Strong-coupling dynamics in Cosmology

Roman Pasechnik

Lund U.

Vacuum in Quantum Physics vs in Cosmology

Vacuum energy

"...the worst theoretical prediction in the history of physics" (Hobson 2006)

in Quantum Physics		in Cosmology
$\epsilon_{vac} \sim 10^{-2}\mathrm{GeV}^4$	**Topological QCD vacuum** **unique strongly-coupled subsystem!**	$\Lambda_{\rm cosm} \sim 10^{-47}\,\mathrm{GeV}^4$
$\sim 10^8 \mathrm{GeV}^4$	**Higgs condensate**	"Old" CC problem: Why such small and positive? "New" CC problem: Why non-zeroth and exists at all?

Vacuum in Quantum Physics has incredibly wrong energy scale!

Quantum-topological (chromomagnetic) vacuum in QCD

$$\varepsilon_{vac(top)} = -\frac{9}{32}\langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x) : |0\rangle + \frac{1}{4}\left(\langle 0| : m_u \bar{u}u : |0\rangle + \langle 0| : m_d \bar{d}d : |0\rangle + \langle 0| : m_s \bar{s}s : |0\rangle\right)$$
$$\simeq -(5 \pm 1) \times 10^9\ \mathrm{MeV}^4.$$

Two possible approaches to this problem:

- **Let's forget about the "bare" vacuum (DE: "phantom", "quintessence", "ghost"… etc)**
  Zero vacuum density in the Minkowski limit, by (Casimir-like) definition, then (Zhitnitsky et al)

  $$\Lambda_{\rm cosm} \equiv \epsilon_{\rm FLRW} - \epsilon_{\rm Mink}$$ **simply imposing a cancellation of the "bare" vacuum by hands!!**

- **Let's look closer at the vacuum state — why/how does it become "invisible" to gravity?**

An illustration: topological vs collective contributions

NPT QCD vacuum

Quantum-topological (instanton) fluctuations

instantons/dyons carrying chromomagnetic and chromoelectric charges

$$\varepsilon_{vac(top)} < 0$$

Quantum-wave (hadronic) fluctuations

have quantum numbers of light hadrons

$$m_h \leq l^{-1}_{g(min)}$$

$$\varepsilon_{vac(h)} > 0$$

<u>exist at the same typical space-time scales</u>

$$l_{g(min)} < l_g < l_{g(max)},$$

$$l_{g(min)} \simeq (1500\ \mathrm{MeV})^{-1}, \qquad l_{g(max)} \simeq (500\ \mathrm{MeV})^{-1}$$

Can they mutually cancel each other? In principle, YES!

Taking into account ONLY metastable hadrons

$$B = \{N,\ \Lambda,\ \Sigma,\ \Xi\}$$

$$M = \{\pi,\ K,\ \eta,\ \eta'\}$$

$$\varepsilon_{vac(h)} = \frac{1}{32\pi^2}\left(2\sum_B (2J_B+1)m_B^4 \ln\frac{\mu}{m_B} - \sum_M (2J_M+1)m_M^4 \ln\frac{\mu}{m_M}\right)$$

$$\mu \simeq l^{-1}_{g(min)}$$

$$\varepsilon_{vac(top)} + \varepsilon_{vac(h)} = 0 \ \text{ for } \ \mu = 1.22\ \text{ GeV}$$

!!!

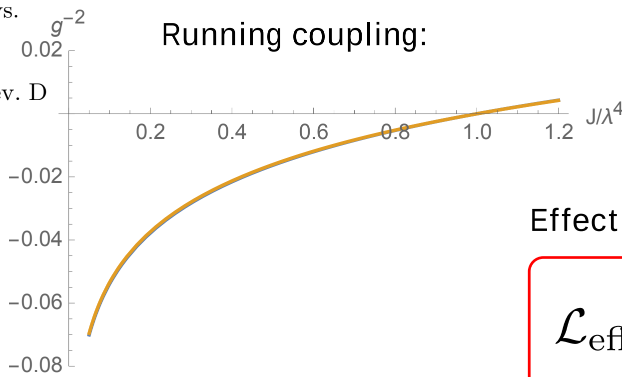
Effective YM action and Savvidy vacuum

At least, for SU(2) gauge symmetry,
the all-loop and one-loop effective Lagrangians
are practically indistinguishable (by FRG approach)

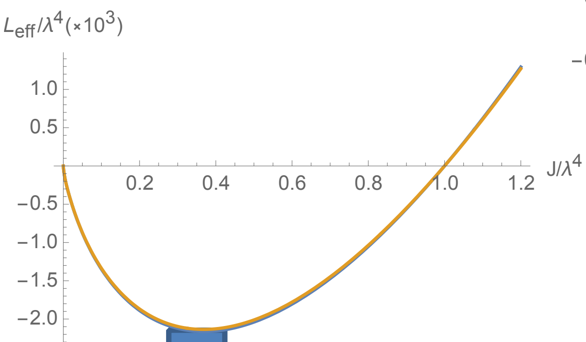
P. Dona, A. Marciano, Y. Zhang and C. Antolini, Phys. Rev. D **93** (2016) no.4, 043012.

A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D **83** (2011) 045014 [Phys. Rev. D **83** (2011) 069903].

Running coupling:



Effective Lagrangian:



gluon condensate (Savvidy vacuum)

Discovery of chromomagnetic condensate:

G. K. Savvidy, Phys. Lett. **71B**, 133 (1977)

G. Savvidy, Eur. Phys. J. C **80** (2020) 165

H. Pagels and E. Tomboulis, Nucl. Phys. B **143**, 485 (1978).

Classical YM Lagrangian:

$$\mathcal{L}_{\rm cl} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_{\rm YM}\, f^{abc} A^b_\mu A^c_\nu$$

$$\mathcal{A}^a_\mu \equiv g_{\rm YM} A^a_\mu$$

Effective YM Lagrangian:

$$\mathcal{F}^a_{\mu\nu} \equiv g_{\rm YM} F^a_{\mu\nu}$$

$$\mathcal{L}_{\rm eff} = \frac{\mathcal{J}}{4\bar{g}^2(\mathcal{J})}, \qquad \mathcal{J} = -\mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu}_a$$

The energy-momentum tensor:

$$T^\nu_\mu = \frac{1}{\bar{g}^2}\left[\frac{\beta(\bar{g}^2)}{2} - 1\right]\left(\mathcal{F}^a_{\mu\lambda}\mathcal{F}^{\nu\lambda}_a + \frac{1}{4}\delta^\nu_\mu \mathcal{J}\right) - \delta^\nu_\mu \frac{\beta(\bar{g}^2)}{8\bar{g}^2}\mathcal{J}$$

Equations of motion:

$$\overrightarrow{\mathcal{D}}^{ab}_\nu \left[\frac{\mathcal{F}^{\mu\nu}_b}{\bar{g}^2}\left(1 - \frac{\beta(\bar{g}^2)}{2}\right)\right] = 0,$$

$$\overrightarrow{\mathcal{D}}^{ab}_\nu \equiv \left(\delta^{ab}\overrightarrow{\partial}_\nu - f^{abc}\mathcal{A}^c_\nu\right),$$

trace anomaly:

$$T^\mu_\mu = -\frac{\beta(\bar{g}^2)}{2\bar{g}^2}\mathcal{J}$$

NOTE: the RG equation

$$\frac{d\ln|\bar{g}^2|}{d\ln|\mathcal{J}|/\mu_0^4} = \frac{\beta(\bar{g}^2)}{2}$$

appears to be invariant under

$$\mathcal{J} \longleftrightarrow -\mathcal{J}$$
$$\bar{g}^2 = \bar{g}^2(|\mathcal{J}|)$$

Real-time evolution of the gluon condensate

$$\mathcal{J} = \frac{2}{\sqrt{-g}} \sum_a (\mathbf{E}_a \cdot \mathbf{E}_a - \mathbf{B}_a \cdot \mathbf{B}_a) \equiv \frac{2}{\sqrt{-g}} (\mathbf{E}^2 - \mathbf{B}^2)$$

FLRW metric in conformal time:

$$g \equiv \det(g_{\mu\nu}), \; g_{\mu\nu} = a(\eta)^2 \mathrm{diag}(1, -1, -1, -1)$$

$$\sqrt{-g} = a^4(\eta)\,, \qquad t = \int a(\eta) d\eta$$

- **Basic qualitative features on the non-perturbative YM action are noticed already at one loop**

Einstein-YM equations of motion for the effective YM theory:

$$\frac{1}{\varkappa}\left(R^\nu_\mu - \frac{1}{2}\delta^\nu_\mu R\right) = \bar{\epsilon}\delta^\nu_\mu + \frac{b}{32\pi^2}\frac{1}{\sqrt{-g}}\left[\left(-\mathcal{F}^a_{\mu\lambda}\mathcal{F}^{\nu\lambda}_a + \frac{1}{4}\delta^\nu_\mu \mathcal{F}^a_{\sigma\lambda}\mathcal{F}^{\sigma\lambda}_a\right)\ln\frac{e|\mathcal{F}^a_{\alpha\beta}\mathcal{F}^{\alpha\beta}_a|}{\sqrt{-g}\,\lambda^4} - \frac{1}{4}\delta^\nu_\mu\,\mathcal{F}^a_{\sigma\lambda}\mathcal{F}^{\sigma\lambda}_a\right],$$

$$\left(\frac{\delta^{ab}}{\sqrt{-g}}\overrightarrow{\partial}_\nu\sqrt{-g} - f^{abc}\mathcal{A}^c_\nu\right)\left(\frac{\mathcal{F}^{\mu\nu}_b}{\sqrt{-g}}\ln\frac{e|\mathcal{F}^a_{\alpha\beta}\mathcal{F}^{\alpha\beta}_a|}{\sqrt{-g}\,\lambda^4}\right) = 0$$

temporal (Hamilton) gauge

$$A^a_0 = 0 \qquad e^a_i A^a_k \equiv A_{ik} \qquad e^a_i e^a_k = \delta_{ik} \qquad e^a_i e^b_i = \delta_{ab}$$

due to local SU(2) ~ SO(3) isomorphism

$$A_{ik}(t,\vec{x}) = \delta_{ik}U(t) + \widetilde{A}_{ik}(t,\vec{x})$$

The resulting equations:

$$\frac{6}{\varkappa}\frac{a''}{a^3} = 4\bar{\epsilon} + T^{\mu,\mathrm{U}}_\mu\,, \qquad T^{\mu,\mathrm{U}}_\mu = \frac{3b}{16\pi^2 a^4}\left[(U')^2 - \frac{1}{4}U^4\right],$$

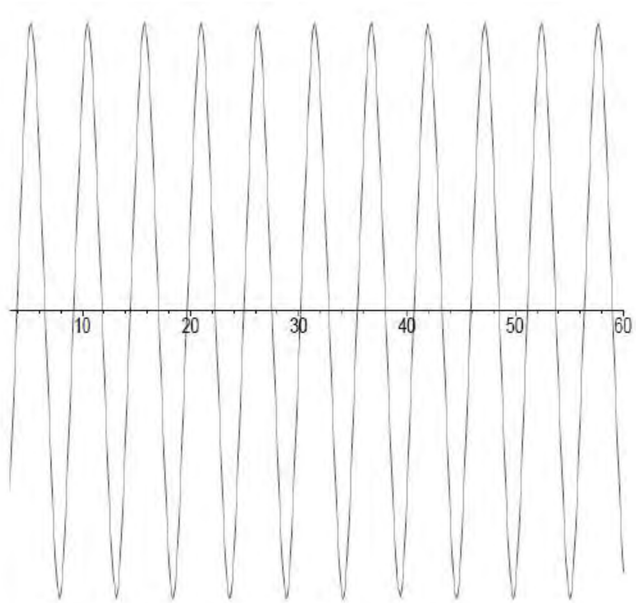
$$\frac{\partial}{\partial\eta}\left(U'\,\ln\frac{6e\big|(U')^2 - \frac{1}{4}U^4\big|}{a^4\lambda^4}\right) + \frac{1}{2}U^3\,\ln\frac{6e\big|(U')^2 - \frac{1}{4}U^4\big|}{a^4\lambda^4} = 0$$

Gluon condensate on non-stationary (FLRW) background

Classical YM condensate

$$Q \equiv \frac{32}{11}\pi^2 e(\xi\Lambda_{\rm QCD})^{-4} T^\mu_\mu[U]$$
$$= 6e\left[(U')^2 - \frac{1}{4}U^4\right] a^{-4}(\xi\Lambda_{\rm QCD})^{-4}$$

Quantum YM vacuum

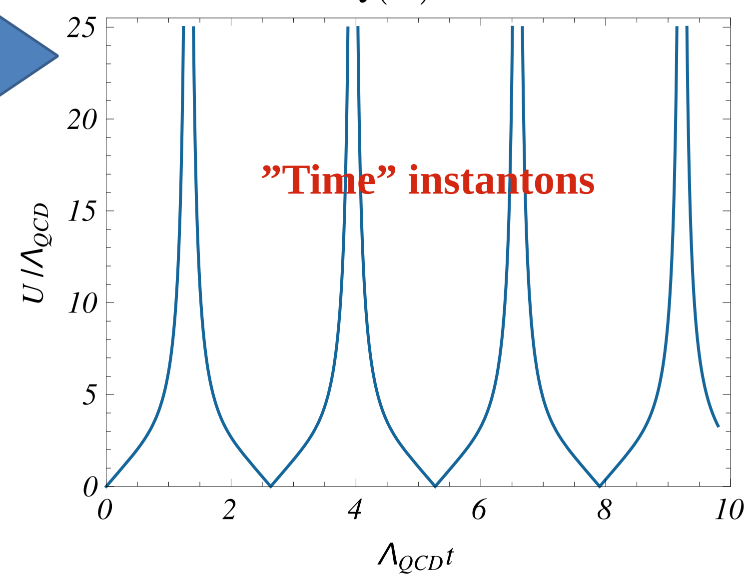


Exact partial solution:

$$|Q| = 1$$

Quantum corrections

$Q(U) = 1$



"Radiation" medium

$$\epsilon_{\rm YM} \propto 1/a^4$$

Unstable solution!

QCD vacuum:
a ferromagnetic undergoing spontaneous magnetisation (Pagels&Tomboulis)

Asymptotic (attractor) solution

$$\epsilon_{\rm CE} \to +{\rm const} \quad t \to \infty$$

Stable solution!

- **In fact, both chromoelectric and chromomagnetic condensates are stable on non-stationary (FLRW) background of expanding Universe**

"Mirror" symmetry of the ground state

In a vicinity of the ground state, the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{\mathcal{J}}{4\bar{g}^2} \qquad \mathcal{J} \simeq \mathcal{J}^*$$

is invariant under

$$\mathbb{Z}_2: \qquad \mathcal{J}^* \longleftrightarrow -\mathcal{J}^*, \quad \bar{g}^2(\mathcal{J}^*) \longleftrightarrow -\bar{g}^2(\mathcal{J}^*), \quad \beta(\bar{g}^2_*) \longleftrightarrow -\beta(\bar{g}^2_*)$$

For pure gluodynamics at one-loop: $\beta_{(1)} = -\frac{bN}{48\pi^2}\,\bar{g}^2_{(1)} \qquad b = 11$

$$\alpha_s = \frac{\bar{g}^2}{4\pi} \qquad \alpha_{\rm s}(\mu^2) = \frac{\alpha_{\rm s}(\mu_0^2)}{1 + \beta_0\,\alpha_{\rm s}(\mu_0^2)\ln(\mu^2/\mu_0^2)} \qquad \mu^2 \equiv \sqrt{|\mathcal{J}|}$$

Choosing the ground state value of the condensate $\mu_0^2 \equiv \sqrt{|\mathcal{J}^*|}$ **as the physical scale**

we observe that the mirror symmetry, indeed, holds provided

$$\mathcal{J} \simeq \mathcal{J}^* \qquad \alpha_s(\mu_0^2) \longleftrightarrow -\alpha_s(\mu_0^2)$$

i.e. in the ground state only!

Heterogenous quantum ground state: two-scale vacuum

The running coupling at one-loop

$$\bar{g}_1^2(\mathcal{J}) = \frac{\bar{g}_1^2(\mu_0^4)}{1+\frac{bN}{96\pi^2}\bar{g}_1^2(\mu_0^4)\ln(|\mathcal{J}|/\mu_0^4)} = \frac{96\pi^2}{bN\ln(|\mathcal{J}|/\lambda_\pm^4)}$$

$$\mathcal{L}_{\rm eff}^{(1)} = \frac{bN}{384\pi^2}\mathcal{J}\ln\left(\frac{|\mathcal{J}|}{\lambda_\pm^4}\right)$$

with two energy scales

$$\lambda_\pm^4 \equiv |\mathcal{J}^*|\exp\left[\mp\frac{96\pi^2}{bN|\bar{g}_1^2(\mathcal{J}^*)|}\right] \qquad |\mathcal{J}^*| = \lambda_+^2\lambda_-^2$$

CE vacuum: $\beta(\bar{g}_*^2) = 2$

e.o.m. is automatically satisfied!

Trace anomaly: $T^\mu_{\mu,{\rm CE}} = -\frac{1}{\bar{g}_*^2}\mathcal{J}^*$

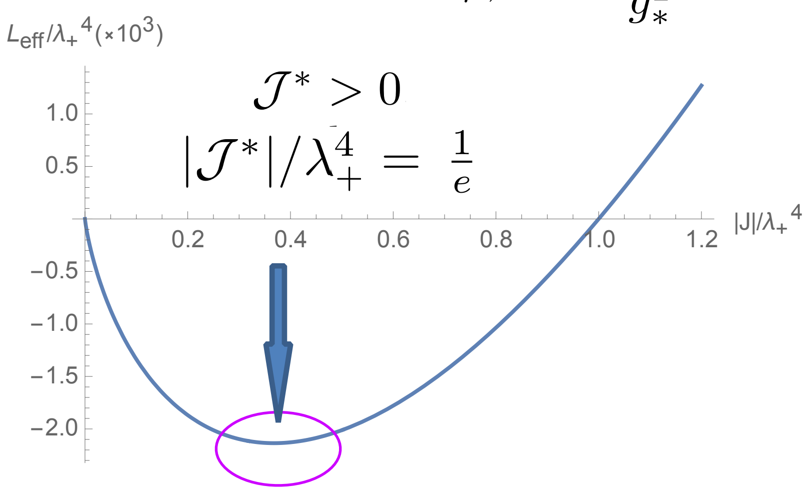
Mirror symmetry

CM vacuum: $\beta(\bar{g}_*^2) = -2$

Reduces to the standard YM e.o.m. discussed in e.g. in instanton theory

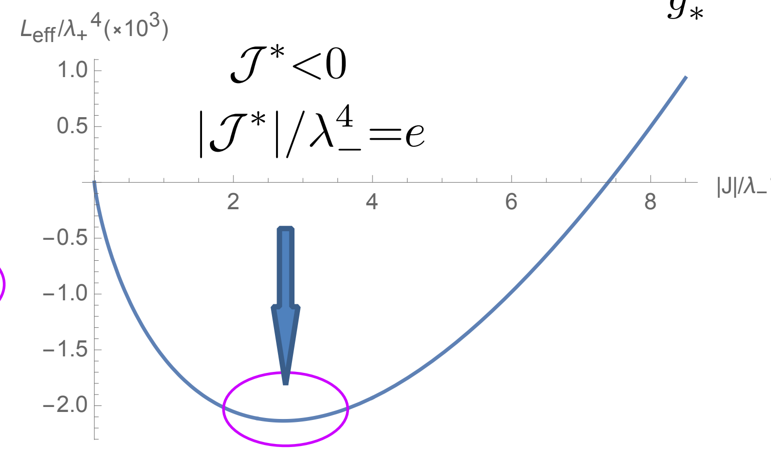
$$\overrightarrow{\mathcal{D}}_\nu^{ab}\left[\frac{\mathcal{F}_b^{\mu\nu}}{\bar{g}^2}\right] = 0, \quad \bar{g}^2 \simeq \bar{g}_*^2$$

Trace anomaly: $T^\mu_{\mu,{\rm CM}} = +\frac{1}{\bar{g}_*^2}\mathcal{J}^*$



One-loop:

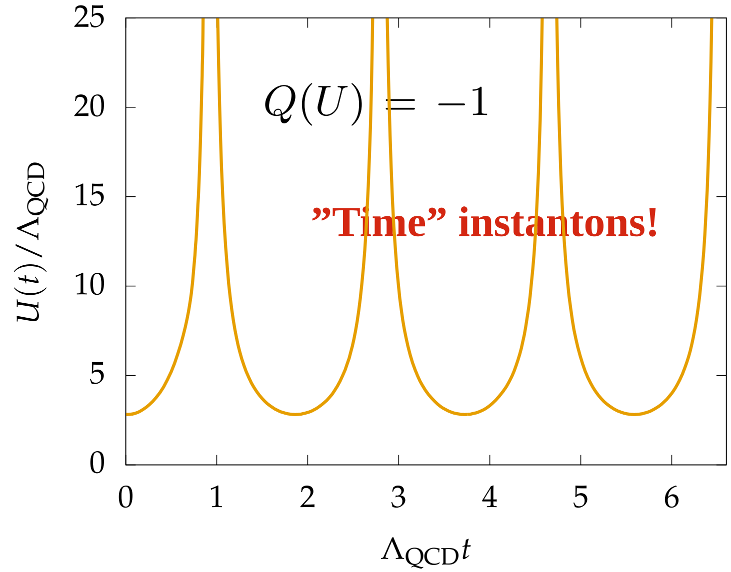
$$\lambda_+^2/\lambda_-^2 = e$$



Cosmological CE attractor

Cosmological CM attractor

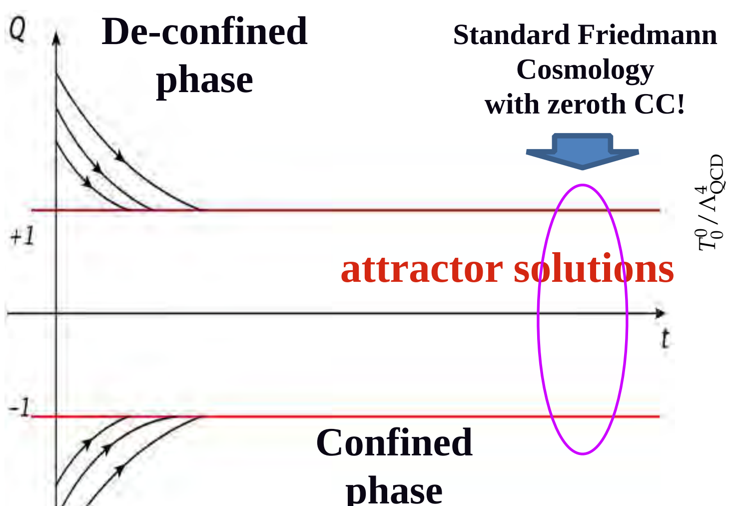
Infrared restoration of conformal invariance



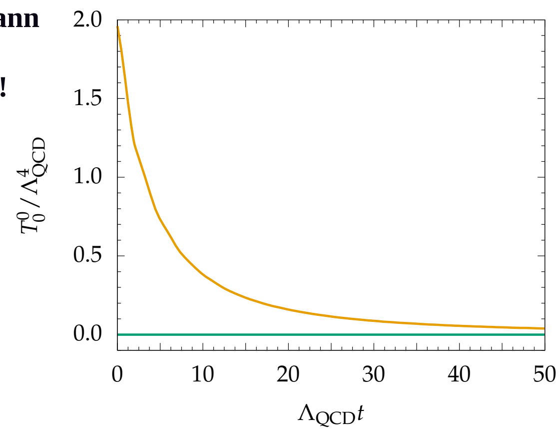
$$\epsilon_{\rm vac} \equiv \frac{1}{4}\langle T^{\mu}_{\mu}\rangle_{\rm vac} = \mp\mathcal{L}_{\rm eff}(\mathcal{J}^*)$$

$$\epsilon^{\rm CE}_{\rm vac}\Big|_{\mathcal{J}^*>0} + \epsilon^{\rm CM}_{\rm vac}\Big|_{\mathcal{J}^*<0} \equiv 0$$

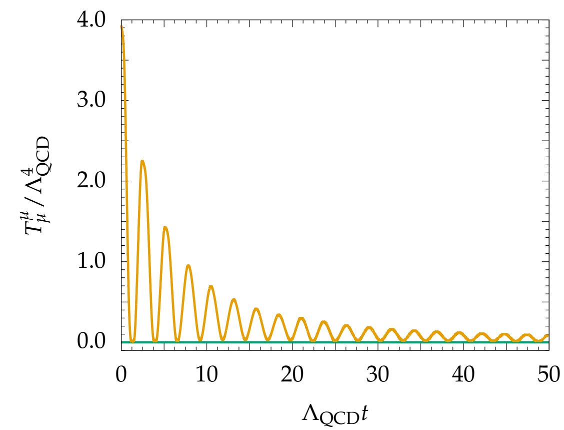
Exact compensation of CM and CE vacua as soon as the cosmological attractor is achieved!



CE energy density



CE EMT trace

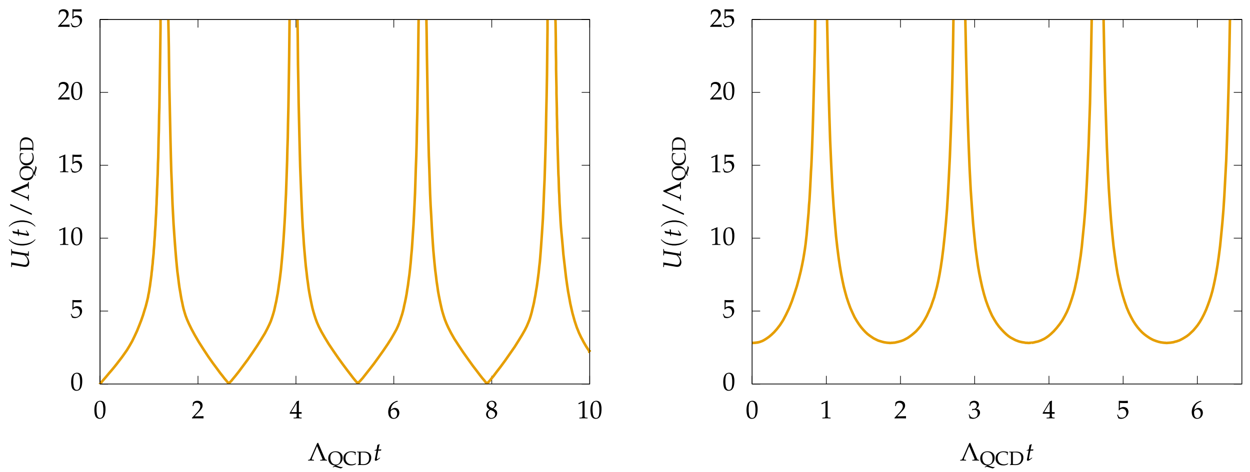


System with very unusual dynamical properties!

Addazi, A.; Marcianò, A.; Pasechnik, R.; Prokhorov, G. Mirror Symmetry of quantum Yang-Mills vacua and cosmological implications. *Eur. Phys. J. C* **2019**, *79*, 251, [arXiv:hep-th/1804.09826].

QCD “time crystal”

- **The emergence of spikes localised in time at a characteristic QCD time lapse** $\Delta t \simeq \Lambda_{\rm QCD}^{-1}$ **and extended in 3-space dimensions reveals the presence of an order state of space-like soliton/domain wall solutions (chronons)**



- **A time-ordered classical solution spontaneously breaking time translational invariance down to a discrete time shift symmetry** $T_n \,:\, t \to t + n\Lambda_{\rm QCD}^{-1}$ **is known as the “time crystal” first discovered by Wilczek in the context of superconductors and superfluids in** F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012)

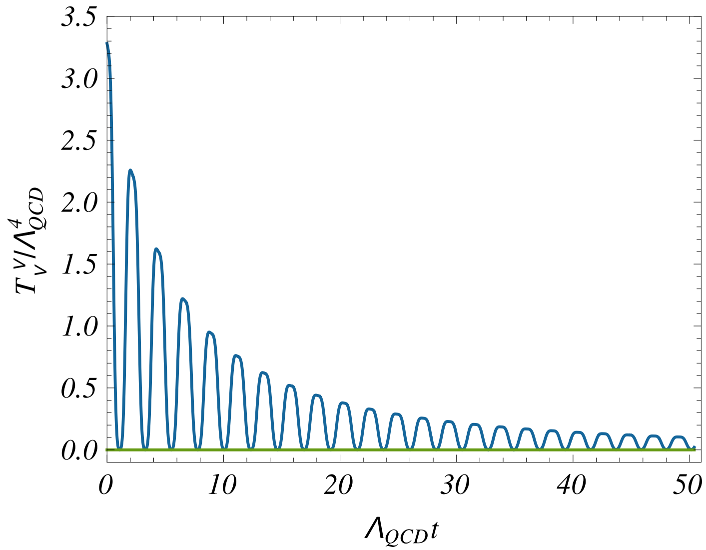
- **The kink (anti-kink) profile localised in time corresponds to a space-like domain wall**

$$U(\eta) \simeq \frac{v}{\sqrt{2}} \tanh^{-1}[\frac{v}{\sqrt{2}}(\eta - \eta_0)] \qquad v \simeq \Lambda_{\rm QCD}$$

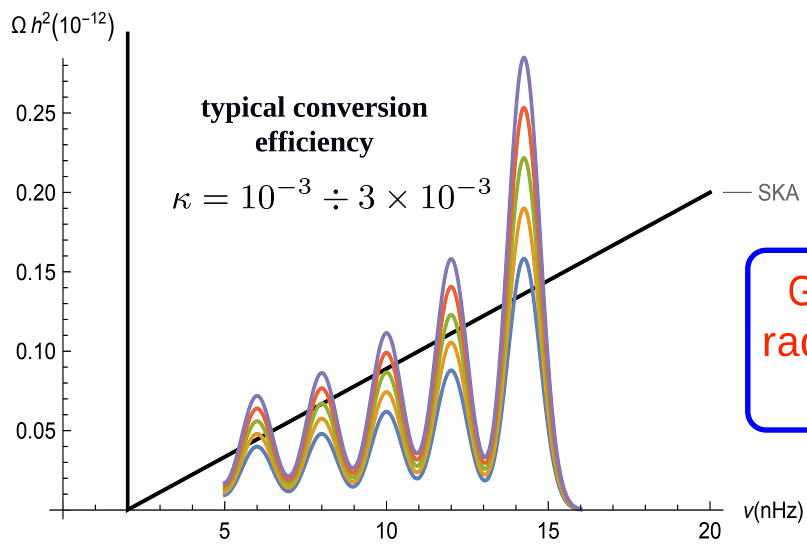
- **As the T-invariance is broken, a massless moduli field** $\eta_0(x,y,z)$ **localised on the domain wall world sheet** x,y,z **arises and corresponds to a Nambu-Goldstone boson**

Gravitational radio-waves from QCD relaxation

The pressure kinks get efficiently transmitted to the primordial plasma inducing shock sound waves and turbulence in it



A. Addazi, A. Marcianò, RP, CPC 43 (2019) 6, 065101 arXiv: 1812.07376



GW signal lies at the radio-astronomy pulsar timing scale

$10^{-9} \div 10^{-8}$ Hz

SKA should be able to probe QCD relaxation through detection of primordial GW radio waves

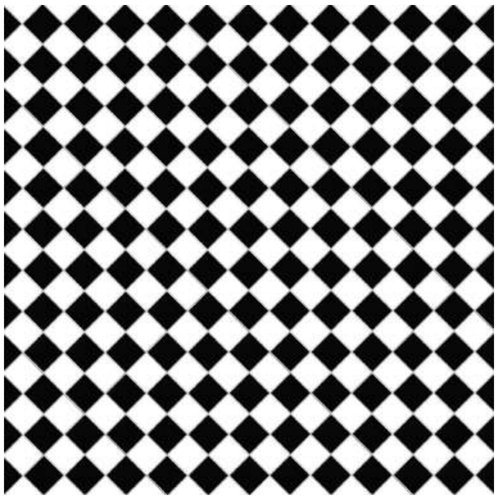
Breaking of Mirror symmetry and Cosmological Constant

Exact mirror symmetry of the YM ground state

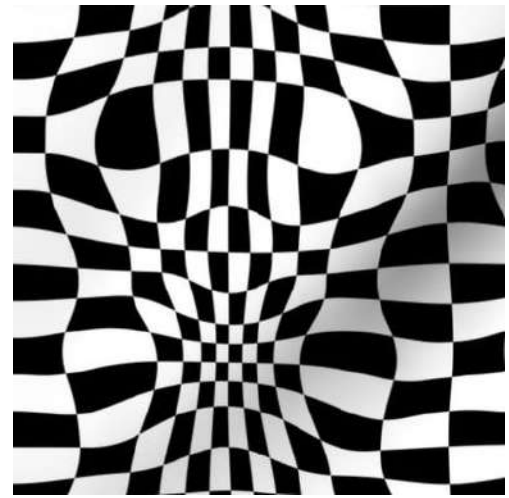
Exact conformal invariance at macroscopic scales

Quantum Gravity in the quasi classical approximation

Mirror symmetry and conformal invariance breakdown at cosmological scales



Gravity



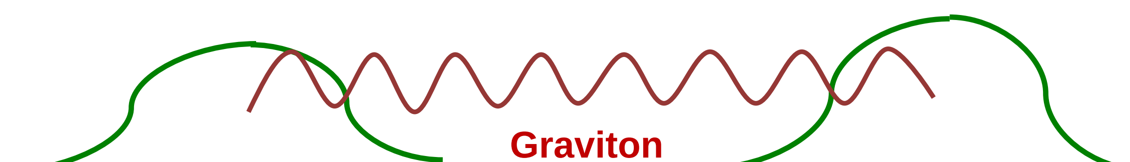
Pasechnik, R.; Beylin, V.; Vereshkov, G. Dark Energy from graviton-mediated interactions in the QCD vacuum. *JCAP* **2013**, *06*, 011, [arXiv:gr-qc/1302.6456].

Ya. Zeldovich (1967):

$$\Lambda \sim G m^6$$

A. Sakharov (1967):

extra terms describing an effect of graviton exchanges between *identical particles (bosons occupying the same quantum state) should appear in the right hand side of Einstein equations (averaged over quantum ensemble)*



$$\varepsilon_\Lambda \sim G \Lambda_{\rm QCD}^6$$

Quasiclassical (semiquantum) gravity

Zeldovich-Sakharov scenario can be realized in the following consistent way:

Action $$S = \int L d^4x, \quad L = -\frac{1}{2\varkappa}\sqrt{-\hat{g}}\hat{g}^{ik}\hat{R}_{ik} + L\left(\hat{g}^{ik}, \chi_A\right)$$

Metric operator $\hat{g}^{ik}$

Macroscopic geometry (c-number part) g^{ik}

Quantum graviton field Φ_i^k

Independent variations over classical and quantum fields:

$$\langle 0|\Phi_i^k|0\rangle = 0$$

$$\delta \int L d^4x = -\frac{1}{2}\int d^4x \left(\sqrt{-g}\,\delta g^{ik}\hat{G}_{ik}\right)_{\Phi_i^k=\mathrm{const}}$$
$$= -\frac{1}{2}\int d^4x \left(\sqrt{-g}\,\delta\Phi^{ik}\hat{G}_{ik}\right)_{g^{ik}=\mathrm{const}}$$

Heisenberg state vector containing info about initial states of all fields exists!

Averaging over initial states

same operator eqns:

$$\hat{G}_i^k = \frac{1}{2}\left(\delta_l^k\delta_i^m + g^{km}g_{il}\right)\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{E}_m^l = 0,$$

$$\hat{E}_m^l = \frac{1}{\varkappa}\left(\hat{g}^{lp}\hat{R}_{pm} - \frac{1}{2}\delta_m^l\hat{g}^{pq}\hat{R}_{pq}\right) - \hat{g}^{lp}\hat{T}_{pm}\left(\hat{g}^{ik}, \chi_A\right)$$

e.o.m. for macroscopic geometry

$$\langle 0|\hat{G}_i^k|0\rangle = 0$$

e.o.m. for graviton field

$$\hat{G}_i^k - \langle 0|\hat{G}_i^k|0\rangle = 0$$

Gluodynamics with vacuum anomaly

Let us now include non-perturbative gluon and quark fields fluctuations into quasiclassical gravity theory!

We need energy-momentum tensor for NPT vacuum fluctuations with conformal anomalies!

Classical conformal symmetry (under rescalings of the background metric and simultaneously fields) is broken by quantum gravity effects!

Basic recipe:

Chromodynamical coupling has to be considered as an operator depending on operators of quantum fields through RG evolution equation

$$\mathcal{A}_i^a = g_s A_i^a$$

$$\mathcal{F}_{ik}^a = \partial_i \mathcal{A}_k^a - \partial_k \mathcal{A}_i^a + f^{abc}\mathcal{A}_i^b \mathcal{A}_k^c$$

stress tensor operator

$$2J\frac{dg_s^2(J)}{dJ} = g_s^2(J)\beta[g_s^2(J)]$$

operator RG equation

$$J = \mathcal{F}_{ik}^a \mathcal{F}_a^{ik}$$

Invariant operator of least dimension

Operator gluodynamics with conformal anomaly:

$$L_{eff} = -\frac{1}{4g_s^2(J)}\mathcal{F}_{ik}^a \mathcal{F}_a^{ik}$$

can now be incorporated into quasiclassical gravity! (after covariant generalization)

$$\left\{\begin{aligned} \hat{T}_{i(g)}^k &= \frac{1}{g_s^2(J)}\left(-\mathcal{F}_{il}^a \mathcal{F}_a^{kl} + \frac{1}{4}\delta_i^k \mathcal{F}_{ml}^a \mathcal{F}_a^{ml} + \frac{\beta[g_s^2(J)]}{2}\mathcal{F}_{il}^a \mathcal{F}_a^{kl}\right) \\ &D_k^{ab}\left\{g_s^{-2}(J)\left(1 - \frac{\beta[g_s^2(J)]}{2}\right)\mathcal{F}_b^{ik}\right\} = 0, \\ &D_k^{ab} = \delta^{ab}\partial_k - f^{abc}\mathcal{A}_k^c. \end{aligned}\right.$$

Λ-term calculation

We start from the Einstein equations for macroscopic geometry:

$$\frac{1}{\varkappa}\left(R_i^k - \frac{1}{2}\delta_i^k R\right) = \langle 0|\hat{T}_i^k|0\rangle \qquad \hat{T}_i^k = \hat{T}_{i(G)}^k + \frac{1}{2}\left(\delta_l^k\delta_i^m + g^{km}g_{il}\right)\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{lp}\hat{T}_{pm}\left(\hat{g}^{ik},\chi_A\right)$$

Trace: $R + 4\varkappa\Lambda = 0$

$$\Lambda = -\frac{b_{eff}}{32}\langle 0|\frac{\alpha_s}{\pi}\left(\frac{\hat{g}}{g}\right)^{1/2}\hat{g}^{il}\hat{g}^{km}\hat{F}_{ik}^a\hat{F}_{lm}^a|0\rangle + \frac{1}{4}\langle 0|\hat{T}_{(G)}|0\rangle$$

Stress tensor in Riemann space is found from YM eqs:

$$\left(\delta^{ab}\frac{\partial}{\partial x^k} - g_s f^{abc}\hat{A}_k^c\right)\sqrt{-\hat{g}}\hat{g}^{il}\hat{g}^{km}\hat{F}_{lm}^b = 0$$

$$\hat{F}_{ik}^a = F_{ik}^a + \frac{1}{2}\psi F_{ik}^a - \psi_i^l F_{lk}^a - \psi_k^l F_{il}^a + O(\alpha_s G)$$

induce interactions of YM field with metric fluctuations

Equation for gravitons turns into:

$$\psi_{i,l}^{k,l} - \psi_{i,l}^{l,k} - \psi_{l,i}^{k,l} + \delta_i^k\psi_{l,m}^{m,l} = \frac{\varkappa b_{eff}\alpha_s}{\pi}\left(-F_{il}^a F_a^{kl} + \frac{1}{4}\delta_i^k F_{ml}^a F_a^{ml}\right)\ln\frac{L_g^{-1}}{\Lambda_{QCD}}$$

After exact cancellation of unperturbed part of EMT tensor we get:

$$\Lambda = -\frac{b_{eff}}{16}\ln\frac{L_g^{-1}}{e\Lambda_{QCD}}\langle 0|\frac{\alpha_s}{\pi}F_{il}^a F_a^{kl}\left(\psi_k^i - \frac{1}{4}\delta_k^i\psi\right)|0\rangle$$

linear in graviton field!

Λ-term calculation

Fock gauge: $\psi^k_{i;\,k} = 0$

Metric fluctuations are induced by QCD vacuum fluctuations!

Exact solution of graviton equation:

$$\psi_i^k(x) = \varkappa b_{eff} \ln \frac{L_g^{-1}}{\Lambda_{QCD}} \int d^4x' \underbrace{\mathcal{G}(x-x')}_{} \times \left(\frac{\alpha_s}{\pi} F^a_{il}(x') F^{kl}_a(x') - \delta_i^k \frac{\alpha_s}{4\pi} F^a_{ml}(x') F^{ml}_a(x') \right)$$

Green function: $\mathcal{G}_{;\,l}^{;\,l} = -\delta(x-x')$

After explicit calculation of averages, we get

$$\Lambda = -\pi G \langle 0| : \frac{\alpha_s}{\pi} F^a_{ik} F^{ik}_a : |0\rangle^2 \times \left(\frac{b_{eff}}{8}\right)^2 \ln \frac{L_g^{-1}}{e\Lambda_{QCD}} \ln \frac{L_g^{-1}}{\Lambda_{QCD}} \int d^4y \mathcal{G}(y) D^2(y) =$$
$$= (1 \pm 0.5) \times 10^{-29} \Delta \ \mathrm{MeV}^4.$$

where

$$\langle 0| \frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x') |0\rangle = \langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(0) F^{ik}_a(0) : |0\rangle D(x-x'),$$

$$\Delta = -\frac{1}{L_g^2} \int d^4y \mathcal{G}(y) D^2(y) \qquad D(x-x') = D_{top}(x-x') - D_h(x-x'), \qquad D(0) = 0.$$

must be established in a dynamical theory of NPT QCD vacuum!

It is expected to be generated by chiral symmetry breaking

In terms of known NPT QCD parameters

$$1/L_{top} \sim 1/L_h \sim 1/L_g,$$

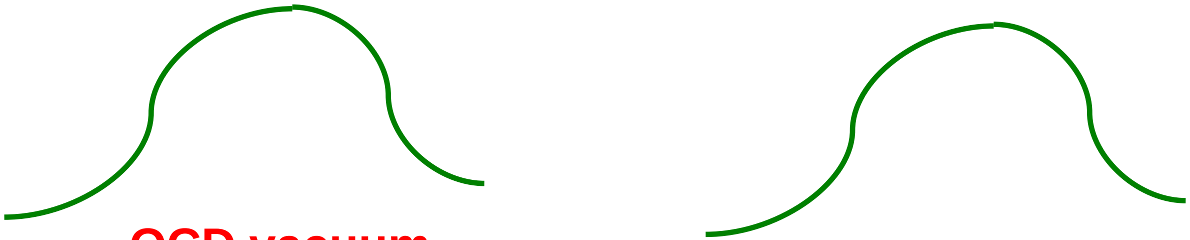
$$|1/L_{top} - 1/L_h| \sim m_u + m_d + m_s$$

$$\Delta = k \cdot \frac{(m_u + m_d + m_s)^2 L_g^2}{(2\pi)^4} \sim 3 \cdot 10^{-6}$$

!!!

Observable Λ-term from QCD?

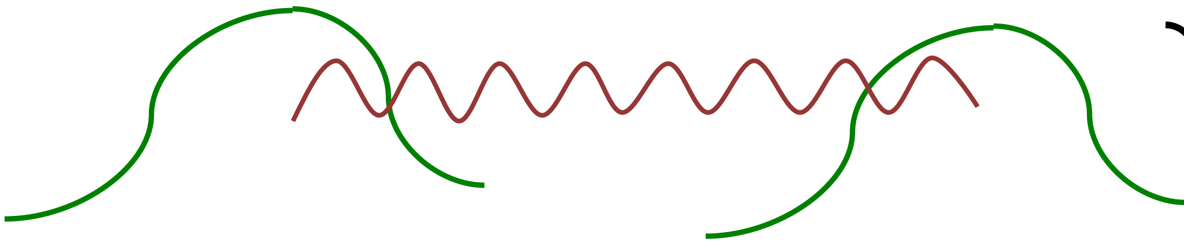
“zeroth” order in QG



QCD vacuum

Must cancel exactly due to QCD confinement
(only a new symmetry of the ground state can do that!)

“first” order in QG



virtual (strong) NPT fluctuations of quark and gluon fields dynamically induce metric fluctuations (gravitons)!

cancel NOT exactly!
(due the chiral SB in QCD)

Observable Λ-term!

$$\varepsilon_\Lambda \sim G\Lambda^6_{\rm QCD}$$

$$\Lambda = \frac{m_\pi^6}{(2\pi)^4 M_{Pl}^2} \simeq 2.98 \times 10^{-35}\ \mathrm{MeV}^4 \qquad m_\pi \simeq 138\ \mathrm{MeV}$$

$$\Lambda_{\rm exp} = (3.0 \pm 0.7) \times 10^{-35}\ \mathrm{MeV}^4$$

Only NPT QCD vacuum fluctuations coupled to Gravity at lowest (hadron) scales of Particle Physics gives rise to Λ-term if UV terms are canceled

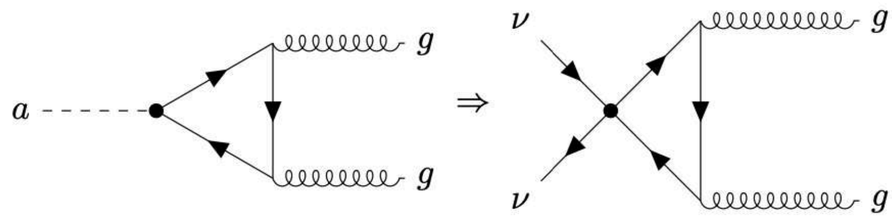
Composite QCD axion as a source of Dark Matter

A. Addazi, A. Marcianò, RP, K. Zeng, Phys.Dark Univ. 36 (2022) 101007; arXiv: 2106.03549

- **Ginzburg, Zharkov 1967**: neutrinos can be non-relativistically produced in a superfluid phase, related to the neutrino mass gap and Dark Energy

- We **postulate** that neutrinos have a Peccei-Quinn (PQ) scale suppressed **portal with the strongly-coupled QCD sector** through quantum anomalies:

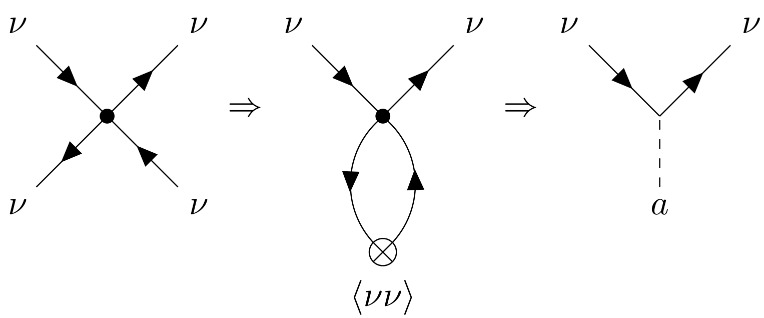
$$\frac{1}{f_{\rm PQ} f_a^2}(\nu^T \mathcal{C}^{-1}\gamma_5\nu)\, G\widetilde{G}$$



- The **QCD axion** is then identified with **the composite pair of neutrinos**:

$$\mathcal{L} = \partial\Phi^\dagger\partial\Phi - V(\Phi)\,, \quad V(\Phi) = (|\Phi|^2 - f_{\rm PQ}^2)^2\,, \quad \Phi = \rho\, e^{\imath \frac{a}{f_{\rm PQ}}}\,, \quad a = \frac{1}{f_a^2}\nu^T\mathcal{C}^{-1}\gamma_5\nu$$

- A four-fermion interaction responsible **for condensation of neutrino pairs and for generation of the neutrino mass scale in the SM**:



Neutrinos can be produced in pairs as cold during the QCD phase transition and condense as non-relativistic Cooper pairs via the misalignment mechanism

- **Neutrino condensate** can naturally account for **the right amount of Dark Matter**

Concluding remarks

- **Local loss of continuous time-translational invariance leads to "time crystal"-type configurations in the QCD vacuum**

- **Nielsen-Olsen proof of instability of CE condensate on a rigid Minkowski in NOT in contradiction with our picture: we consider YM evolution on a dynamical (FLRW) spacetime while equilibrium is achieved only asymptotically.**

- **A possible decay of CE condensate into an anisotropic vacuum after a cosmological relaxation time would be exponentially suppressed and is practically never realised**

- **Even starting from an initial non-zero energy-density, the evolution of localised 3-space "pockets" of the CE and CM condensates trigger a mutual screening, flowing towards a zero-energy density attractor and accompanying by a formation of the domain walls corresponding to an asymptotic restoration of the Z_2 (Mirror) symmetry and effectively protecting the "false" CE vacua pockets from further decay**

- **The vacua cancellation mechanism seems to naturally marry the existing confinement pictures related to a formation of a network of t'Hooft monopoles or chromovortices. In this approach, the scalar kink profile may correspond the J-invariant whose change may be related to the presence of monopole or vortex solutions localise inside the space-time domain walls. This implies the existence of space-time solitonic objects of a new type.**

Concluding remarks

- **Breaking of the Mirror symmetry by gravitational interactions induces non-vanishing leading order contribution to the QCD ground state energy compatible with the observed cosmological constant value that must be taken into account in any model of DE**

- **Pressure oscillations during the QCD relaxation epoch trigger multi-peaked primordial gravitational wave spectrum in the radio-frequency range that can be potentially probed by the SKA telescope**

- **Cold neutrino pairs can be produced during the QCD transition and condense into axions through a possible four-fermion neutrino interaction and a coupling to the QCD anomaly enabling neutrino mass gap and Dark Matter generation**