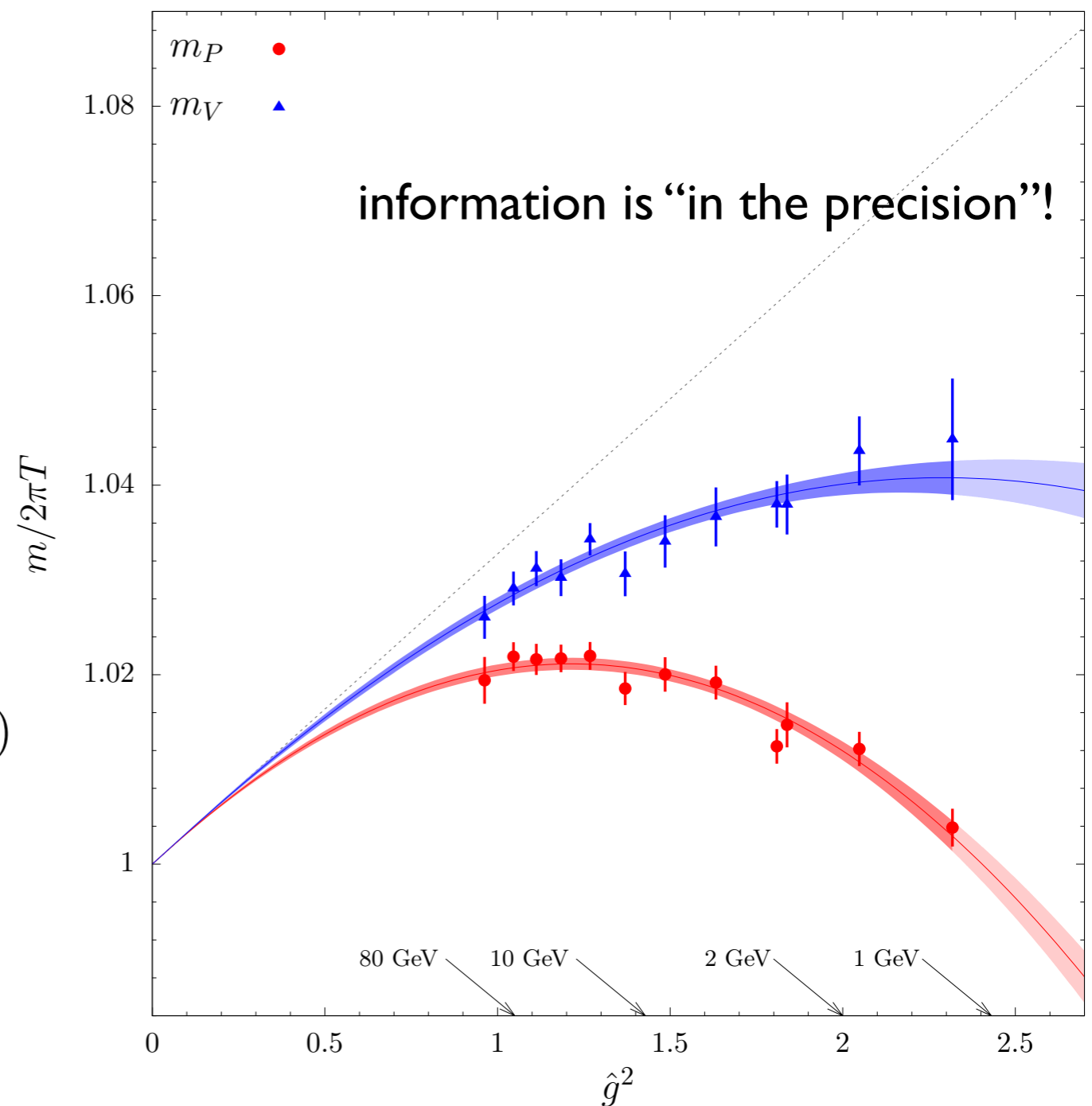
$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4 \hat{g}^4(T)$$

$$p_2 = 0.032739961$$

[Laine, Vepsäläinen., JHEP 04]

$p_3, p_4, s_4$  fitted, excellent  $\chi_{\text{dof}}^2$

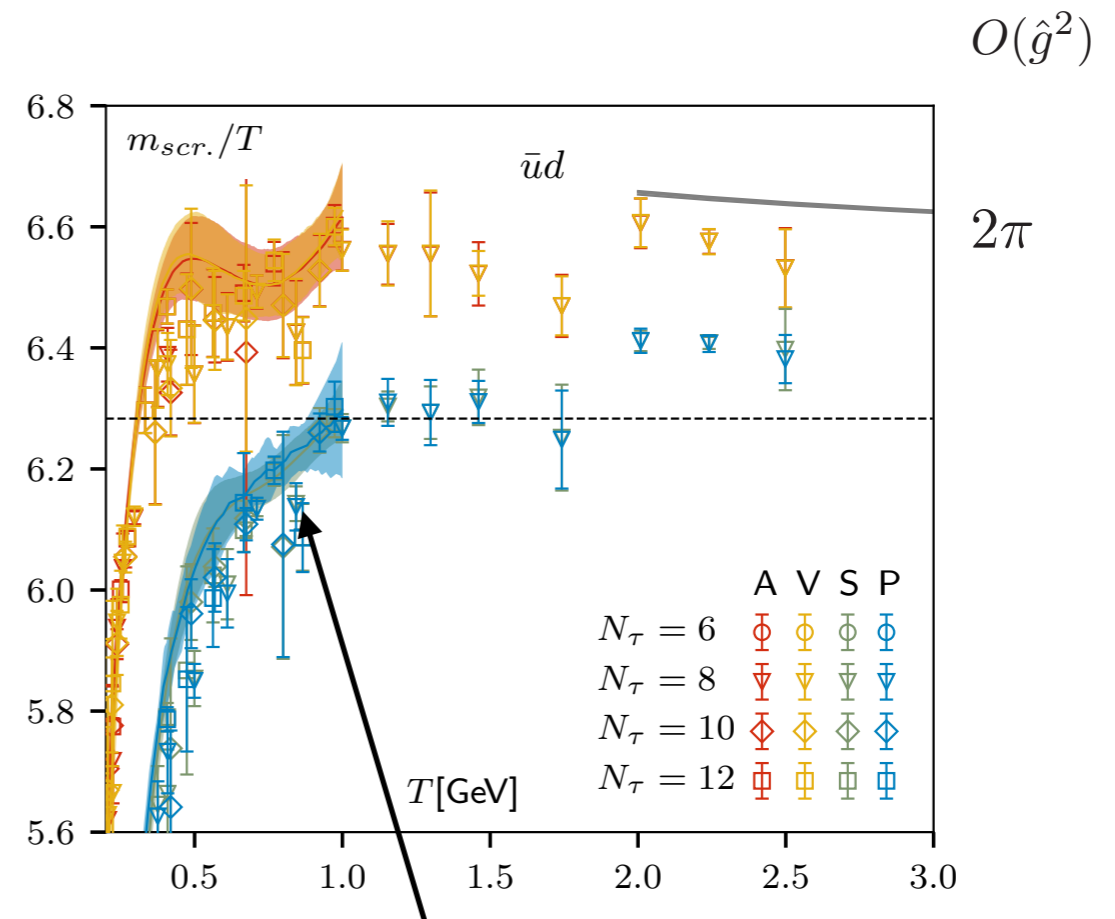
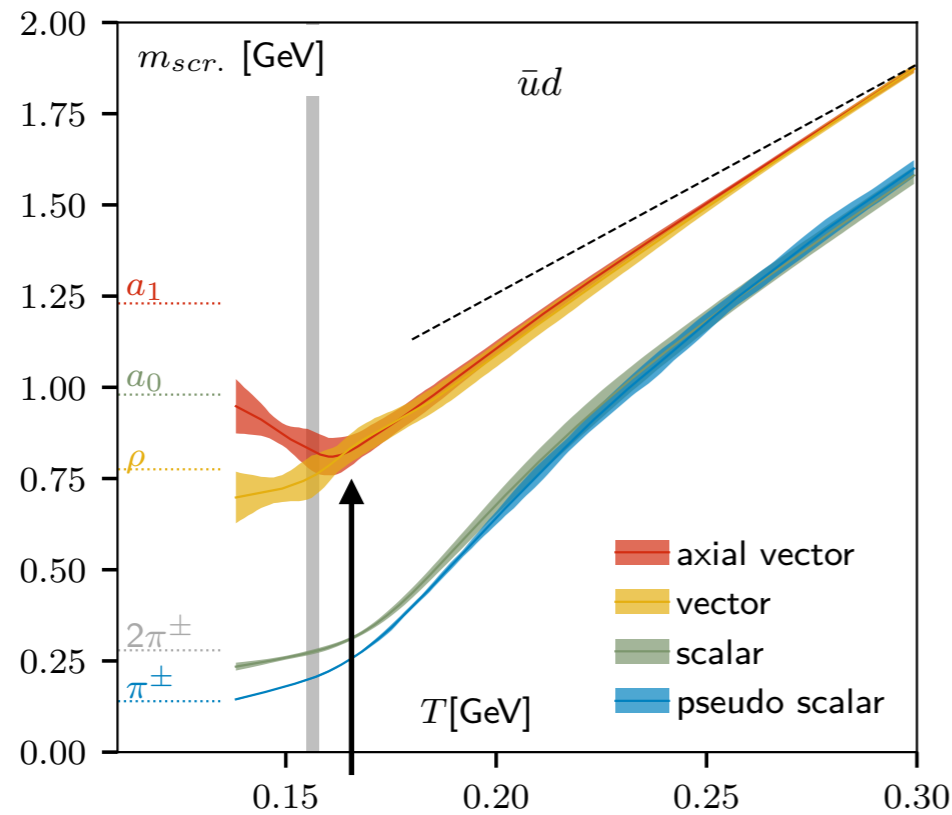
Quark hadron duality holds



$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{\text{MS}}}} + \frac{4}{9\pi^2} \ln \left( 2 \ln \frac{2\pi T}{\Lambda_{\overline{\text{MS}}}} \right)$$

# Meson screening masses at intermediate temperatures

[HotQCD, PRD 19]



Chiral symmetry restoration

Heavy chiral partners “come down”  
in all flavour combinations

➔ pressure increases

Drastic change: “vertical” - “horizontal”

Remember resummed pert. theory:

$$\frac{m_{PS}}{2\pi T} = 1 + p_2 \hat{g}^2(T) + p_3 \hat{g}^3(T) + p_4 \hat{g}^4(T) ,$$

$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4 \hat{g}^4(T) ,$$

Cannot describe the “bend”

No quark hadron duality for  $T < 0.5$  GeV in 12 lightest meson channels! CS symmetry!

# Finite density

- Finite density:  $\mu\bar{\psi}\gamma_0\psi$  is **CS invariant**; regime must continue to finite density

- Upper “boundary” of CS band: screening mass at “bend” (one possible def.)

$$r_V^{-1} \equiv m_V(\mu_B = 0, T_s) = C_0 T_s \quad \rightarrow \quad \begin{array}{l} T < T_s \text{ unscreened} \\ T > T_s \text{ screened} \end{array}$$

- For small  $\mu_B$

$$\frac{m_V(\mu_B)}{T} = C_0 + C_2 \left(\frac{\mu_B}{T}\right)^2 + \dots \quad \rightarrow \quad \frac{dT_s}{d\mu_B} = -\frac{2C_2}{C_0} \frac{\mu_B}{T} - \frac{2C_2^2}{C_0^2} \left(\frac{\mu_B}{T}\right)^3 + \dots$$

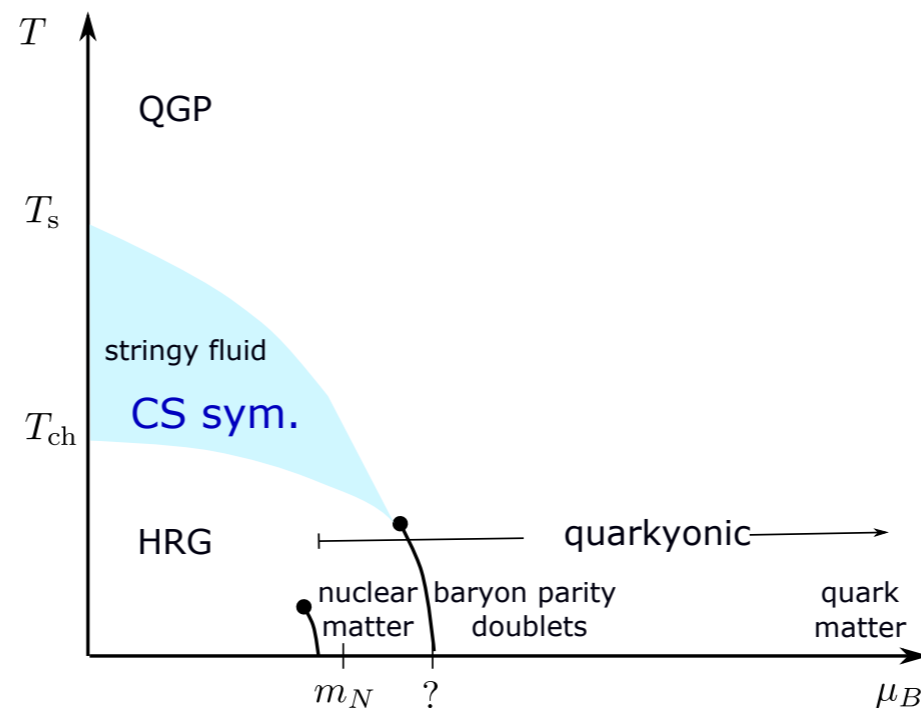
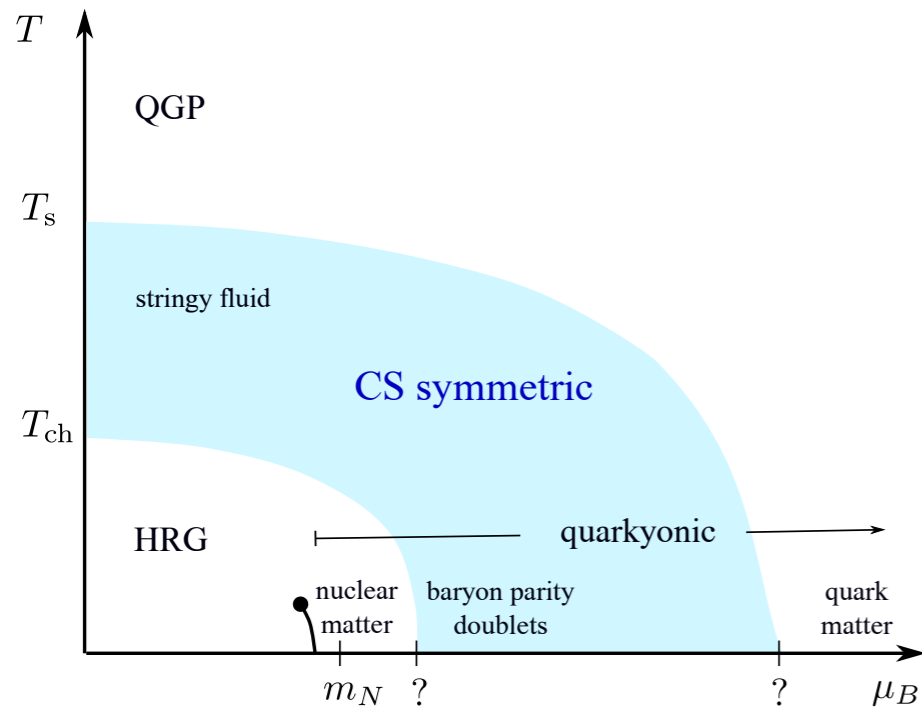
$C_2 > 0$

- Lower “boundary” of CS band: (this is a lower bound only)

$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 - 0.016(5) \left(\frac{\mu_B}{T_{pc}(0)}\right)^2 + \dots \quad \approx \quad \frac{T_{ch}(\mu_B)}{T_{ch}(0)}$$

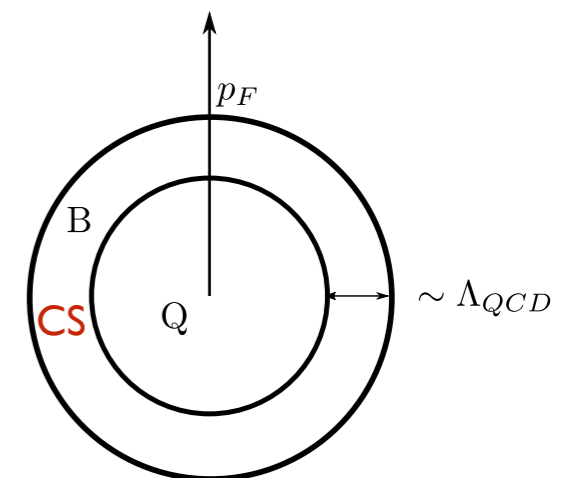
Separate order parameters for  $SU(2)_A, U(1)_A, SU(4)$  ?

# The QCD phase diagram



...

- Cold and dense candidate: baryon parity doublet models **CS symmetric** [Glozman, Catillo PRD 18]
- Quarkyonic matter [McLerran, Pisarski, NPA 07; O.P., Scheunert JHEP 19]  
Contains regime with chirally symmetric baryon matter  
Fully consistent with transient intermediate CS regime!
- Can be realized with or without true chiral phase transition!



# Effective degrees of freedom...? $\rightarrow$ Spectral functions

Based on micro-causality of scalar, local quantum fields at finite T:

[Bros, Buchholz., NPB 94, Ann. Inst. Poincare Phys.Theor. 96]

$$\rho_{PS}(p_0, \vec{p}) = \int_0^\infty ds \int \frac{d^3 \vec{u}}{(2\pi)^2} \epsilon(p_0) \delta(p_0^2 - (\vec{p} - \vec{u})^2 - s) \tilde{D}_\beta(\vec{u}, s)$$

Exact, goes to Källen-Lehmann representation for  $T \rightarrow 0$

↑  
thermal spectral density

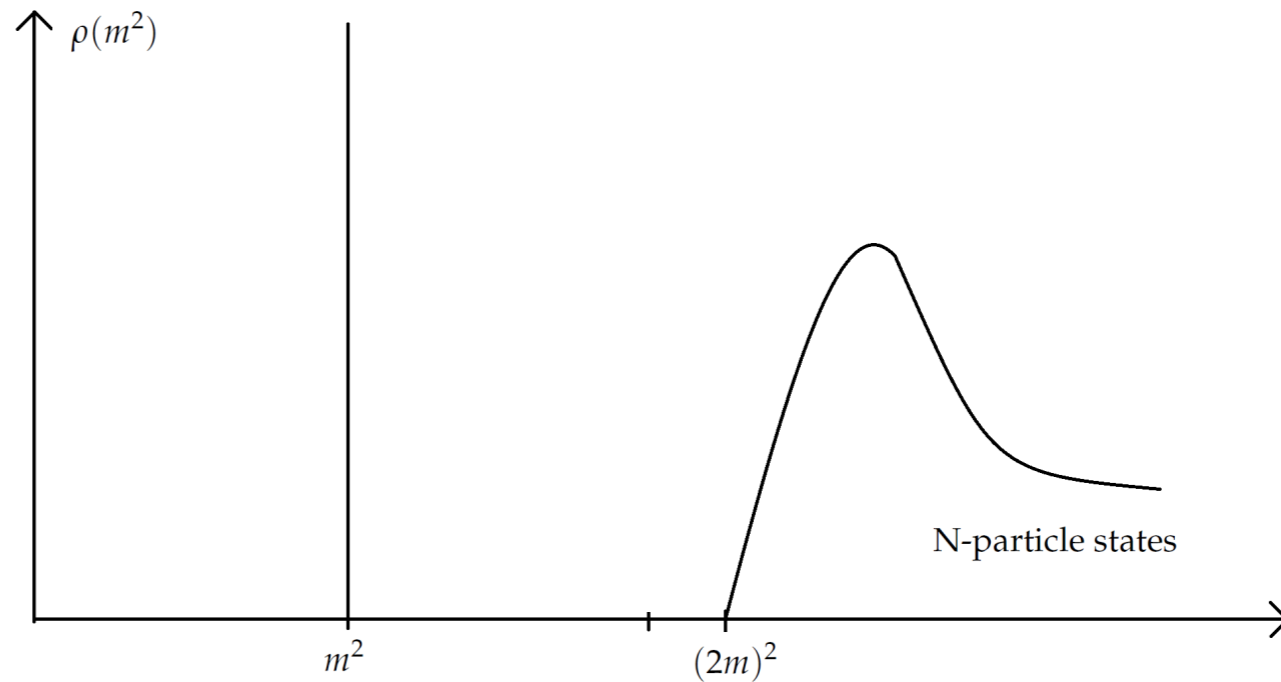
$\rightarrow$  Relation between spatial correlators and thermal spectral density

$$C_{PS}^s(z) = \frac{1}{2} \int_0^\infty ds \int_{|z|}^\infty dR e^{-R\sqrt{s}} D_\beta(R, s)$$

[Lowdon, O.P., JHEP 22 ]

For stable massive particle with gap to continuum states (QCD pions!):

Vacuum spectral function:



**Ansatz**  $\tilde{D}_\beta(\vec{u}, s) = \tilde{D}_{m,\beta}(\vec{u}) \delta(s - m^2) + \tilde{D}_{c,\beta}(\vec{u}, s)$  [Bros, Buchholz., NPB 02]

Analytic structure inherited from vacuum in absence of phase transition

➔ low T behaviour dominated by vacuum particle states

# The pion spectral function

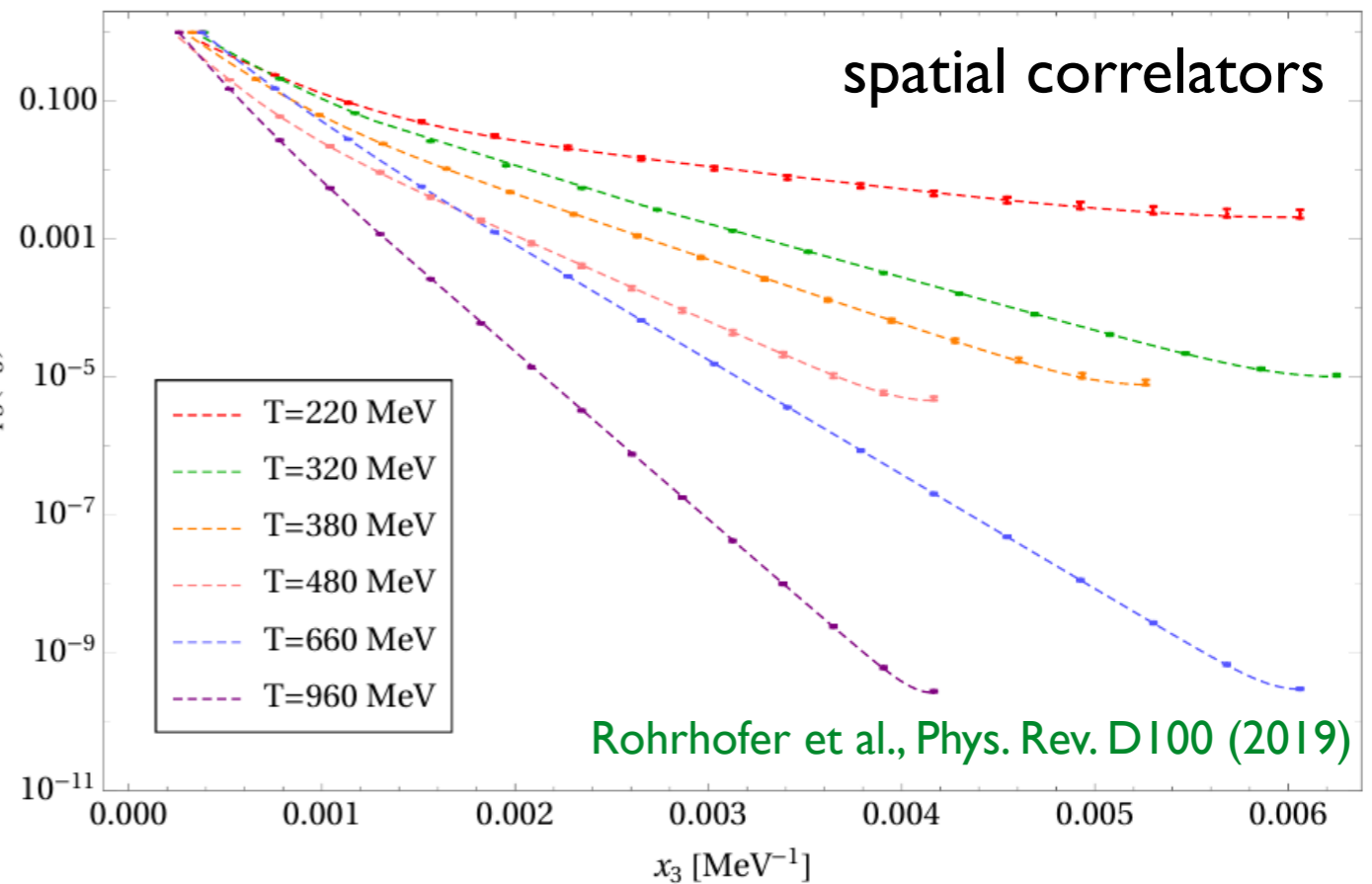
[Lowdon, O.P., JHEP 22]

2-state fits  $\pi, \pi^*$

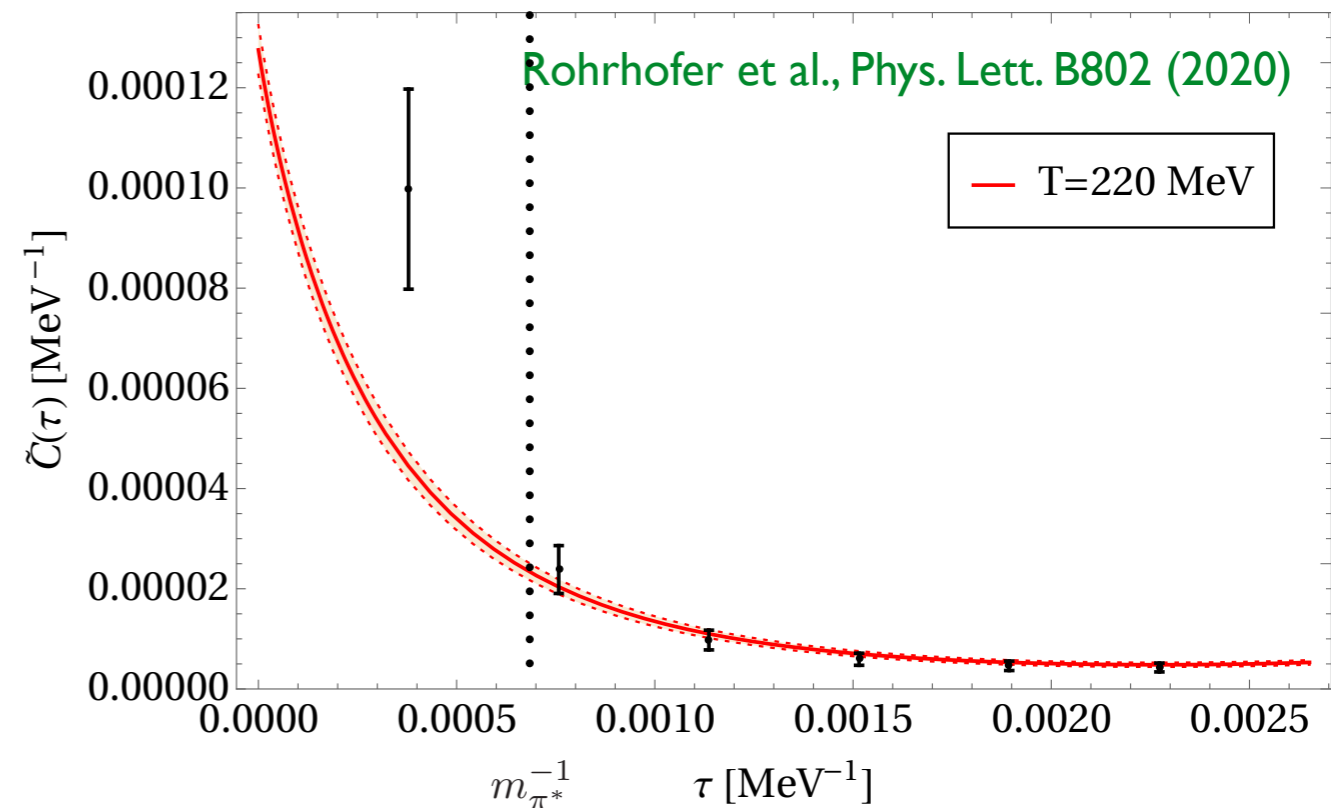
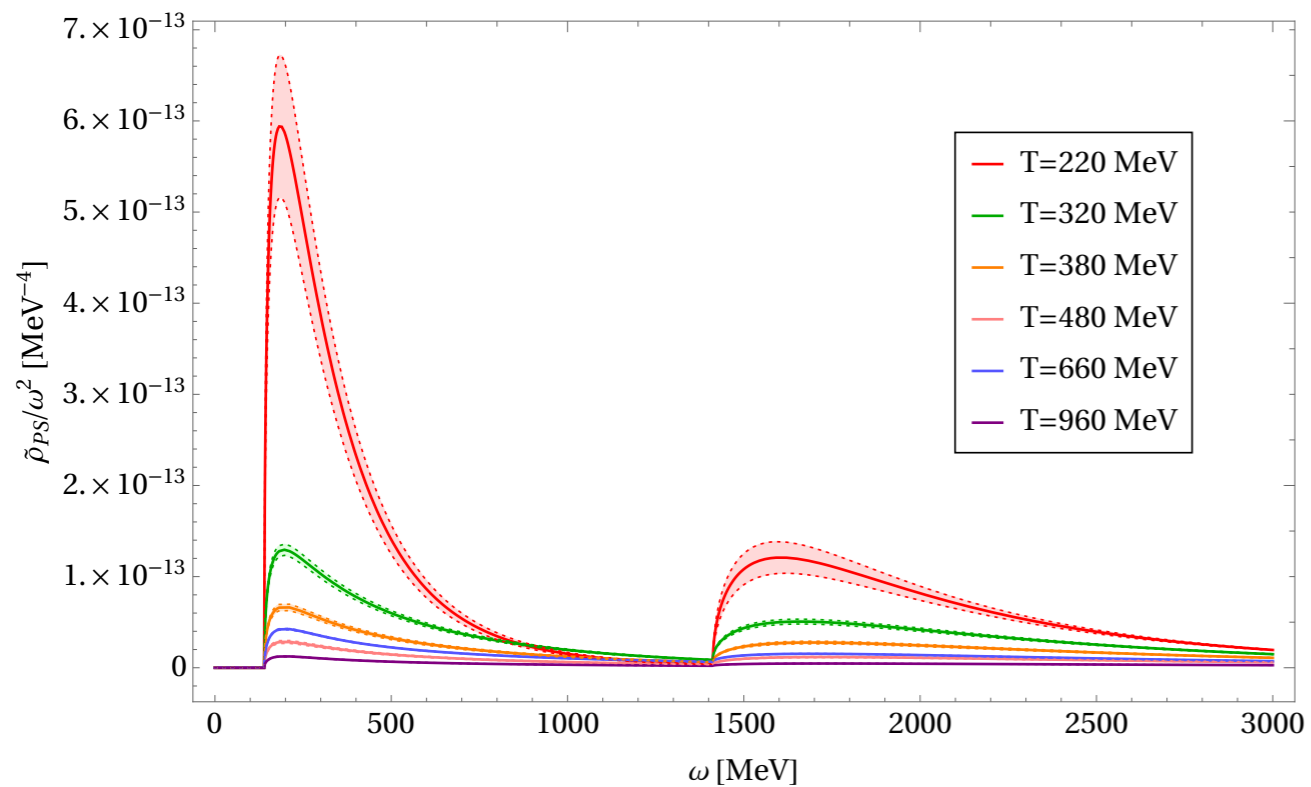
$$D_{m_{\pi^{(*)}}, \beta} = \alpha_{\pi^{(*)}} e^{-\gamma_{\pi^{(*)}} x_3}$$

spectral functions

$C_{PS}(x_3)$



predict temporal correlators, compare with data



# Comparison with plasmon ansatz

Bros+Buchholz Ansatz

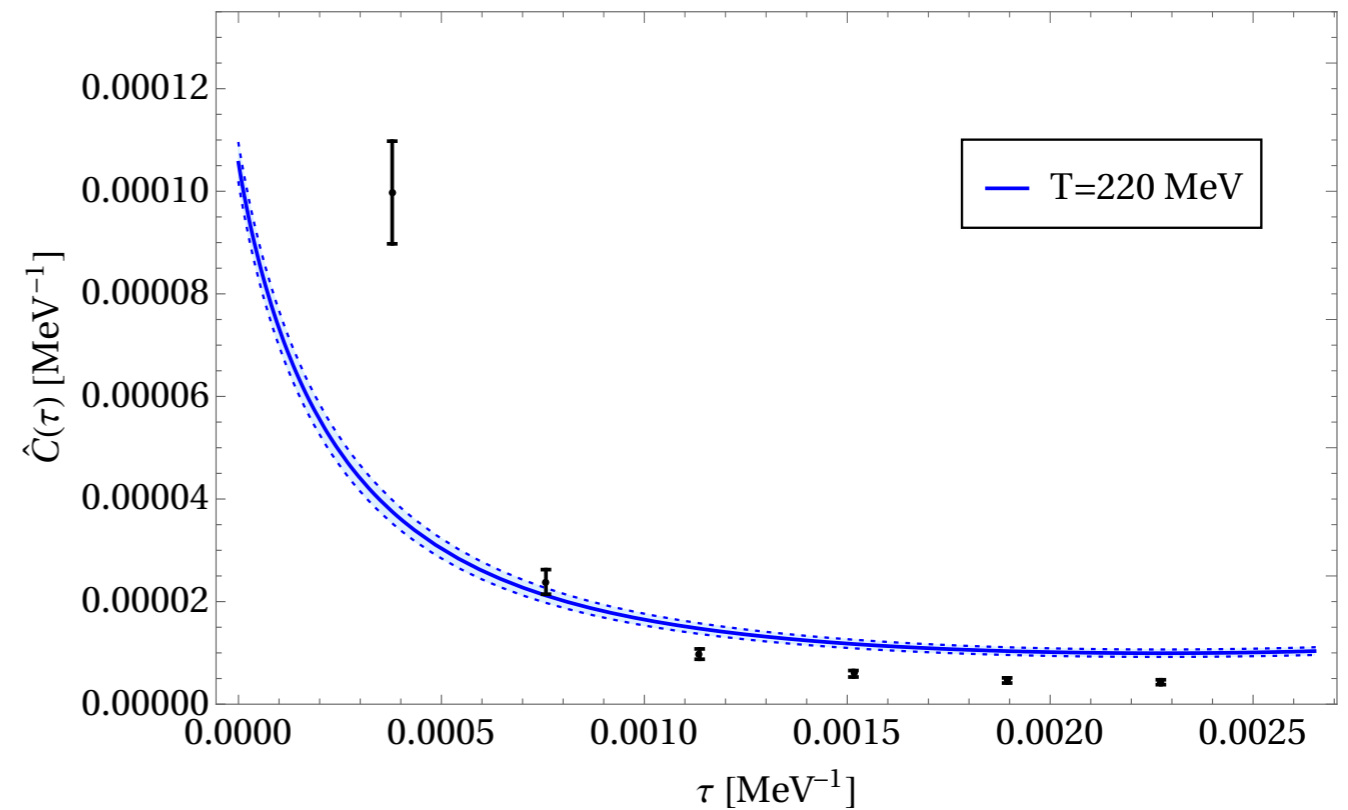
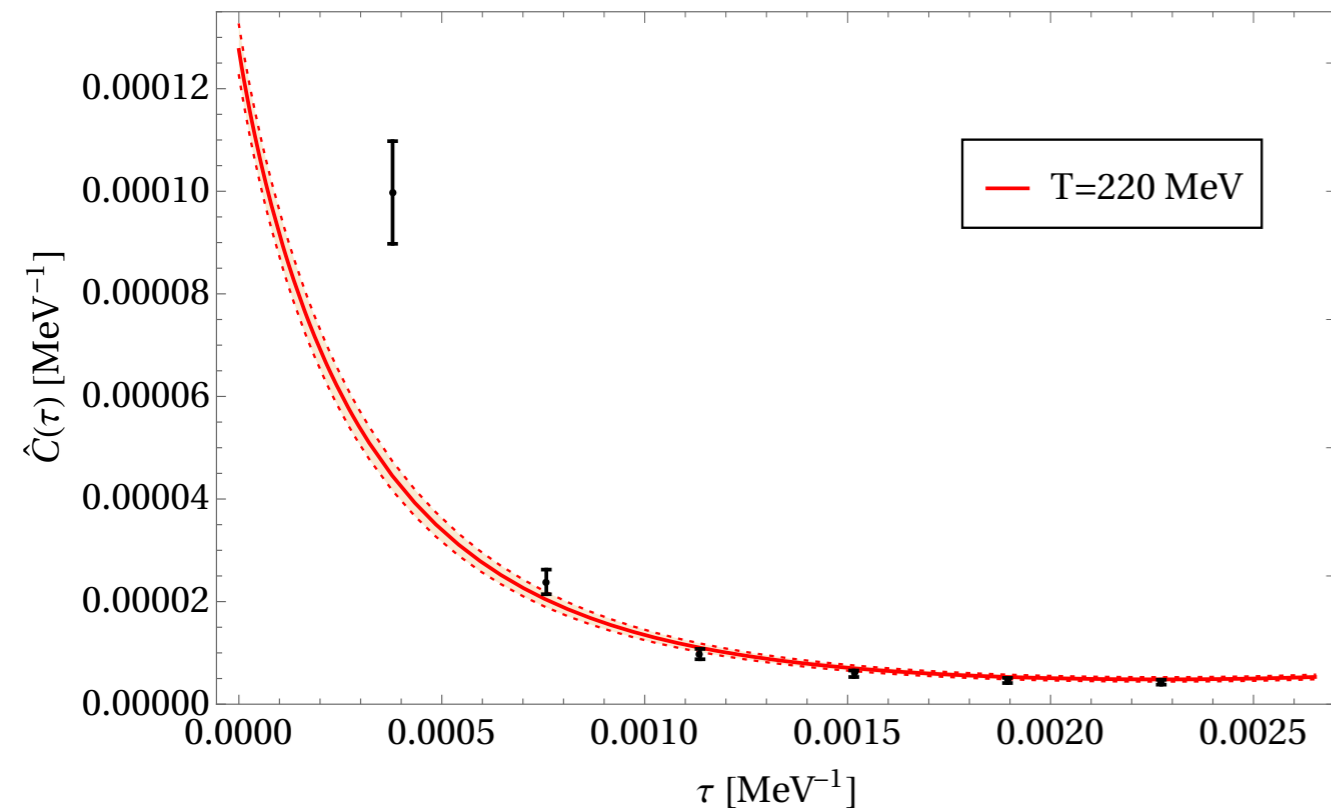
Perturbative plasmon: Breit-Wigner shape

Both fit spatial correlator

$$\rho_{PS}(\omega, \mathbf{p} = 0) = \epsilon(\omega) \left[ \theta(\omega^2 - m_\pi^2) \frac{4 \alpha_\pi \gamma_\pi \sqrt{\omega^2 - m_\pi^2}}{(\omega^2 - m_\pi^2 + \gamma_\pi^2)^2} + \theta(\omega^2 - m_{\pi^*}^2) \frac{4 \alpha_{\pi^*} \gamma_{\pi^*} \sqrt{\omega^2 - m_{\pi^*}^2}}{(\omega^2 - m_{\pi^*}^2 + \gamma_{\pi^*}^2)^2} \right]$$

$$\rho_{PS}^{BW}(\omega, \mathbf{p} = 0) = \frac{4 \alpha_\pi \omega \Gamma_\pi}{(\omega^2 - m_\pi^2 - \Gamma_\pi^2)^2 + 4 \omega^2 \Gamma_\pi^2} + \frac{4 \alpha_{\pi^*} \omega \Gamma_{\pi^*}}{(\omega^2 - m_{\pi^*}^2 - \Gamma_{\pi^*}^2)^2 + 4 \omega^2 \Gamma_{\pi^*}^2}$$

Predicted temporal correlators:



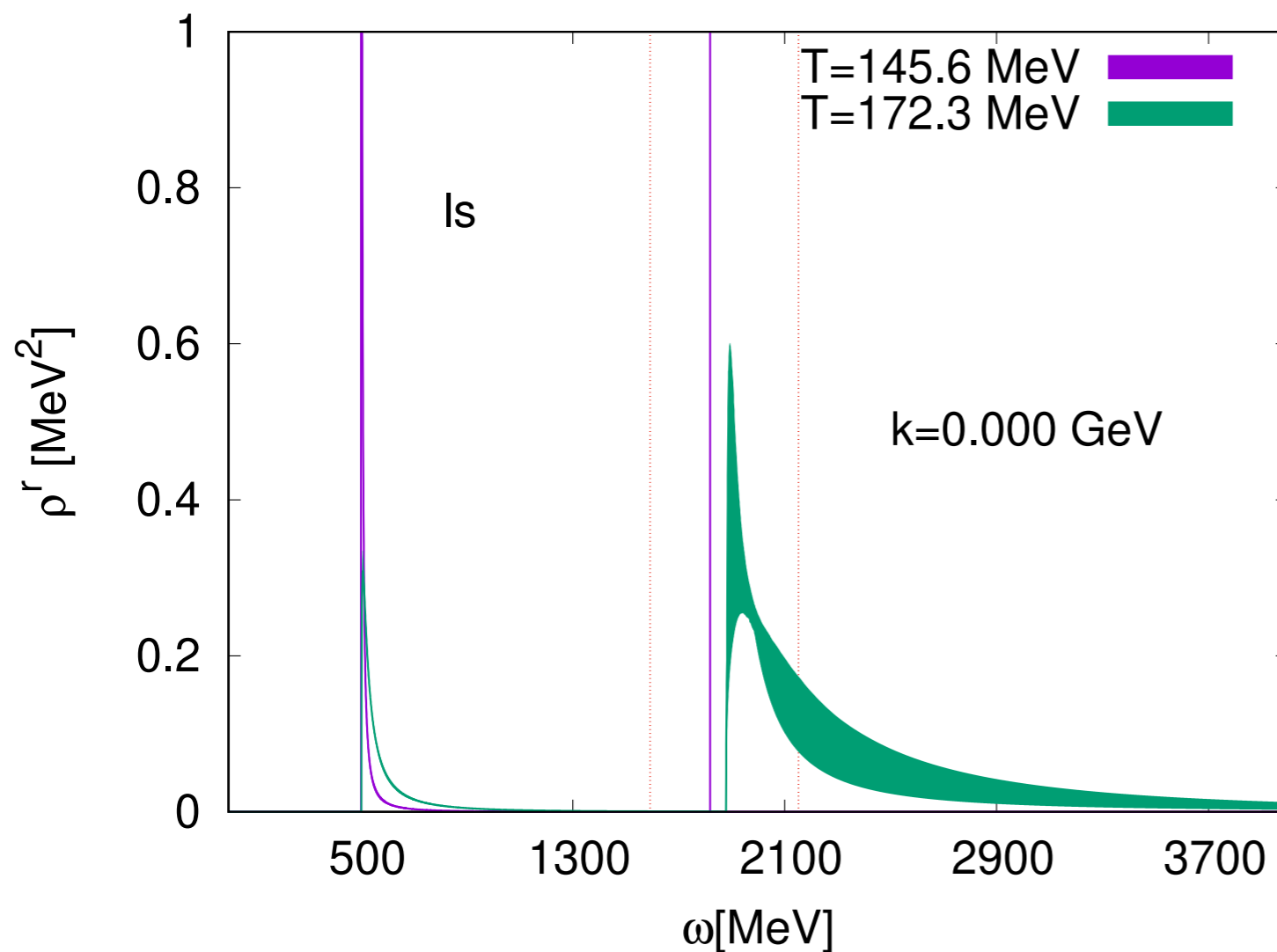
# In progress: same analysis for additional states

[Bala, Kaczmarek, Lowdon, O.P., Ueding]

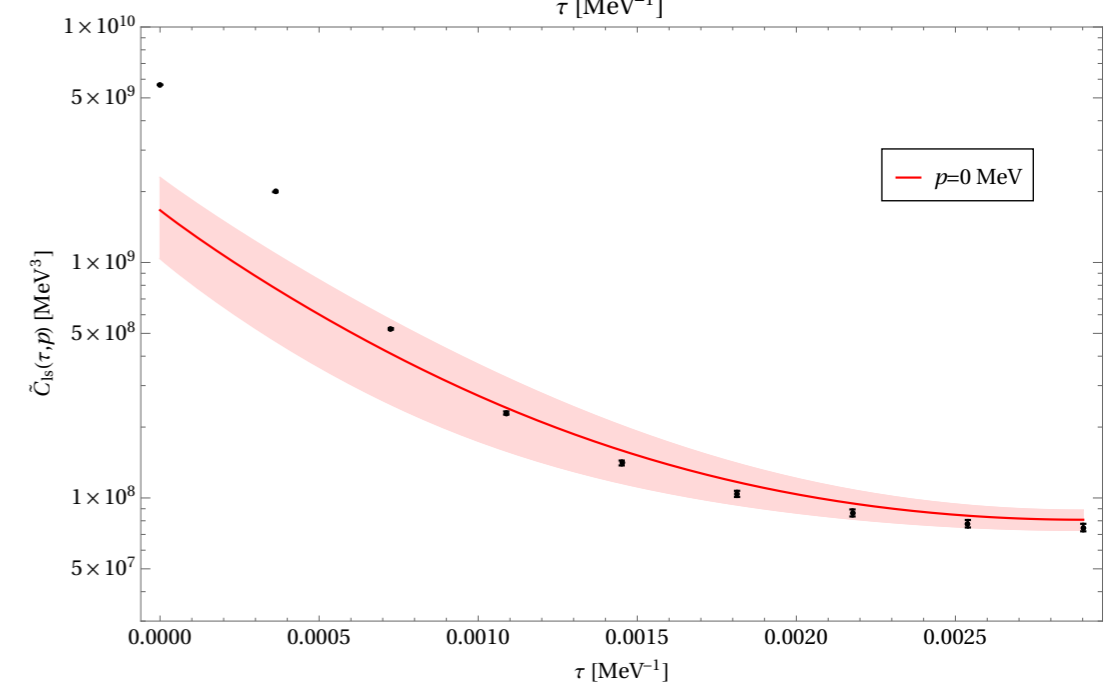
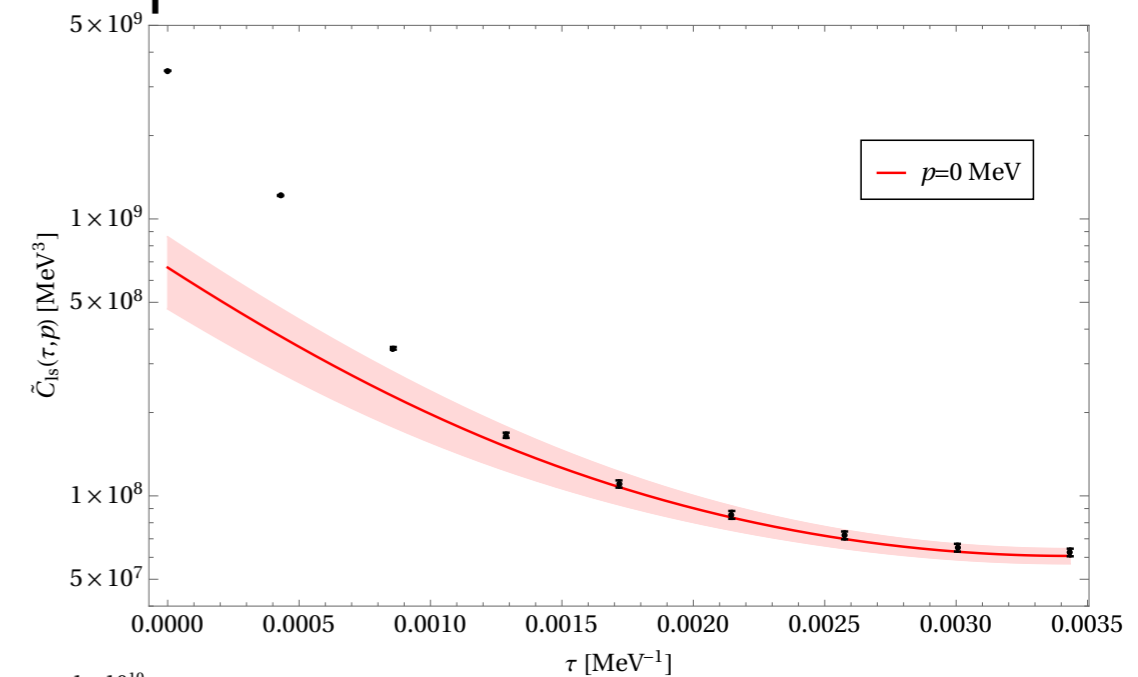
$N_f = 2 + 1$  HISQ sea + domain wall valence quarks, physical masses on  $64^3 \times 16$

Goal: analyse **all** scalar and pseudo-scalar correlators, here:  $\bar{l}_S$  - channel

spectral function from z-correlator



predicted + measured t-correlator

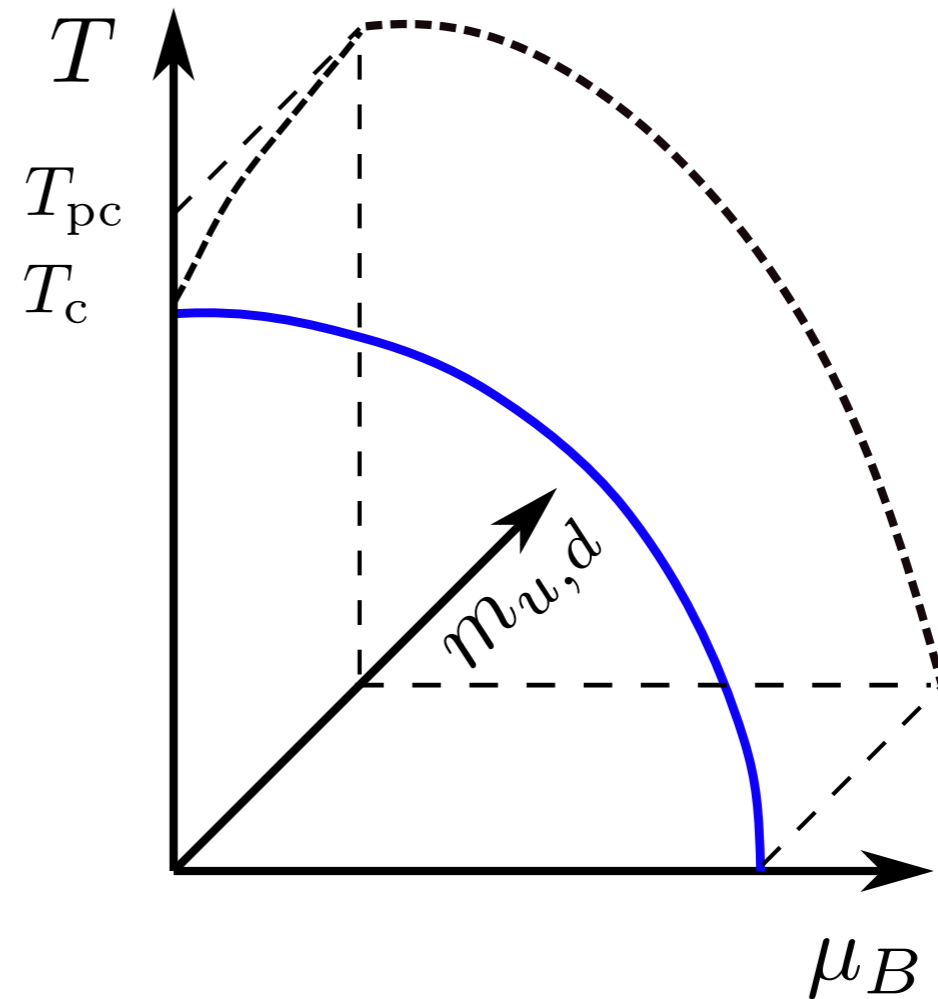
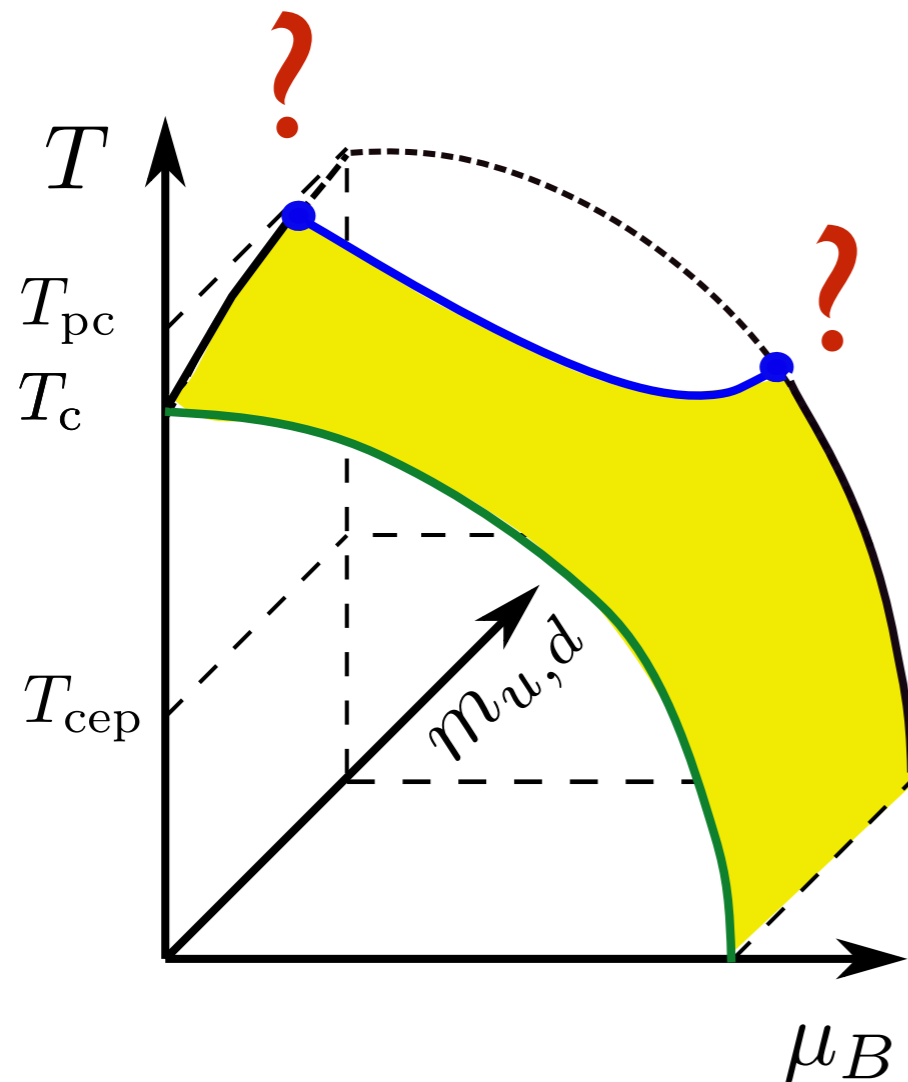


# Conclusions

- Zero density, unimproved staggered,  $N_f=2-6$ ,  $O(a)$ -improved Wilson  $N_f=3$ :  
1st order chiral transition region not connected to continuum limit
- Chiral transition second order (probably up to the conformal window)
- Chiral spin symmetric regime in a band  $T_{pc} < T < 3T_{pc}$
- CS-band must continue to finite density
- Lowest excitations in CS-band hadron/resonance - like,  
no perturbative/partonic description!

**Backup slides**

# Other possibilities, mostly ignored

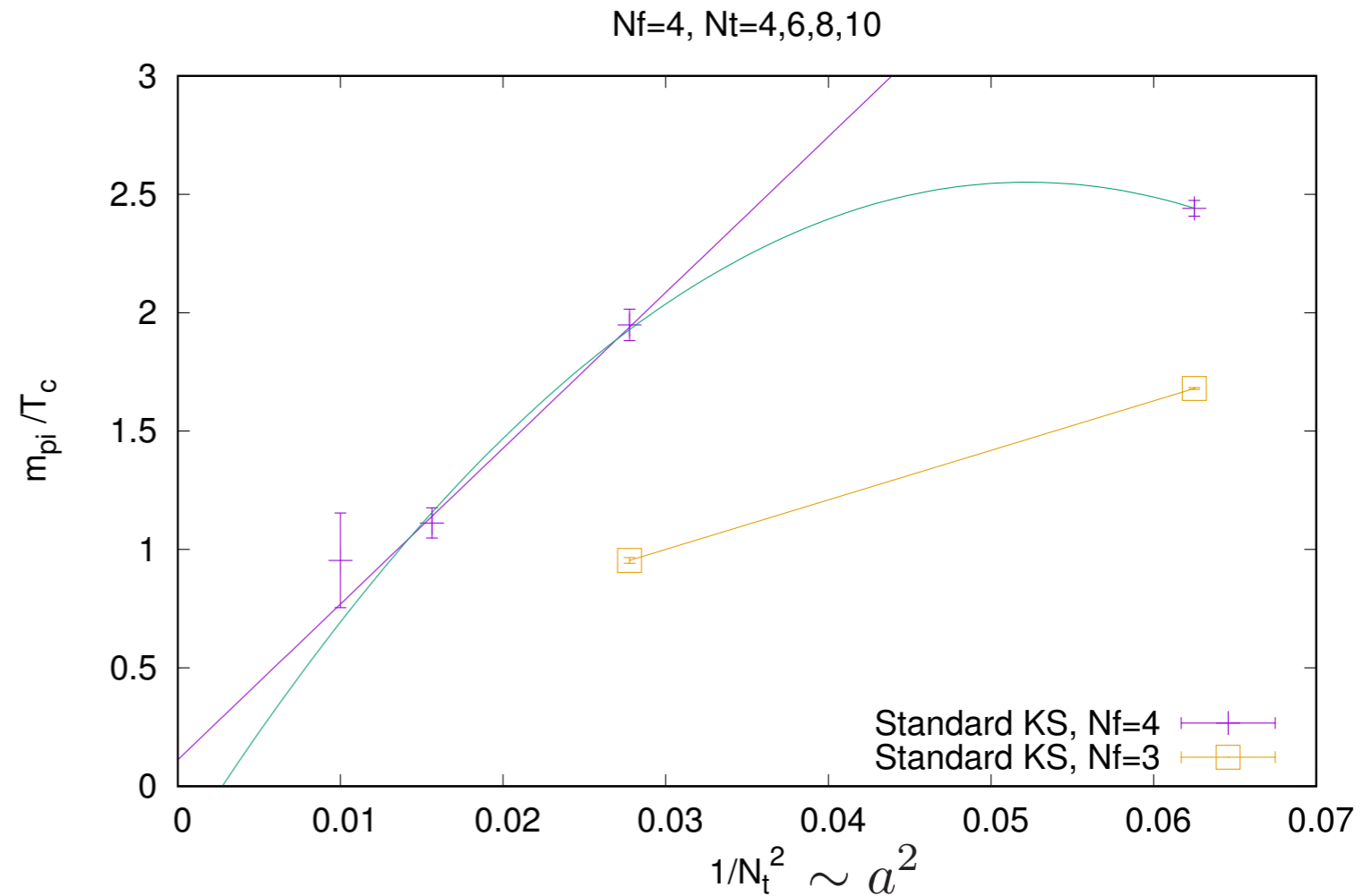
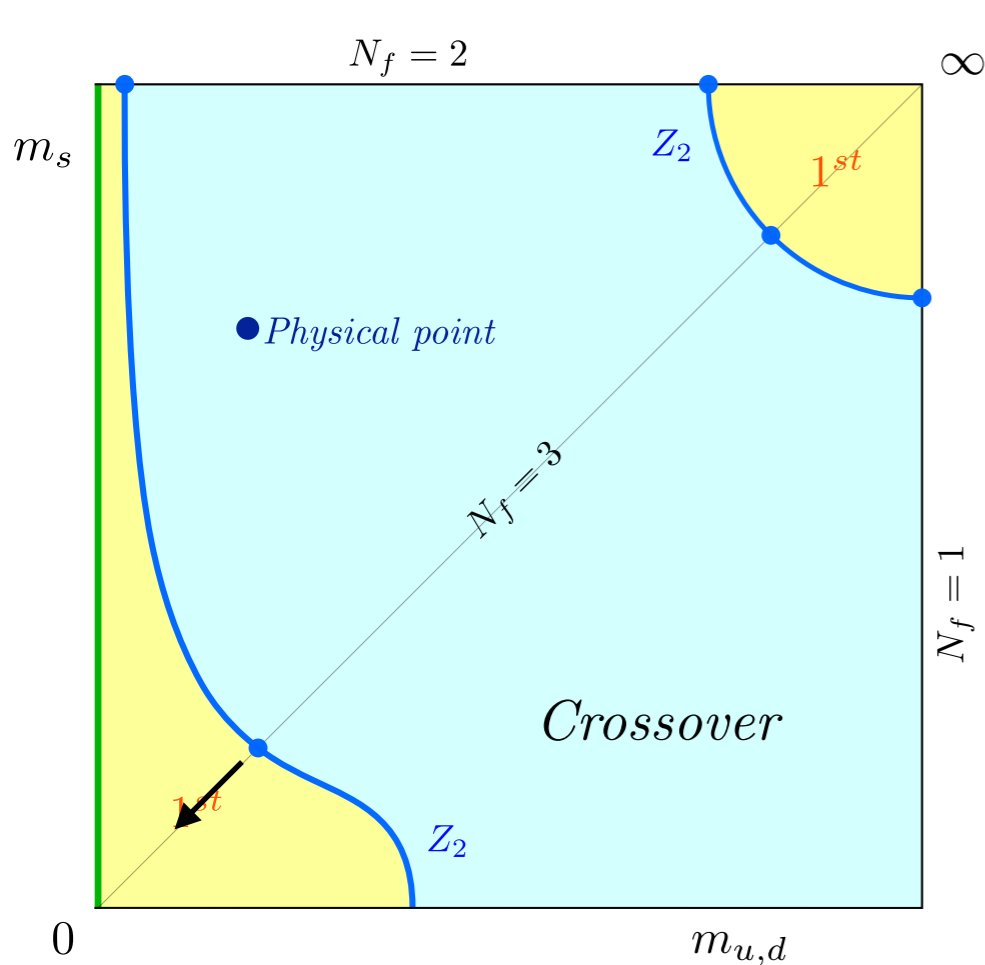


**Note:** universality is NOT an argument to deduce the QCD phase diagram from models!

e.g. 3d Ising model vs. water: **same** universality at critical point, **different** phase diagrams

# The nature of the QCD chiral transition, $N_f=3,4$

...has enormously large cut-off effects!



Unimproved staggered:

1st order region shrinks for  $a \rightarrow 0$ , both for  $N_f = 3, 4$  [de Forcrand, D'Elia, PoS LAT 17]

No 1st order seen for improved staggered actions, even for  $N_f = 3$  [HotQCD PRD 17,21]

# What about Pisarski, Wilczek?

- Investigated 3d sigma model, i.e.  $\phi^4$ - Ginzburg-Landau-Wilson theory for chiral condensate
- Results based on epsilon expansion about  $\epsilon = 1$
- All conclusions confirmed by [Butti, Pelissetto, Vicari, JHEP 03] (High order perturbative expansion in fixed d)
- Support also from simulation of 3d sigma model [Gausterer, Sanielovici, PLB 88]
- Conformal bootstrap methods: fixed point also for  $O(4) \times O(2)$  [Nakayama, Ohtsuki PRD 14]

Suggested resolution:

$\phi^6$  term should be included in 3d, renormalisable

FRG: 3d  $\phi^6$  has infrared fixed points and 2nd order transitions [Litim, Tetradis, NPB 96]

[Fejos, PRD 22] 3d  $\phi^6$  with t'Hooft term: 2nd order transition for restored anomaly!

# Why dominance by particle states is plausible

V,A correlators in the chiral limit using PCAC,  $\epsilon = T^2/(6f_\pi^2)$

[Dey, Eletsky, Ioffe PLB 90]

$$C_V(p, T) = (1 - \epsilon)C_V(p, 0) + \epsilon C_A(p, 0)$$

$$C_A(p, T) = (1 - \epsilon)C_A(p, 0) + \epsilon C_V(p, 0)$$

Vacuum pole structure of correlators reflected in finite T correlators

General spectral decomposition of spatial correlators with eigenstates of H

$$C_\Gamma(\tau, \mathbf{0}, T) = \sum_{m,n} |\langle m | O_\Gamma(0, \mathbf{0}) | n \rangle|^2 e^{-\tau(E_n - E_0)} e^{-(T-\tau)(E_m - E_0)}$$

Analytic structure of vacuum dominant for **low** temperatures