New Trends in Thermal Phases of QCD

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Prague

From the thermal QCD transition to the zero temperature QCD transition

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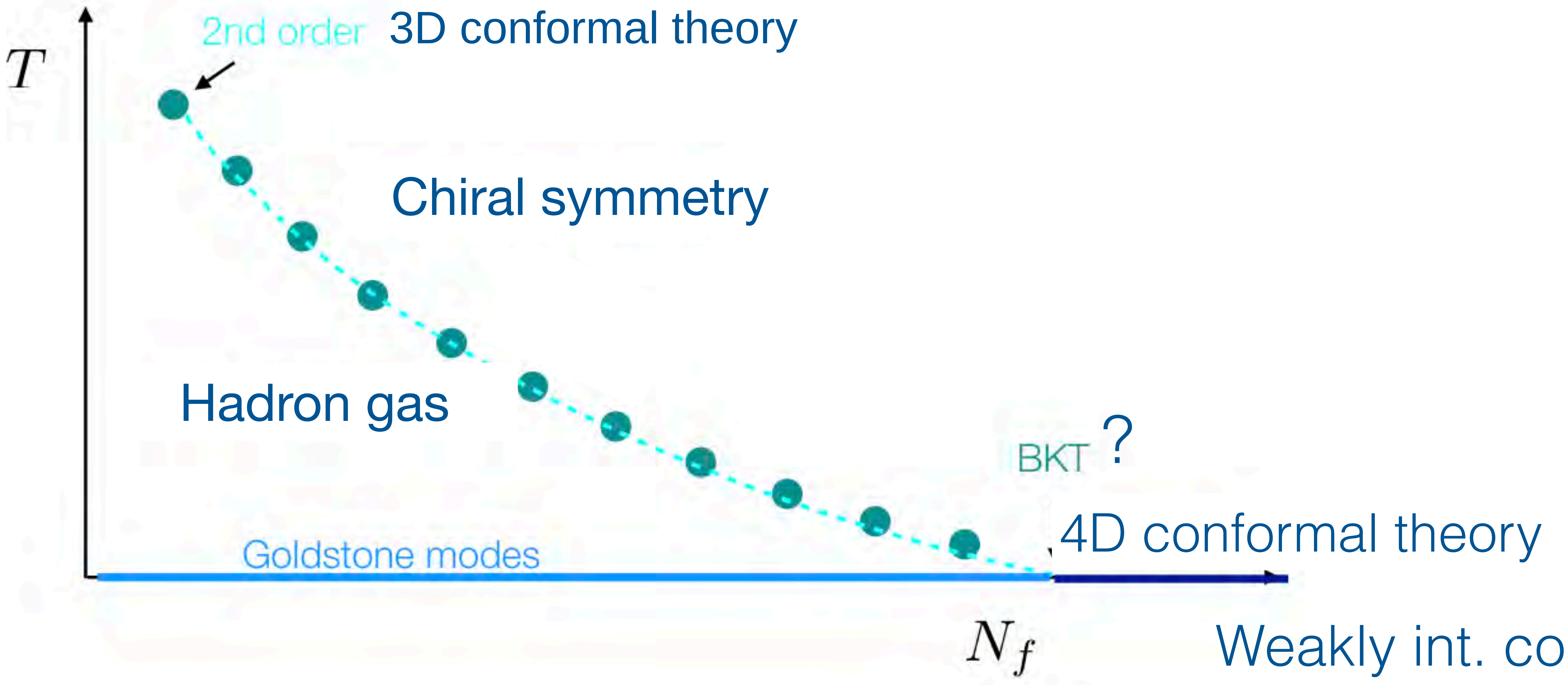
AdS/CFT



Weakly int. conformal

Strongly interacting conformal

.. prologue..

Euclidean finite temperature field theory

Imaginary time

and

Inverse
Temperature

pbc: bosons
abc: fermions

d-dimensional space

Continuous transition: diverging correlation length - dimensional reduction

Mode expansion and Decoupling

$$\phi(x,t) = \sum_{\omega_n = 2n\pi T} e^{i\omega_n t}\phi_n(x)$$

Bosons

$$\psi(x,t) = \sum_{\omega_n = (2n+1)\pi T} e^{i\omega_n t}\psi_n(x)$$

Fermions

In the expression for the Action

$$S(\phi,\psi) = \int_0^{1/T} dt \int d^d x \mathcal{L}(\phi,\psi)$$

the integral over time can then be traded with a sum over modes, and we reach the c
that a d+1 statistical field theory at $T > 0$ is equivalent to a d-dimensional theory with an
infinite number of fields.

When dimensional reduction is possible, only one boson (zero frequency) field survives

$$U(n)_L \times U(n)_R \cong SU(n) \times SU(n) \times U(1)_V \times U(1)_A$$

Spontaneously Broken, $(n^2 - 1)$ GB

Experimental Evidence

baryon number

Explicitely broken

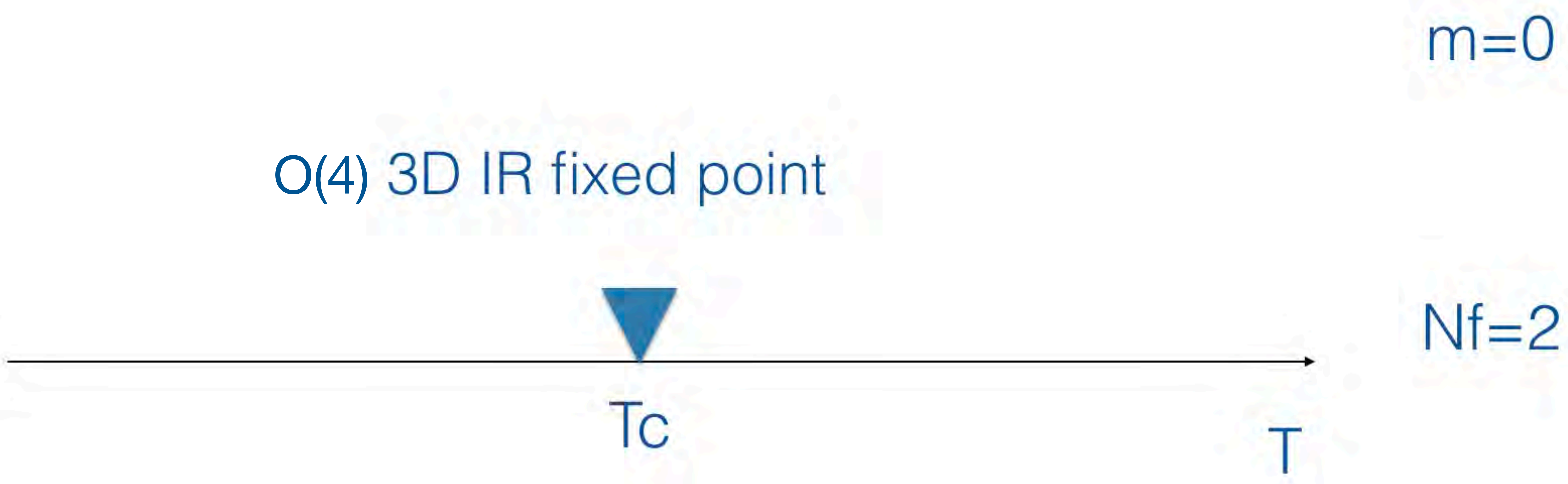
Universality class of the high T transition: theoretical (*lack of*) guidance

Parisen Toldin, Pelissetto, Vicari 2003

N_f	$\mathrm{U}(1)_A$ broken	$\mathrm{U}(1)_A$ restored
1	crossover or 1^{st}ord	$\mathrm{O}(2) \to \mathbb{Z}_2$ or 1^{st}ord
2	$\mathrm{O}(4) \to \mathrm{O}(3)$ or 1^{st}ord Pisarski, Wilczek 1984	$\mathrm{U}(2)_L \otimes \mathrm{U}(2)_R \to \mathrm{U}(2)_V$ or 1^{st}ord
≥ 3	~~1^{st}ord~~ ?	1^{st}ord

Challenged by Cuteri, Philipsen, Sciarra 2020-2022

Nf=2



The magnetic equation of State:

$$h = M^{\delta} f(t/M^{1/\beta})$$

$M \equiv \bar{\psi}\psi$, $h \equiv m_q$, $t \equiv T - T_c$, m_q is the quark mass and T_c is the critical temperature

Three strategies to identify the scaling behaviour:

- direct comparison with the Equation of State
- the study of the dependence of the pseudo-critical temperatures on the breaking field, also known as scaling of pseudo-critical temperatures
- definition of RG invariant quantities, which do not depend on the breaking field at the critical point.

Byproduct: critical temperature in the chiral limit

Significant source of scaling violations:

additive linear mass corrections to $\bar{\psi}\psi$

Analysis tool : scaling of the singular part of the Free Energy

Assumption: Free Energy = Singular + Regular

Mass

Ginzburg region
Non-trivial
criticality

$$t \equiv (T - Tc)/Tc$$

$$t < G$$

Correlation dominated

$\xi^2 G$ Large

Analysis tool : scaling of the singular part of the Free Energy

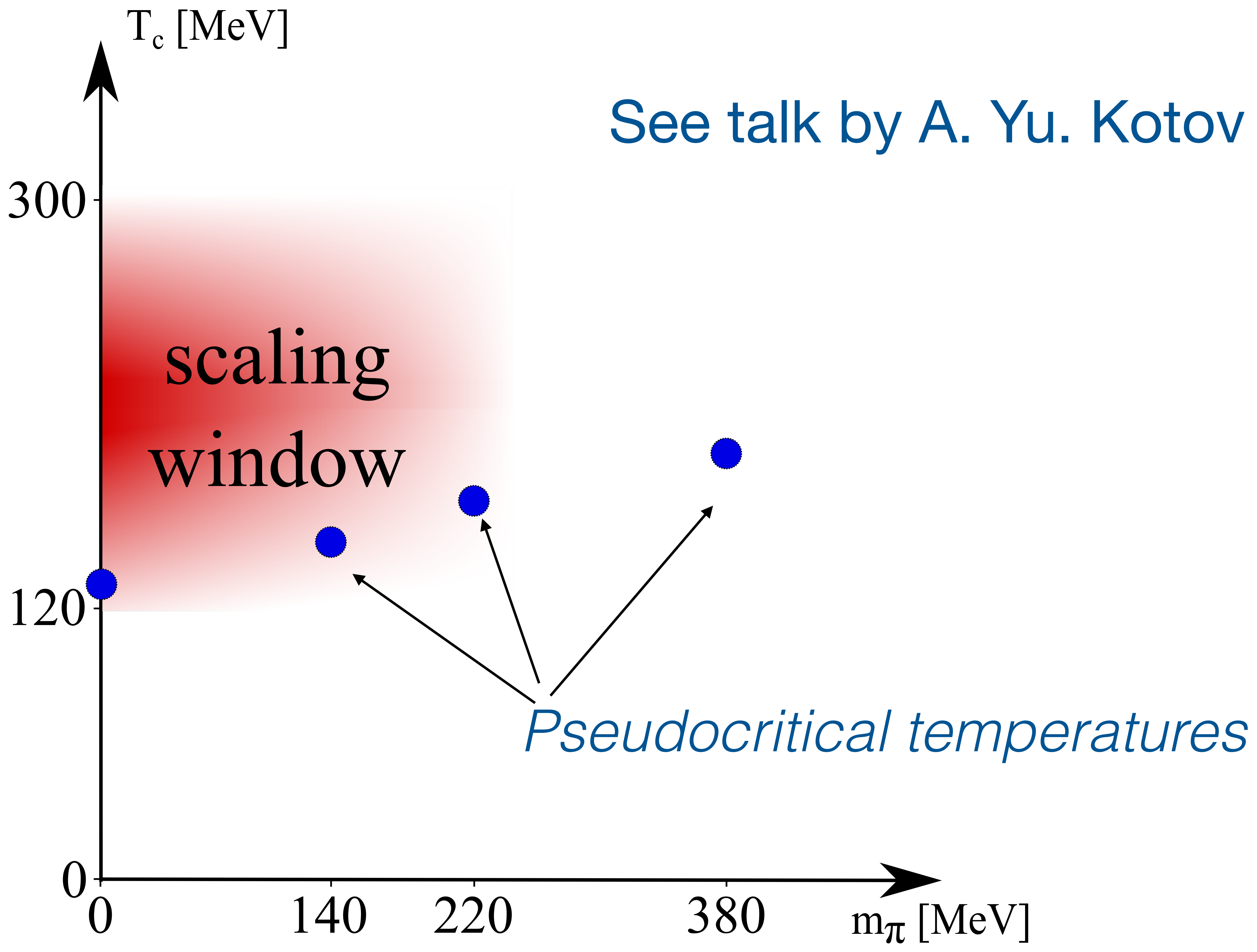
Assumption: Free Energy = Singular + Regular

Mass

$$|t| > G$$

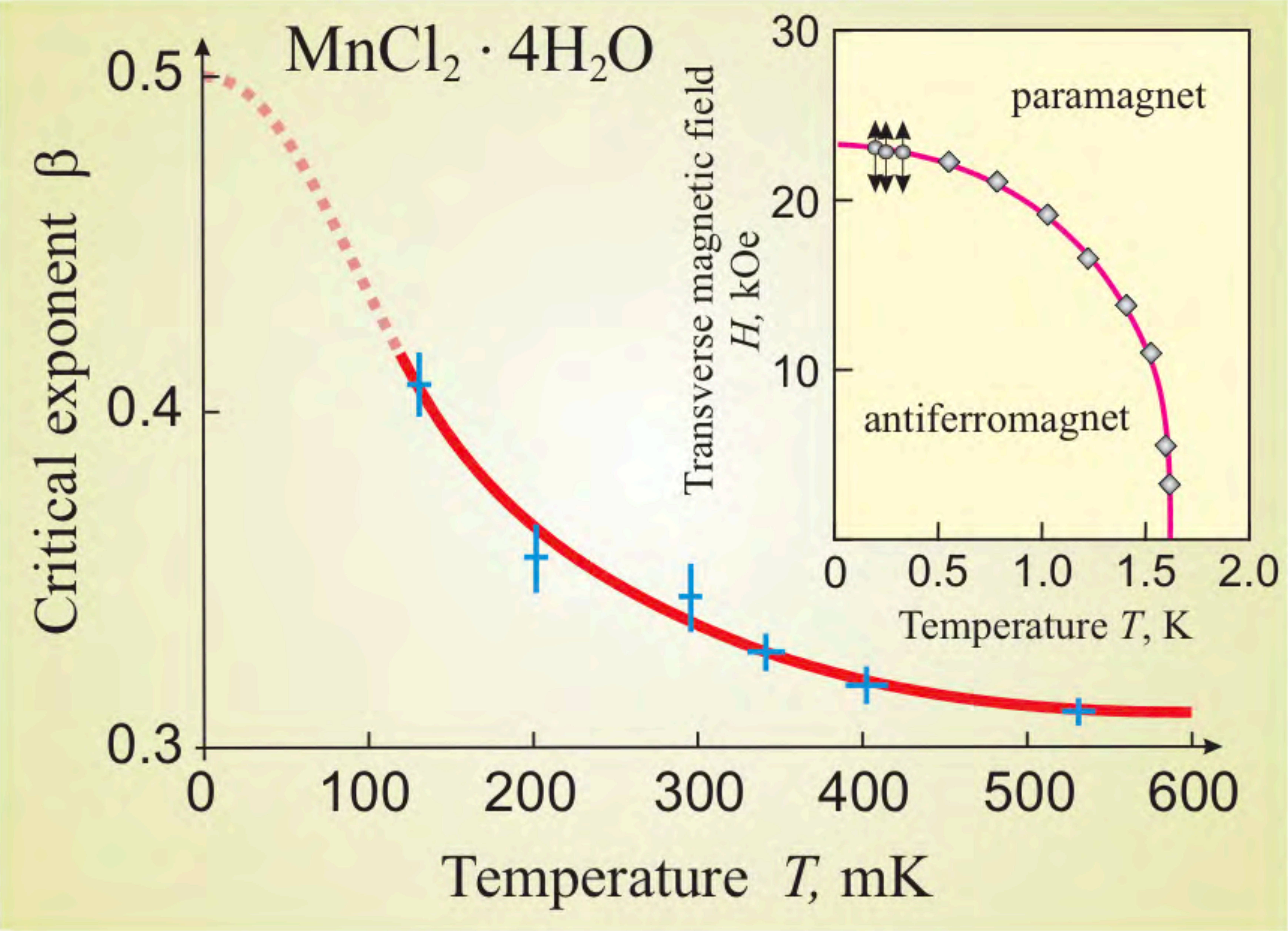
Mean field:
Interaction dominated,
$\xi^2 G$ Small

A sketch of the O(4) 3D scaling window for high T QCD



See talk by A. Yu. Kotov

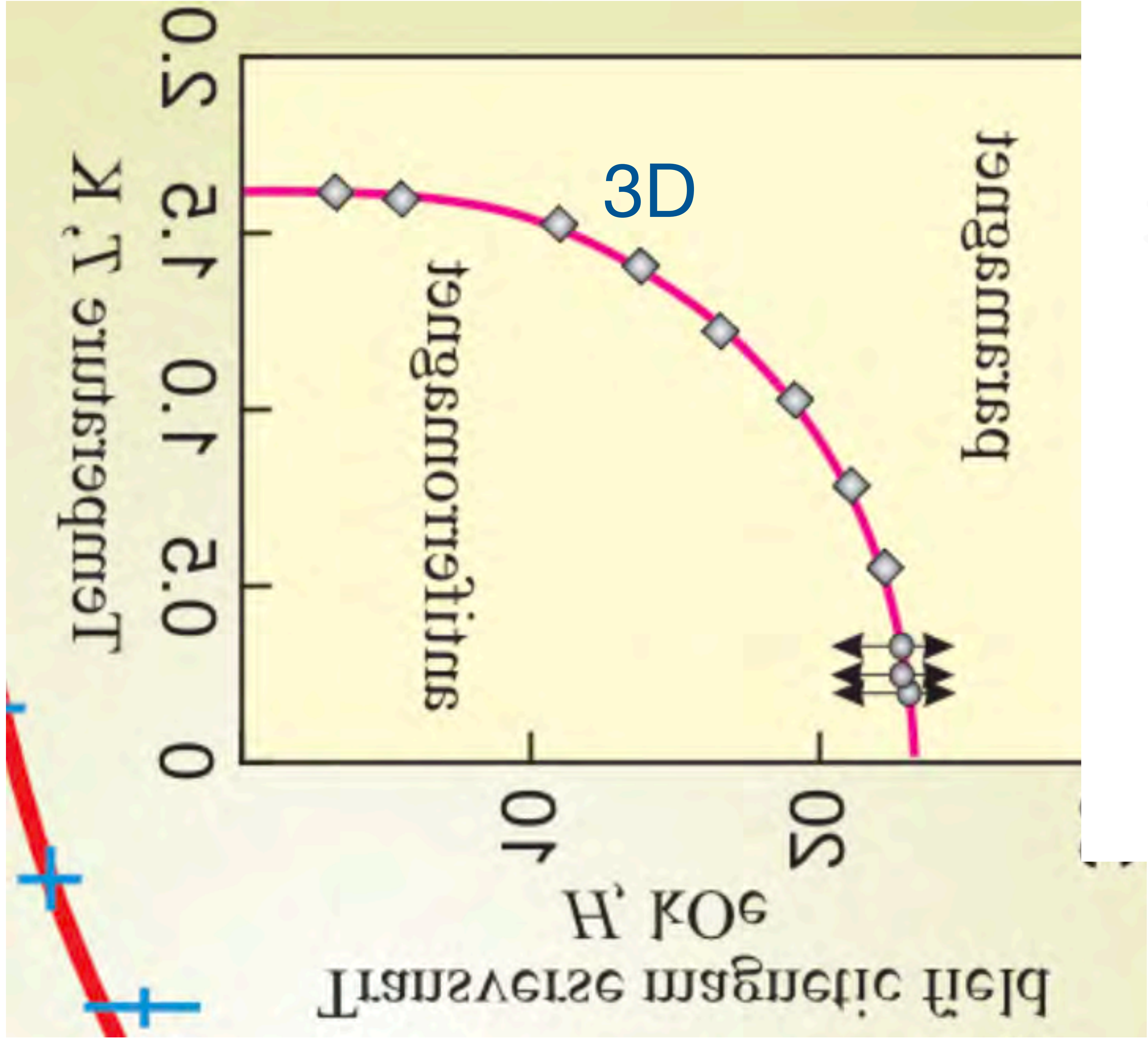
Kotov, Trunin, MpL, 2021



Vasin, Ryzhov, Vinokur
2015, Nature Sc.Rep.

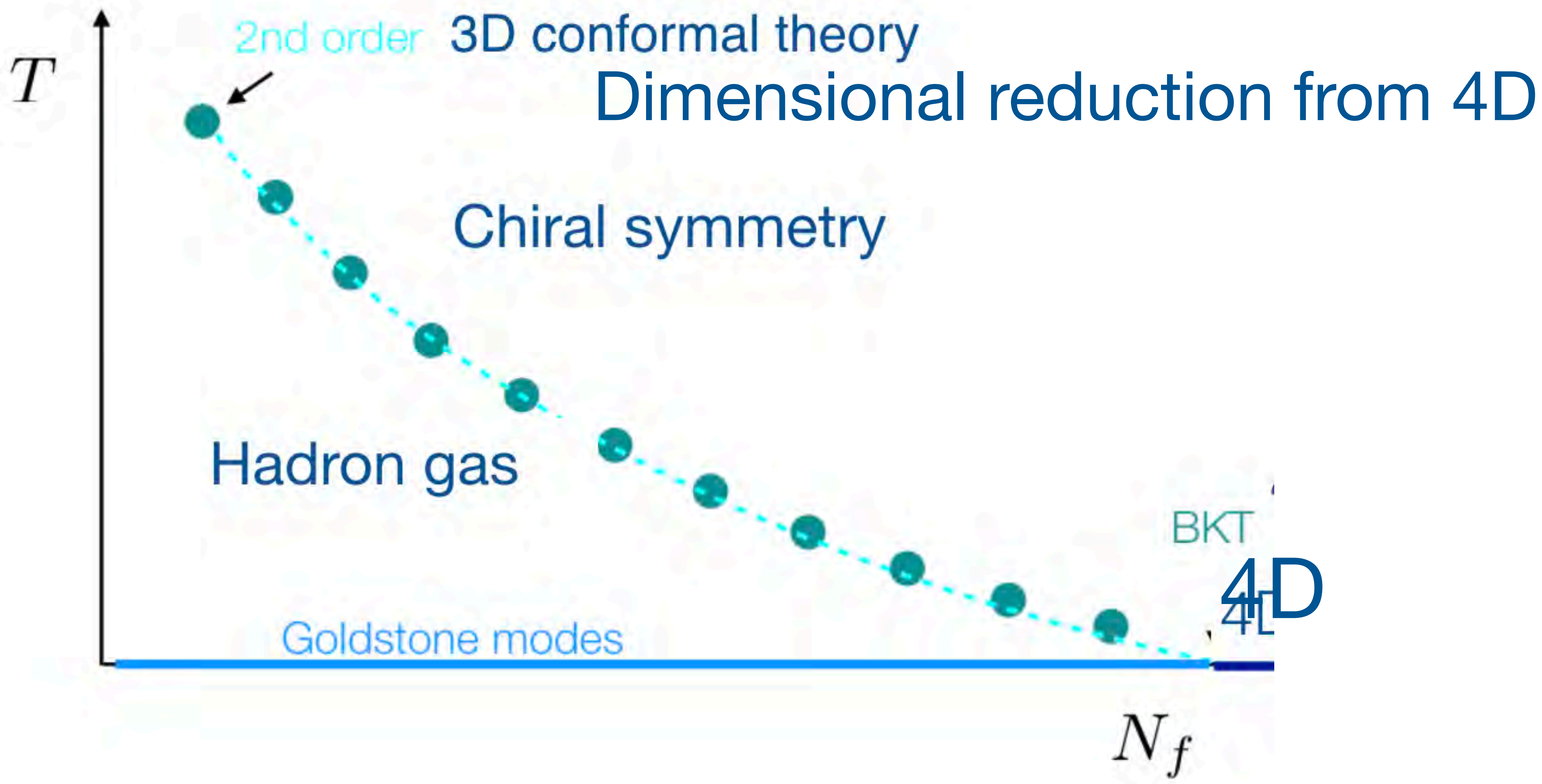
Continuous evolution of critical exponents along the critical line

T



Magnetic field

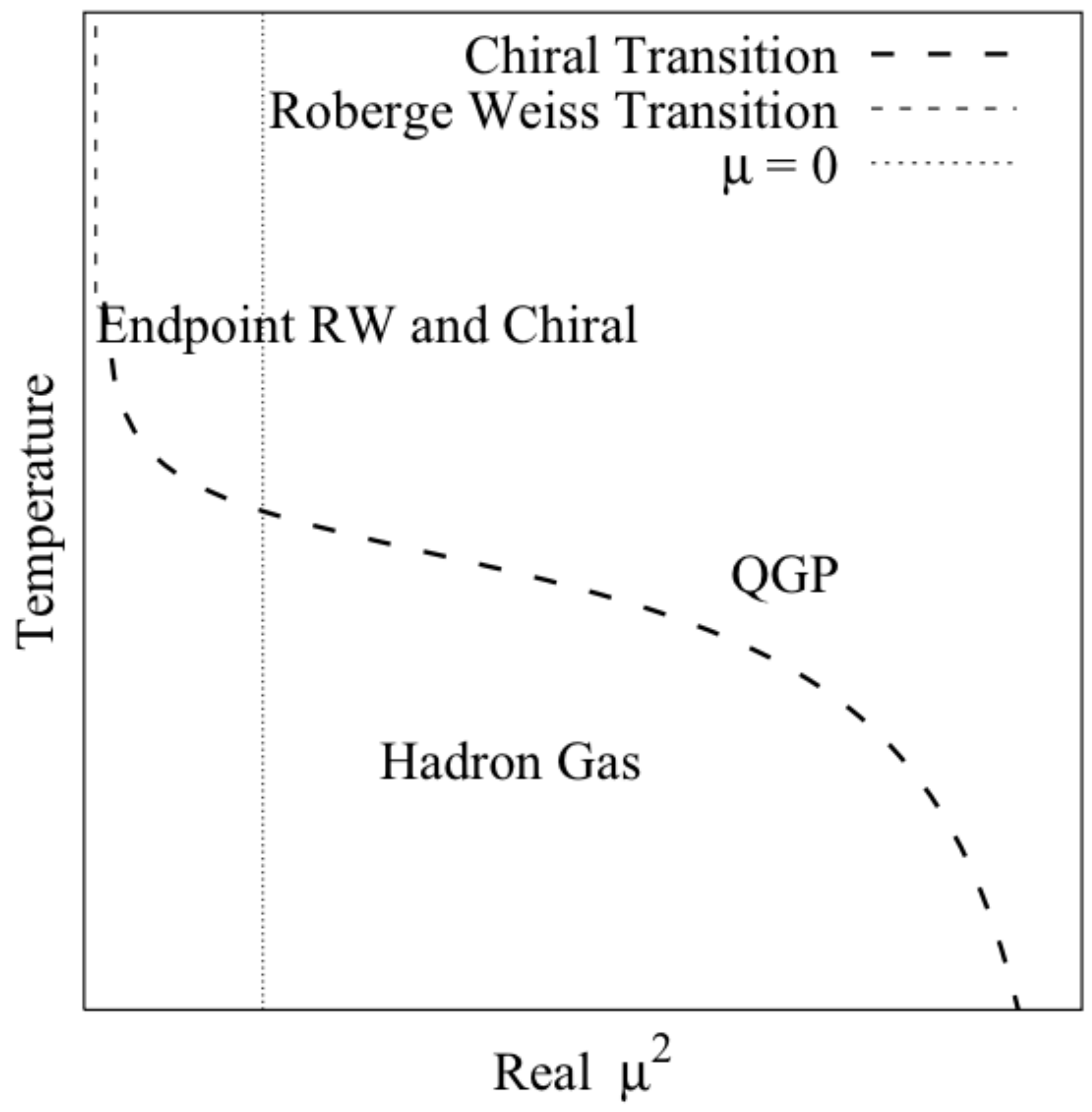
AdS/CFT



Mapping to 3D+1
4th dimension:
imaginary time

Strongly coupled QGP and singularities

T_{RW} approx. 207 MeV



Di Renzo, D'Elia, MpL 2007

$T = T_{RW}$ $$n(\mu_I) = A\mu_I({\mu_I^c}^2 - \mu_I^2)^\alpha$$

The conformal window

The QCD beta function, the infrared fixed point and

The zero temperature phase transition

$$\mu\frac{\partial}{\partial\mu}\alpha(\mu) = \beta(\alpha) \equiv -b\,\alpha^2(\mu) - c\,\alpha^3(\mu) - d\,\alpha^4(\mu) - \dots\,,$$

$$b = \frac{1}{6\pi}\,(11N - 2N_f)$$

$$c = \frac{1}{24\pi^2}\left(34N^2 - 10NN_f - 3\frac{N^2-1}{N}N_f\right)\;.$$

Asymptotically free: $b > 0\ (N_f < \frac{11}{2}N)$

For $N = 3$ c changes sign when $N_f > 8.05$

Infrared fixed point: $\alpha_* = -\frac{b}{c}\;.$

Running coupling

$$b \log\left(\frac{q}{\mu}\right) = \frac{1}{\alpha} - \frac{1}{\alpha(\mu)} - \frac{1}{\alpha_*} \log\left(\frac{\alpha\left(\alpha(\mu) - \alpha_*\right)}{\alpha(\mu)\left(\alpha - \alpha_*\right)}\right) ,$$

Close to Nf = 16 the infrared fixed point is small

defining

$$\Lambda = \mu \exp\left[\frac{-1}{b\,\alpha_*} \log\left(\frac{\alpha_* - \alpha(\mu)}{\alpha(\mu)}\right) - \frac{1}{b\alpha(\mu)}\right] ,$$

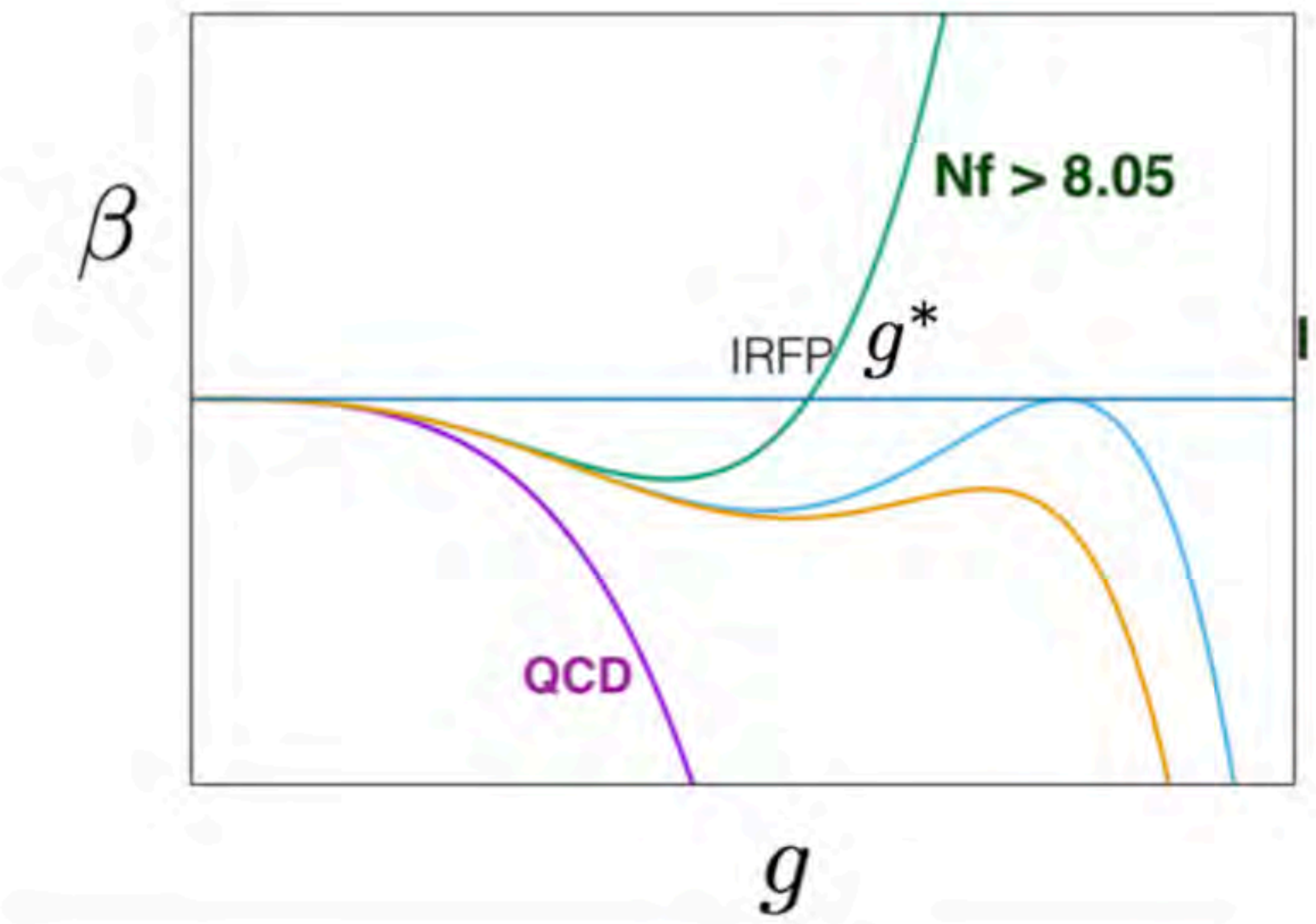
Running coupling:

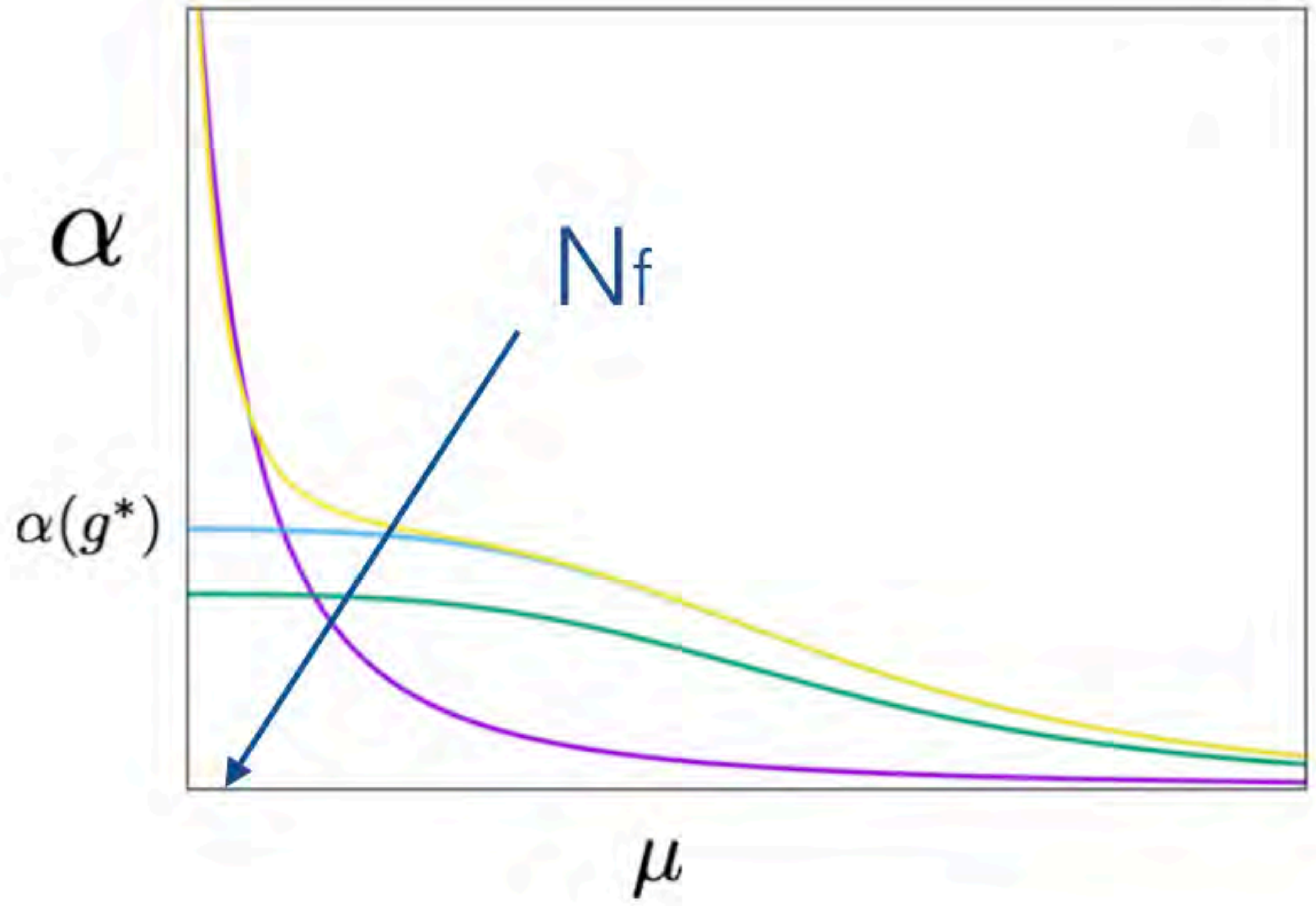
$$\alpha \approx \frac{1}{b \log\left(\frac{q}{\Lambda}\right)} ;$$

UV

$$\alpha \approx \frac{\alpha_*}{1 + \frac{1}{e}\left(\frac{q}{\Lambda}\right)^{b\alpha_*}} .$$

Close to IR fixed point





$$\alpha_c \equiv \frac{\pi}{3\,C_2(R)} = \frac{2\pi\,N}{3\,(N^2-1)}$$

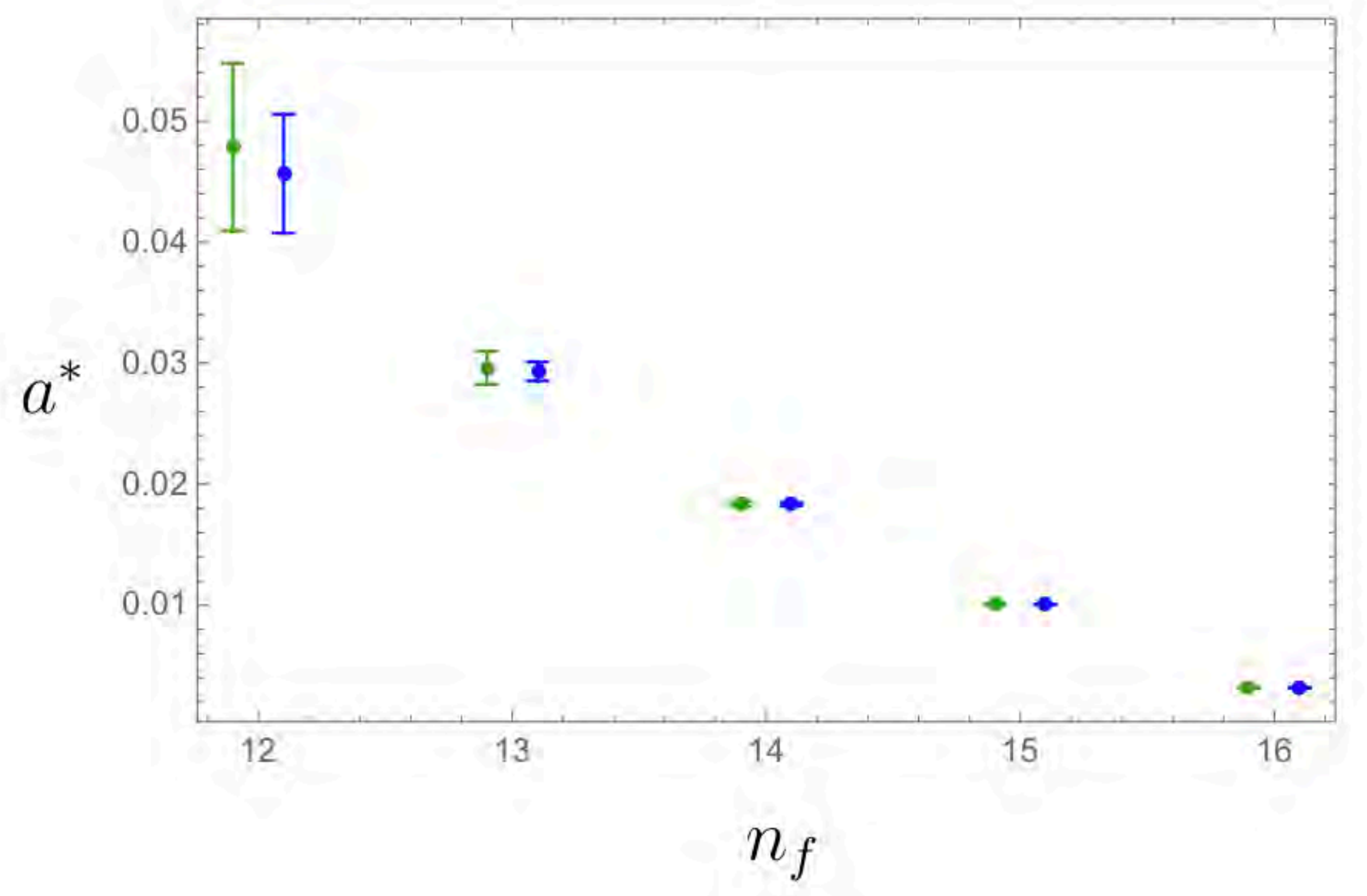
When

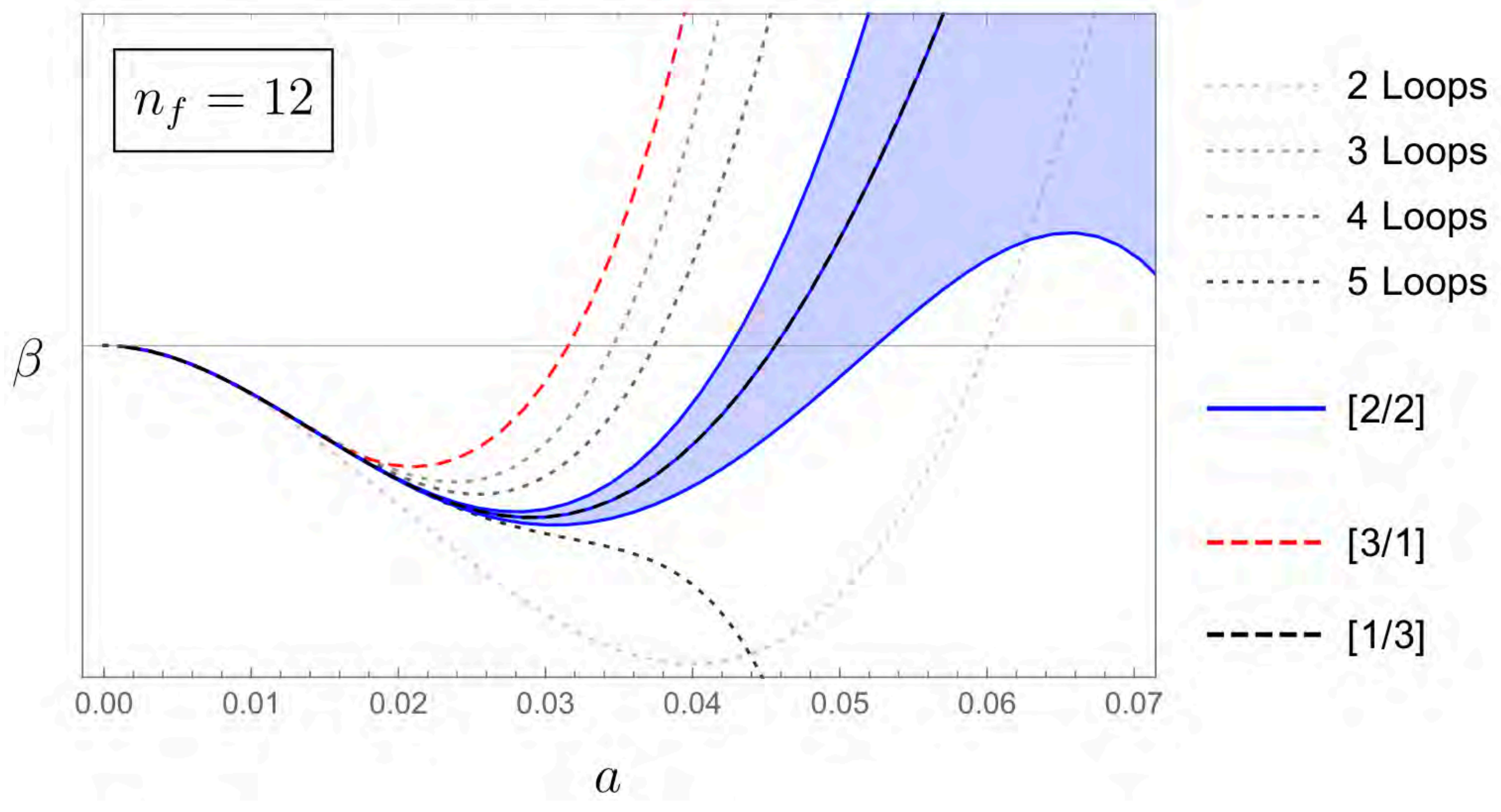
$$N_f > \quad N_f^c = N\left(\frac{100N^2-66}{25N^2-15}\right)$$

$$\alpha(g^\star) < \alpha_c$$

chiral symmetry is unbroken

IR fixed point

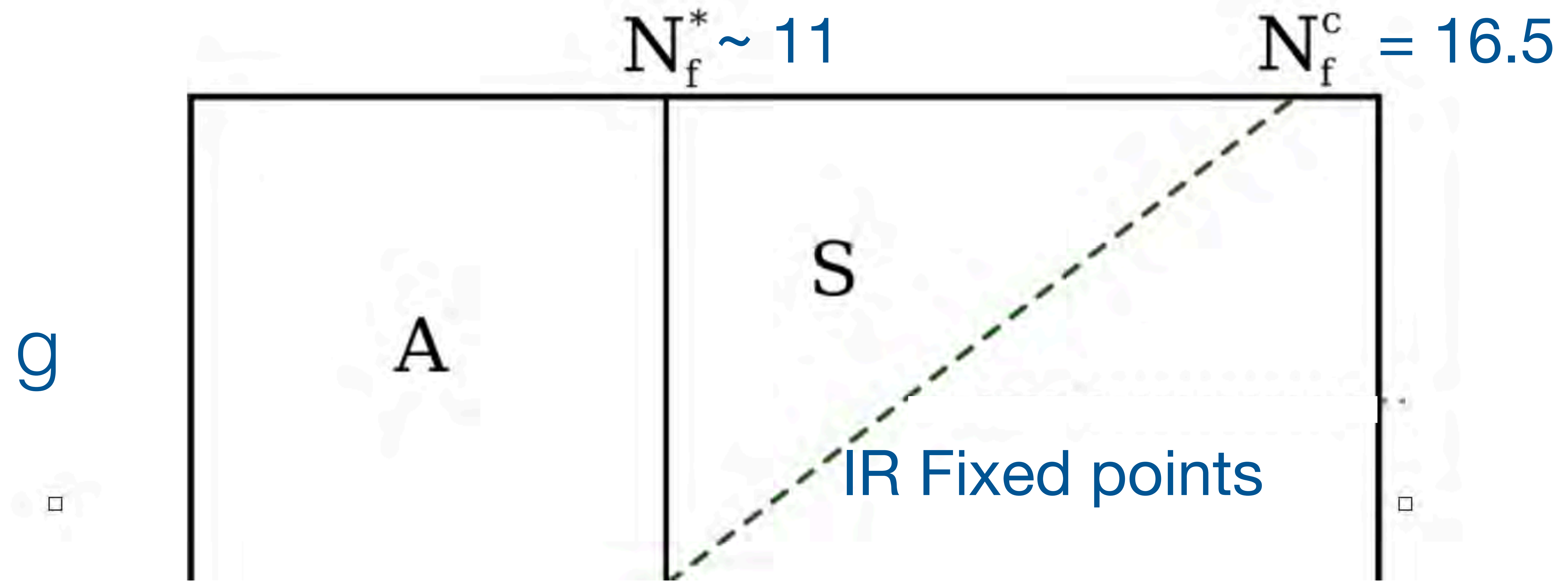




Di Pietro, Serone 2020

similar results Shrock, Dietrich 2014-2022

Phases of Nc=3 QCD at zero temperature



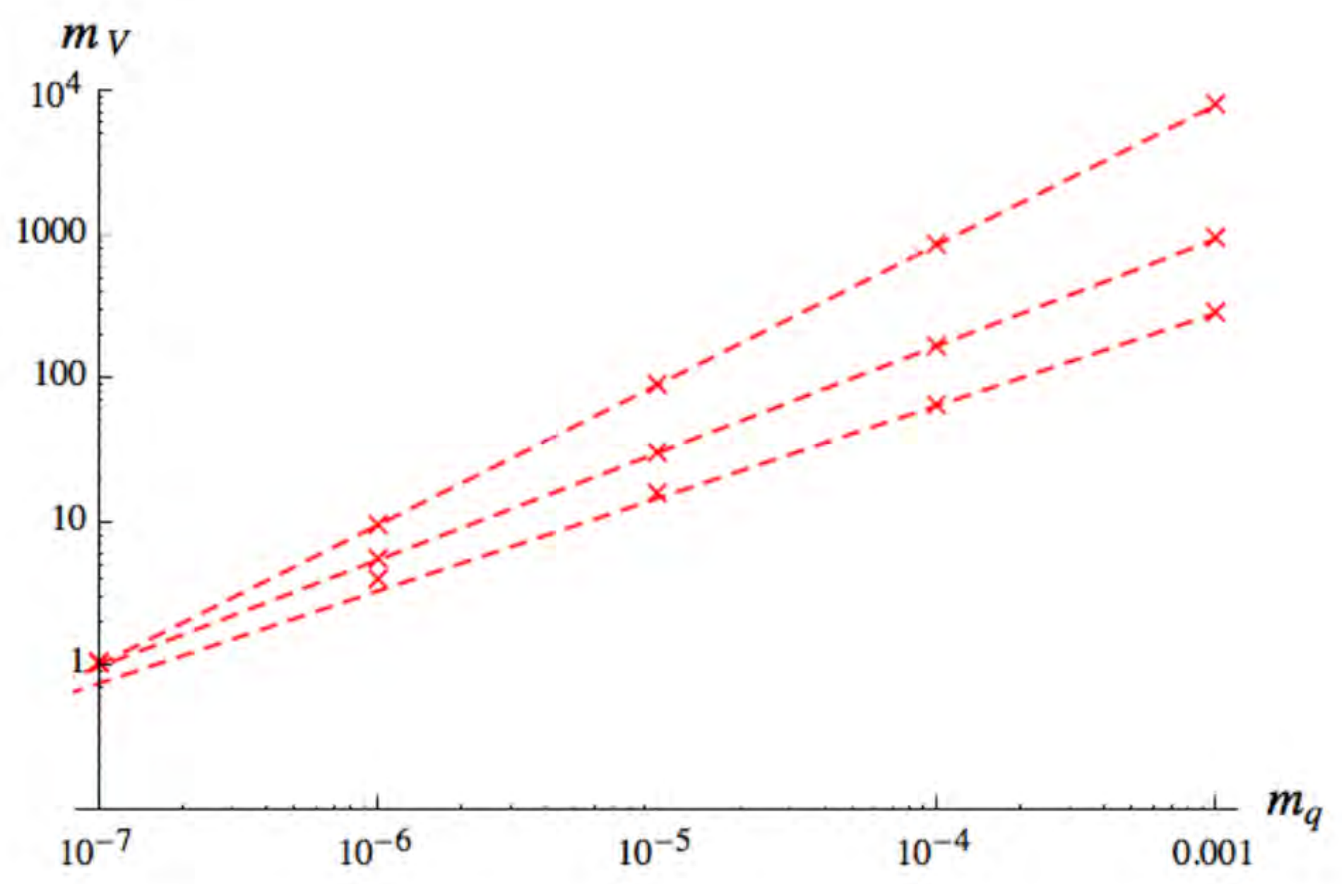
The chiral condensate in the conformal, mass deformed theory

$$\langle \bar{q}q \rangle \sim m^{\eta_{\bar{q}q}}$$

$$\eta_{\bar{q}q} = (3-\gamma_*)/(1+\gamma_*)$$

Del Debbio, Zwicky 2010

Conformal scaling
(valid also for the masses)



$$M_H = c_H m^{1/y_h}$$

Status Nf = 12

Slide from M.Serone at “Newton 1665”

Ref.	$\|\gamma^*\|$	Method
This work	0.320(85)	PB coupling
	0.345(47)	PB conformal
[44]	0.23(6)	Gradient flow
[45]	0.47(10)	Top. susceptibility
	0.33(6)	
[46]	0.32(3)	Dirac eigenmodes
[47]	0.235(46)	Finite-size scaling
[48]	0.235(15)	
[49]	0.45(5)	
[50]	0.403(13)	

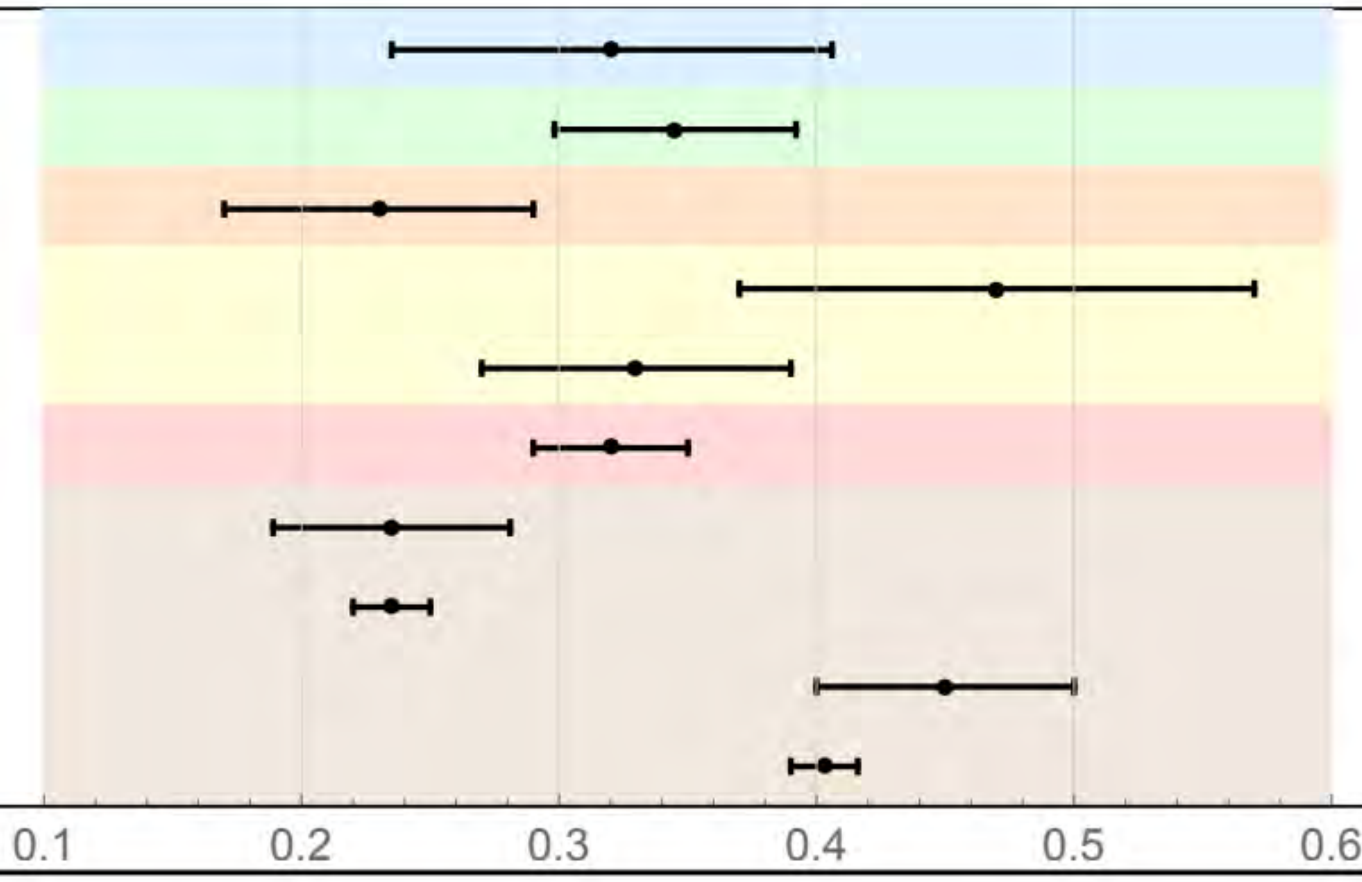


Table 2: Comparison between the results of our Padé-Borel (PB) resummation for the mass-anomalous dimension for QCD with $n_f = 12$ –both using the coupling expansion and the Banks-Zaks conformal expansion, and averaging over all available Padé approximants in each case– and lattice results.

[44] - Carosso, Hasenfratz, Neil 1806.01385

[45] - Aoki et al. 1601.04687

[46] - Cheng, Hasenfratz, Petropoulos, Schaich, 1301.1355

[47] - Lombardo, Miura, Nunes da Silva, Pallante, 1410.0298

[48] - Cheng et al, 1401.0195

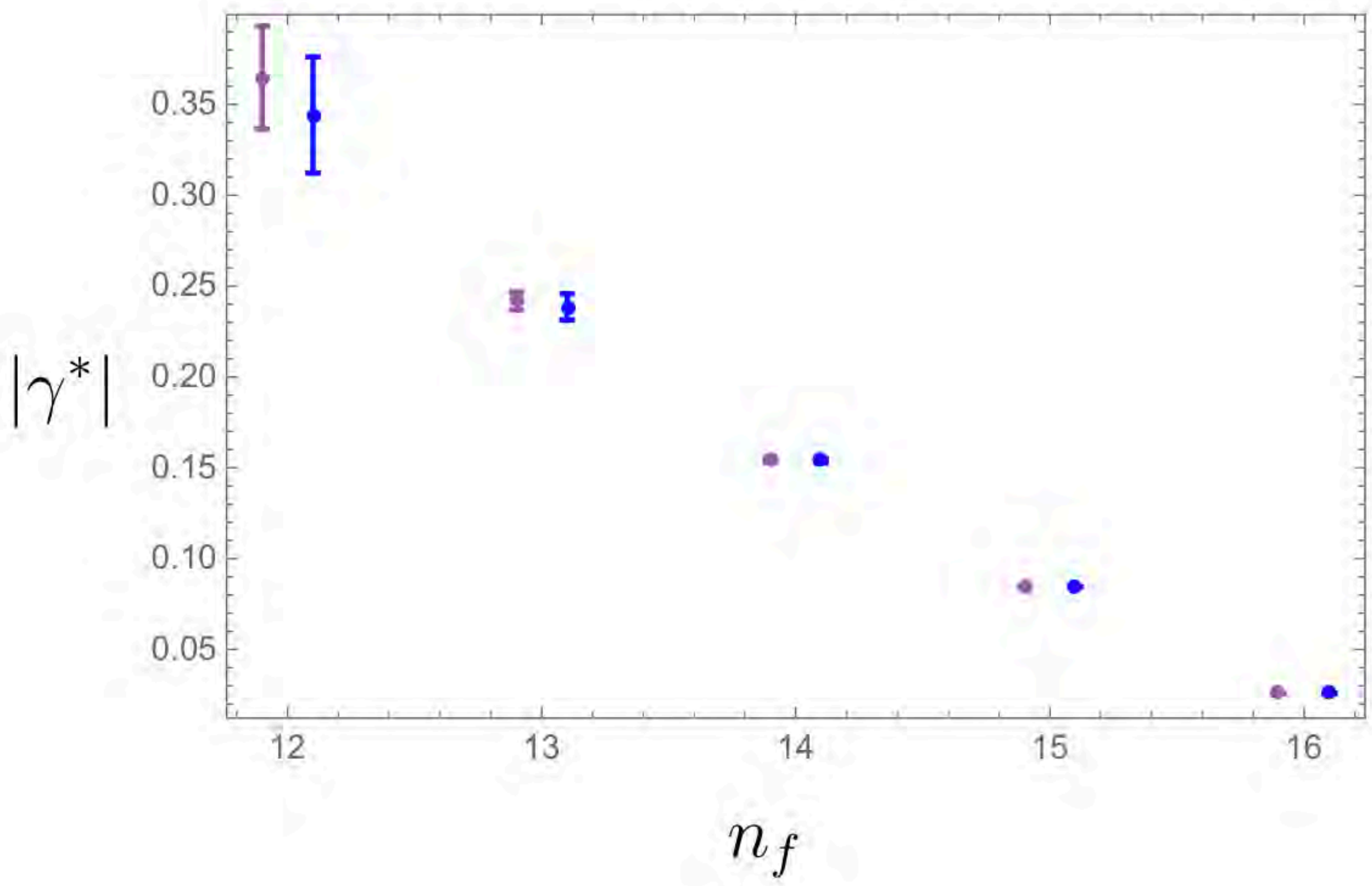
[49] - Aoki et al, 1207.3060

[50] - Appelquist et al, 1106.2148

[from 2003.01742]

One group claims non-conformality

Anomalous dimension

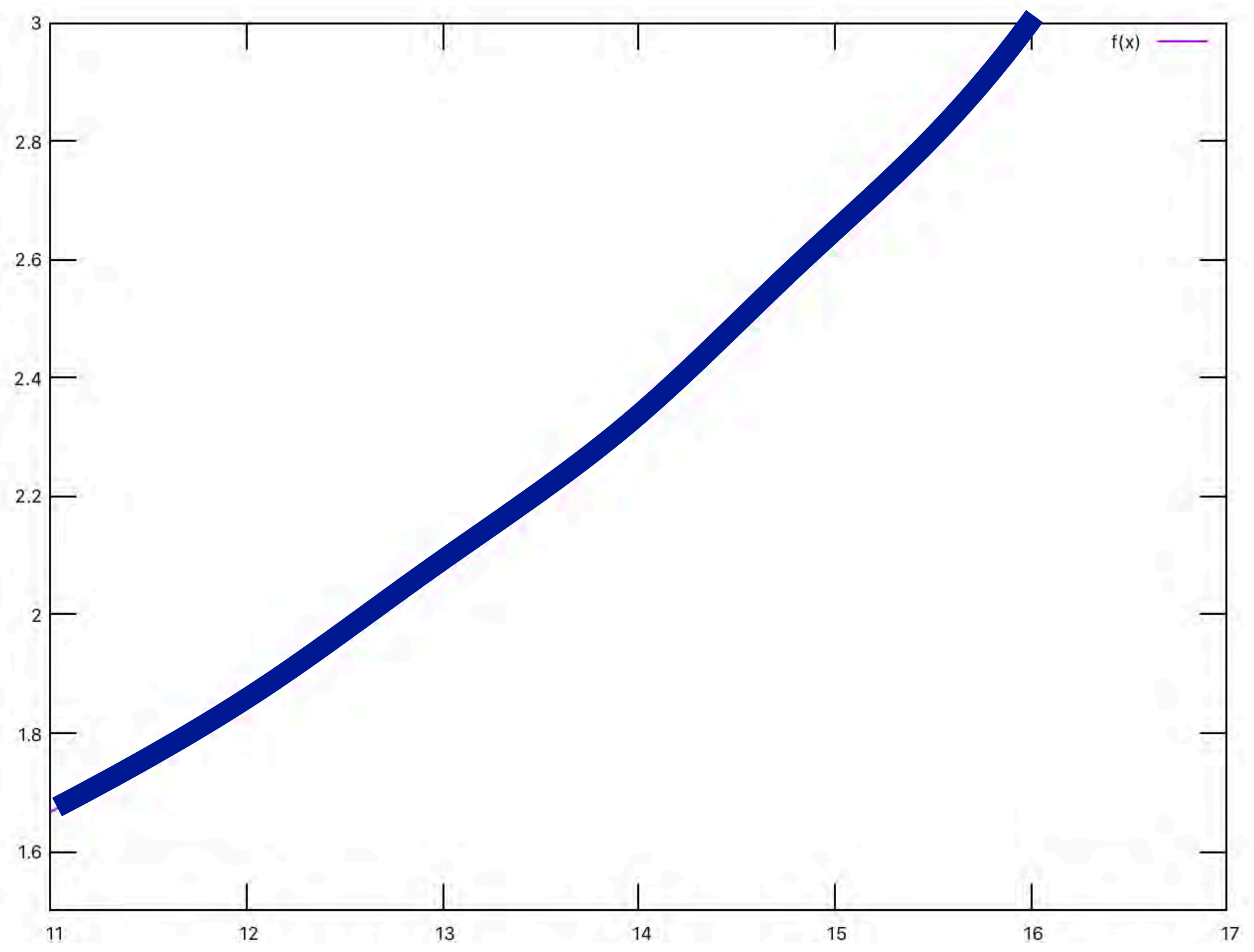


The chiral condensate in the conformal mass deformed theory

$\eta_{\bar{q}q}$

$\eta_{\bar{q}q} = (3-\gamma_*)/(1+\gamma_*)$

3



1.7

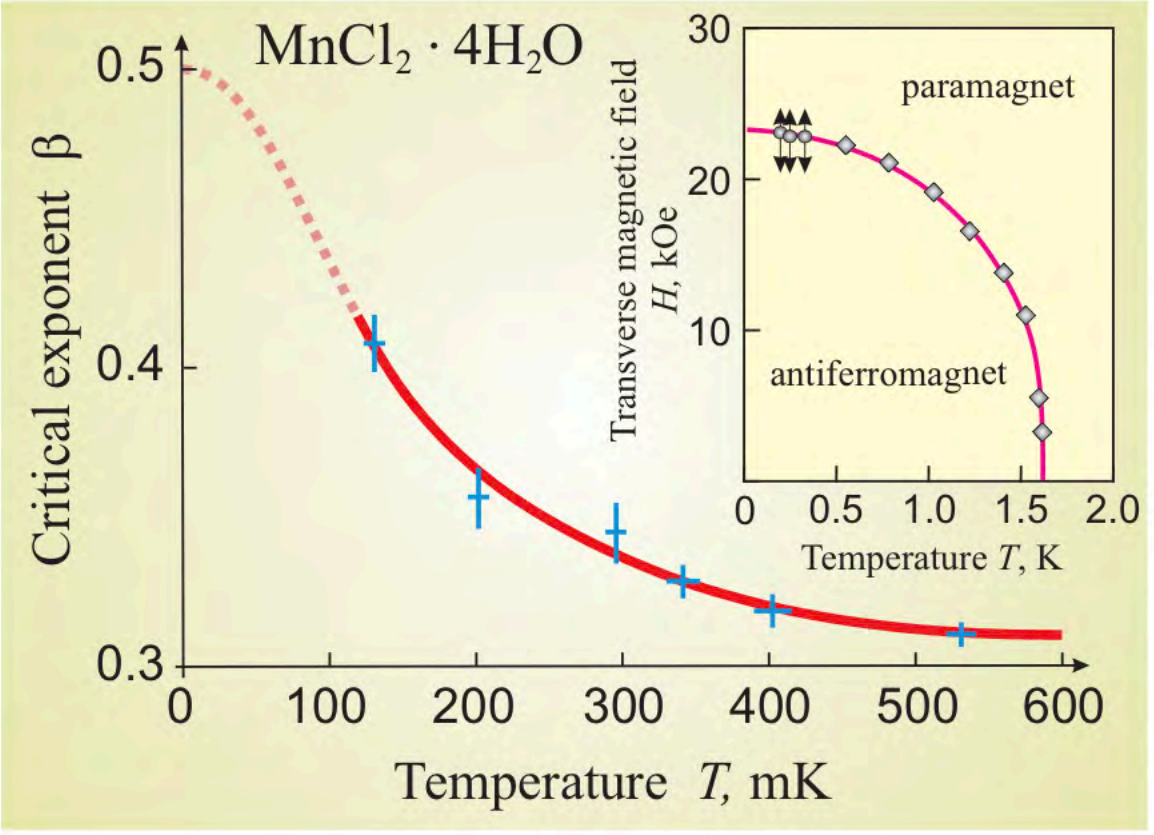
11

Nf

$$\langle \bar{q}q \rangle \sim m^{\eta_{\bar{q}q}}$$

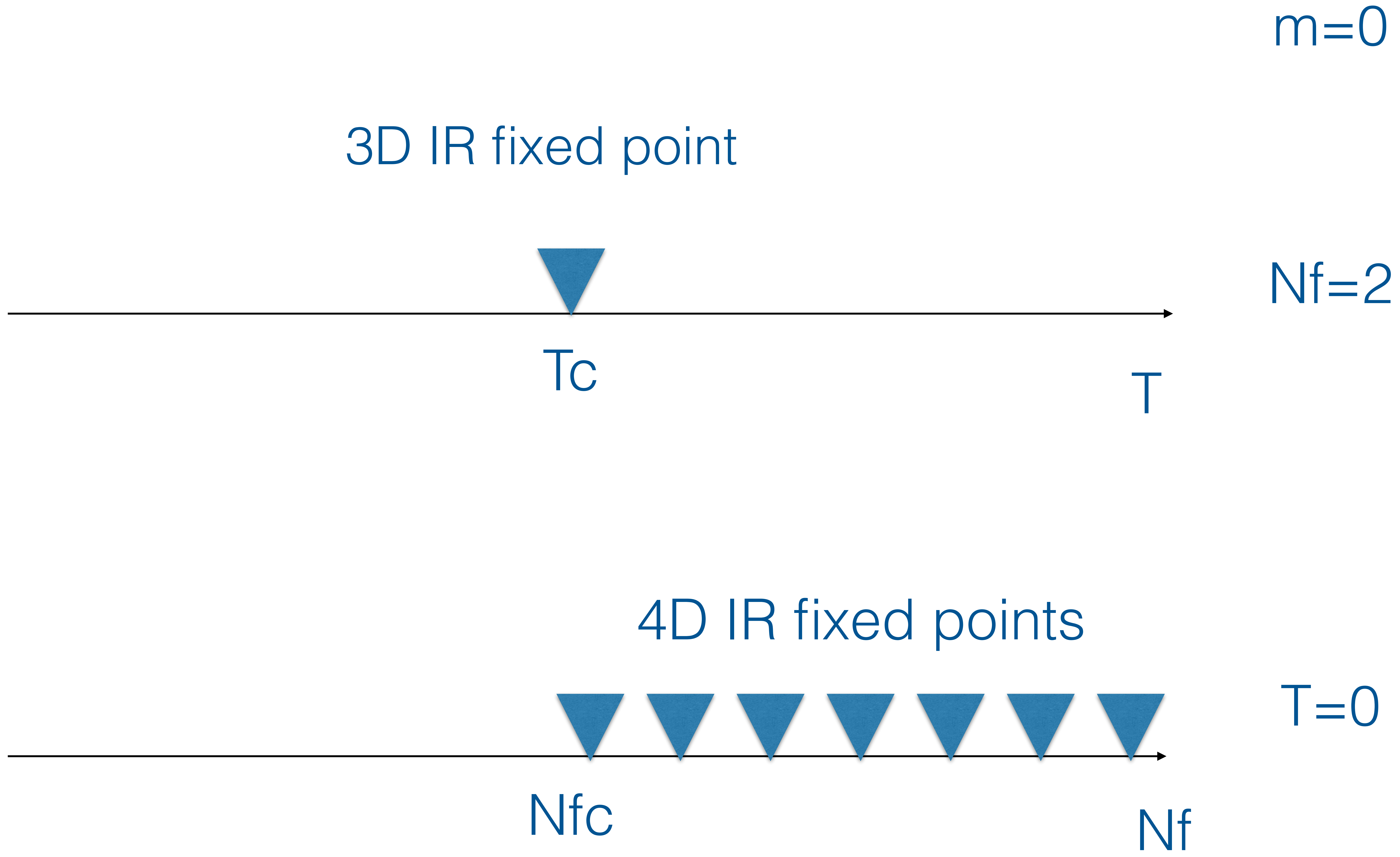
$$1 \leq \eta_{\bar{q}q} \leq 3$$

Expected $\eta_{\bar{q}q} = 1$
at the transition



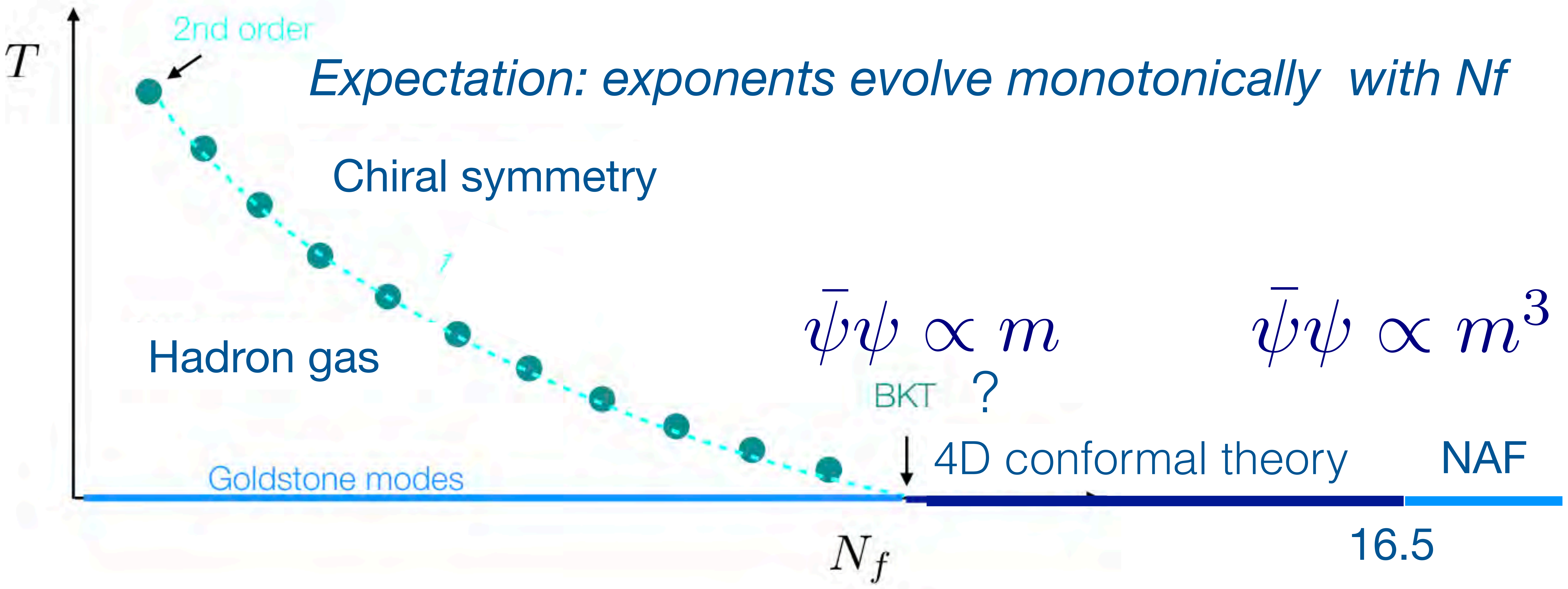
Critical line in the T, Nf plane:

From the high temperature critical region to the conformal window?


m=0
3D IR fixed point
Nf=2
Tc
T
4D IR fixed points
T=0
Nfc
Nf

Scaling of the condensate : no match btwQCD at Tc <-> conformal window

$$\bar{\psi}\psi \propto m^{1/\delta} \simeq m^{0.21}$$

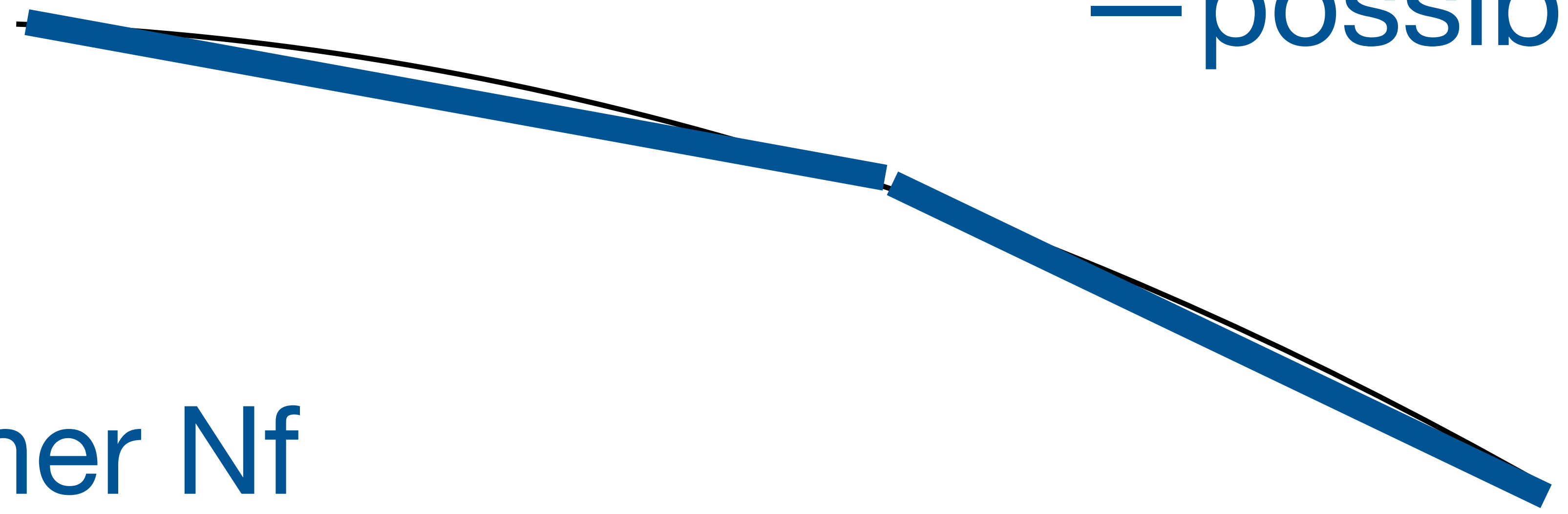


Different (integer) Nf and Nf+1 with own's universality classes interpolated by quark mass

Simplest plausible scenario continuous crossover

—possibly with intermediate mean field regions?

C1



C2

C1 Wilson-Fisher Nf

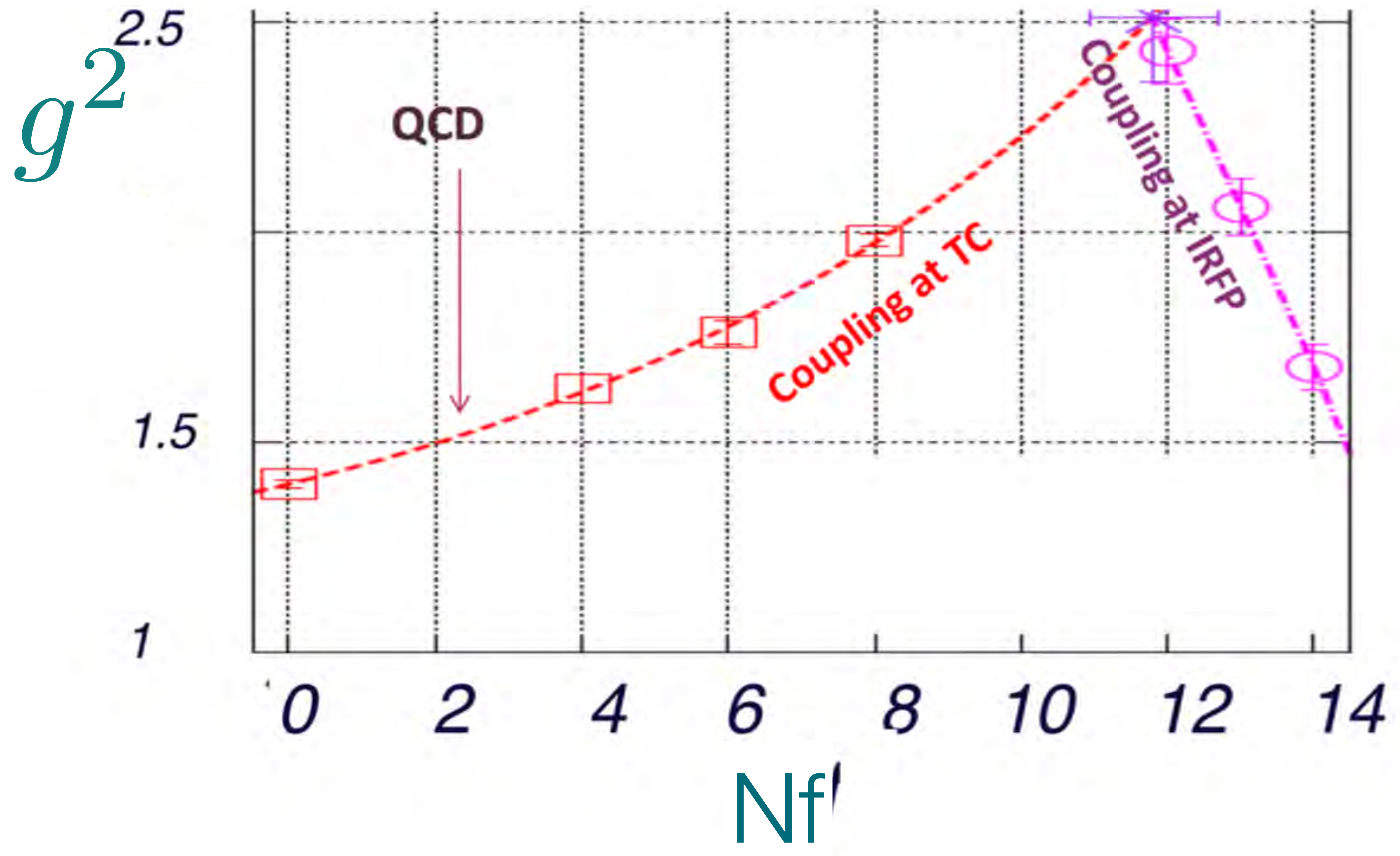
SU(Nf)XSU(Nf)

C2 : W-F Nf+1

SU(Nf+1)XSU(Nf+1)

Coupling at the transition and conformal dynamics

“Analogy” between thermal transition for Nf=2 and Nf=14 ?

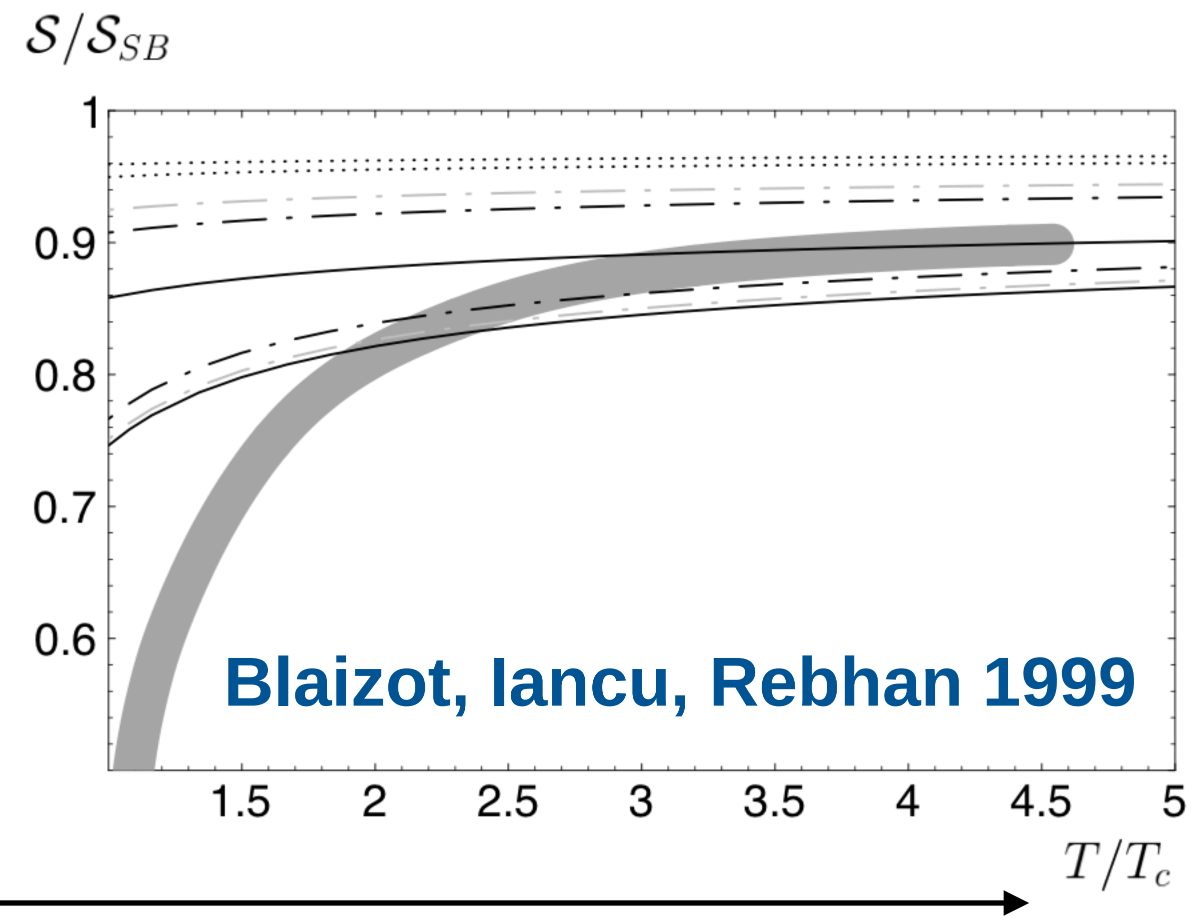


Miura, MpL 2013

The coupling **increases with Nf** in the broken phase and **decreases with Nf** in the conformal window

Thermal behaviour clearly different

m=0



3D IR fixed point

Temperature

Tc

Nf=2

4D IR fixed points

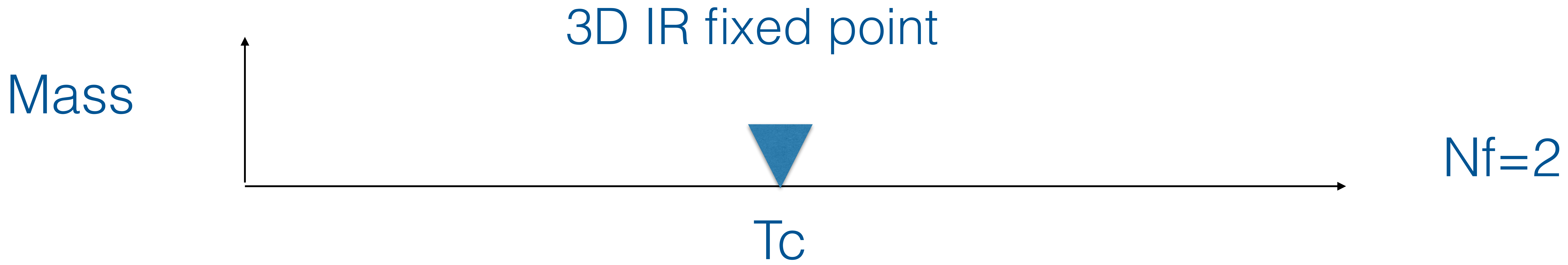
Temperature

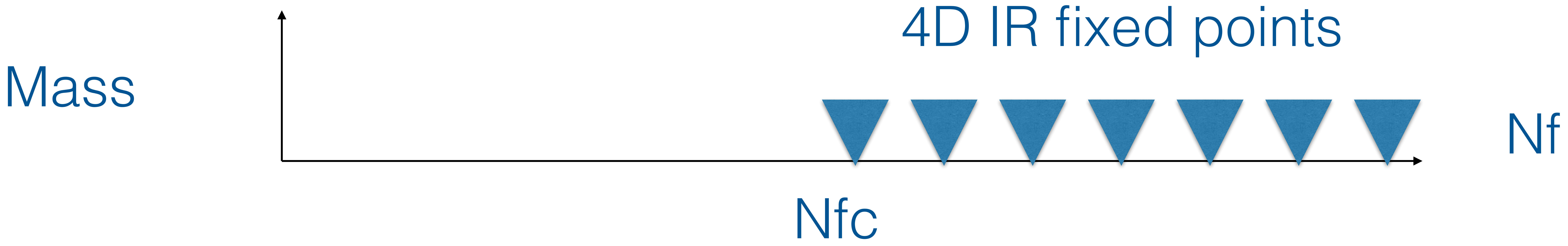
Nf

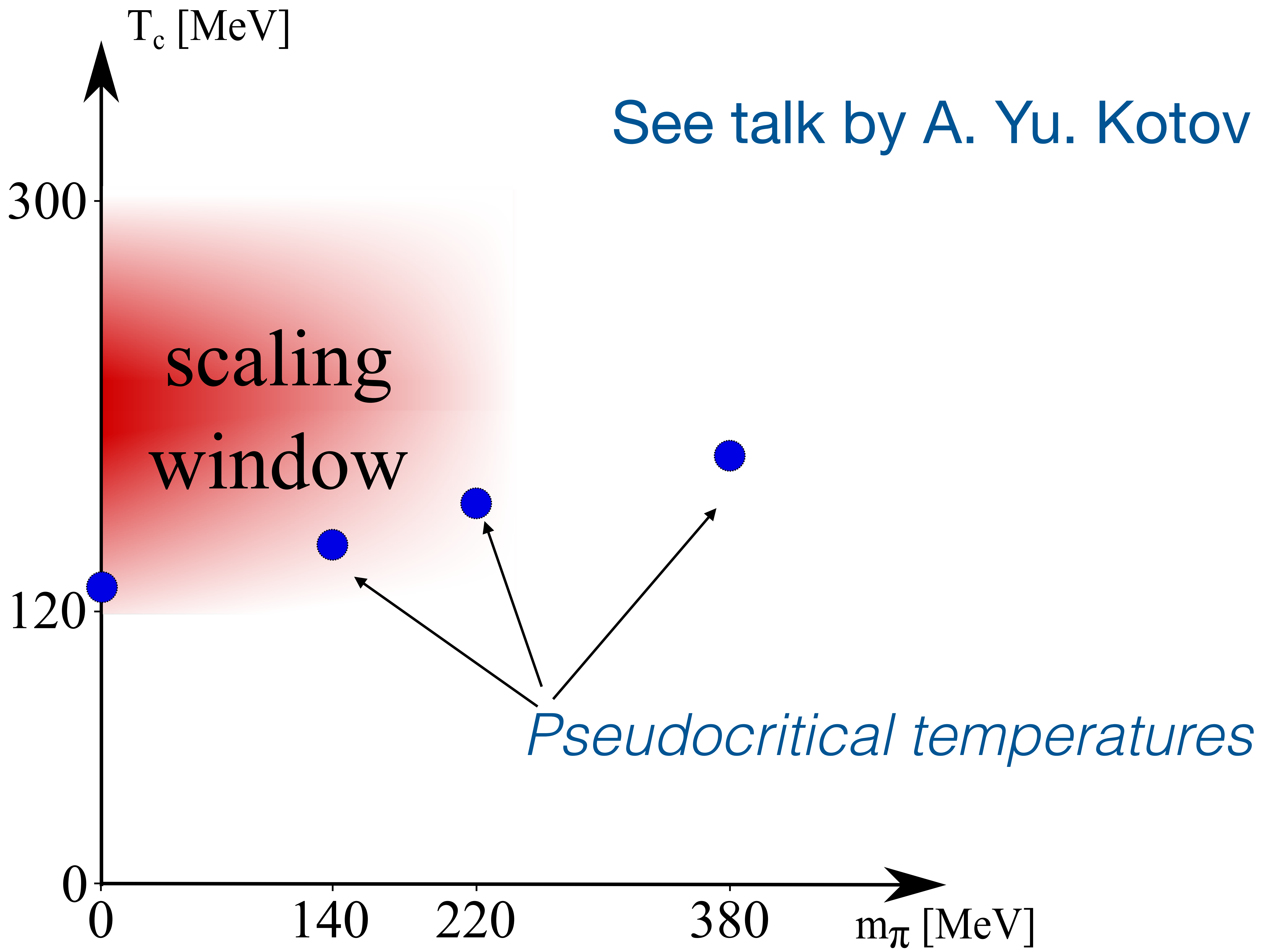
Nfc

$$S \propto T^3$$

Add a mass term : conformality broken → compare theories within the scaling window







See talk by A. Yu. Kotov

Kotov, Trunin, MpL

The chiral condensate in the conformal, mass deformed theory

$$\langle \bar{q}q \rangle \sim m^{\eta_{\bar{q}q}} \qquad \eta_{\bar{q}q} = (3-\gamma_*)/(1+\gamma_*)$$

Del Debbio, Zwicky 2010

The chiral condensate in QCD close to Tc

$$< q\bar{q} > \approx m^{\frac{1}{\delta}} - A(T - T_c) m^{\frac{1}{\delta} - \frac{1}{\beta\delta}}$$

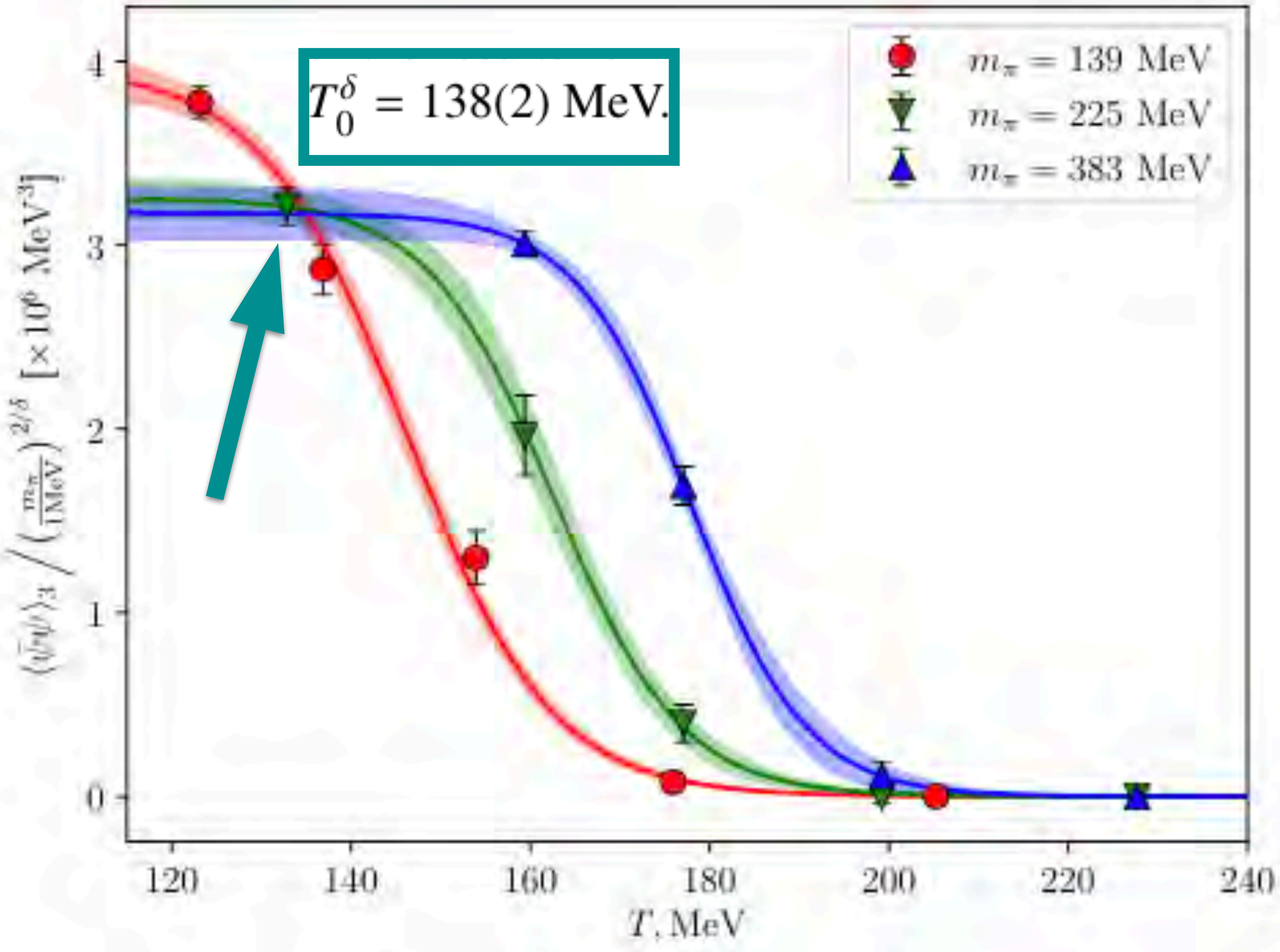
Perhaps for T > Tc: $$< \bar{\psi}\psi > \propto m^{1/\delta_{eff}}$$

$$m_1 < m < m_2$$

Baseline

Scaling at the critical point: searching for $<\bar{\psi}\psi>_3 (T=T_0) = A m_\pi^{2/\delta}$

$$\delta = \delta(O(4)3D) = 4.8$$



Kotov, Trunin, MpL

$R_\pi = \chi_T^{-1}/\chi_L^{-1}$

(reminiscent of the Kouvel-Fisher parameter)

- For a continuous symmetry $h = M^\delta f(t/M^{1/\beta})$ becomes:

$$h_a = M_a M^{\delta-1} f(t/M^{1/\beta})$$

$t = T - T_c$

- The response to an external field h_a is given by the inverse susceptibility $(\chi^{-1})_{ab} = \partial h_a/\partial M_b$

$$(\chi^{-1})_{ab} = \delta_{ab} M^{\delta-1} f(x) + (\delta-1)\frac{M_a M_b}{M^2} M^{\delta-1} f(x) - \frac{x}{\beta}\frac{M_a M_b}{M^2} M^{\delta-1} f'(x)$$

$x = t/M^{1/\beta}$

- We can separate longitudinal and transverse susceptibilities $\chi_L = \partial M/\partial h$ and $\chi_T = M/h$

$$\chi_T^{-1} = M^{\delta-1} f(x), \qquad \chi_L^{-1} = M^{\delta-1}\left(\delta f(x) - \frac{x}{\beta} f'(x)\right)$$

- Define $R_\pi = \chi_T^{-1}/\chi_L^{-1}$

$$\frac{1}{R_\pi(t,m)} = \delta - \frac{x}{\beta}\frac{f'(x)}{f(x)}, \qquad R_\pi(0,m) = \frac{1}{\delta}$$

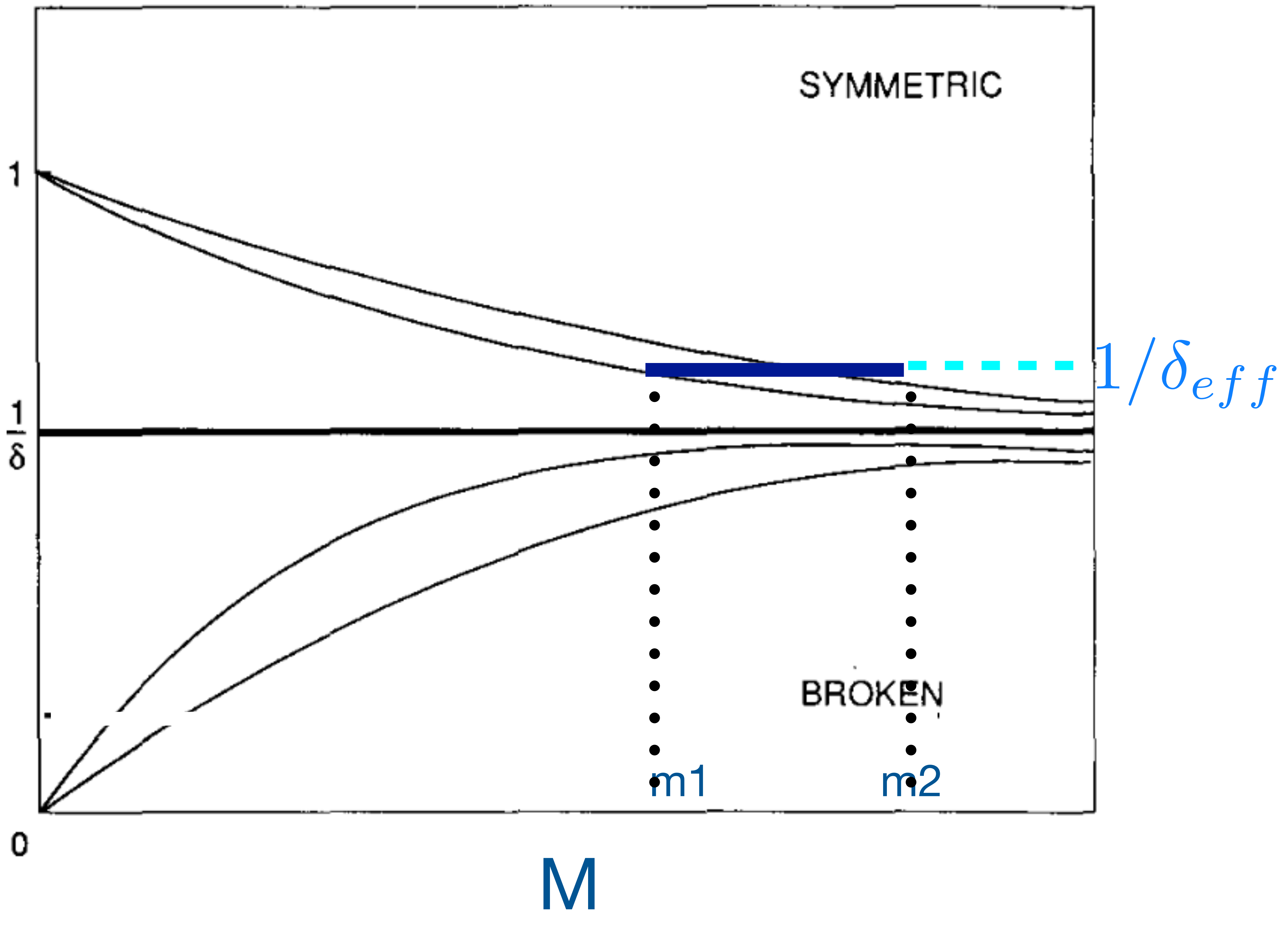
Kogut, Kocic, MpL, 1992
Karsch, Laermann, 1992
HotQCD, 2020 - 2021

$$R_\pi = \chi_T^{-1}/\chi_L^{-1}$$ and an effective δ exponent

Chiral symmetry

R_π

Goldstone

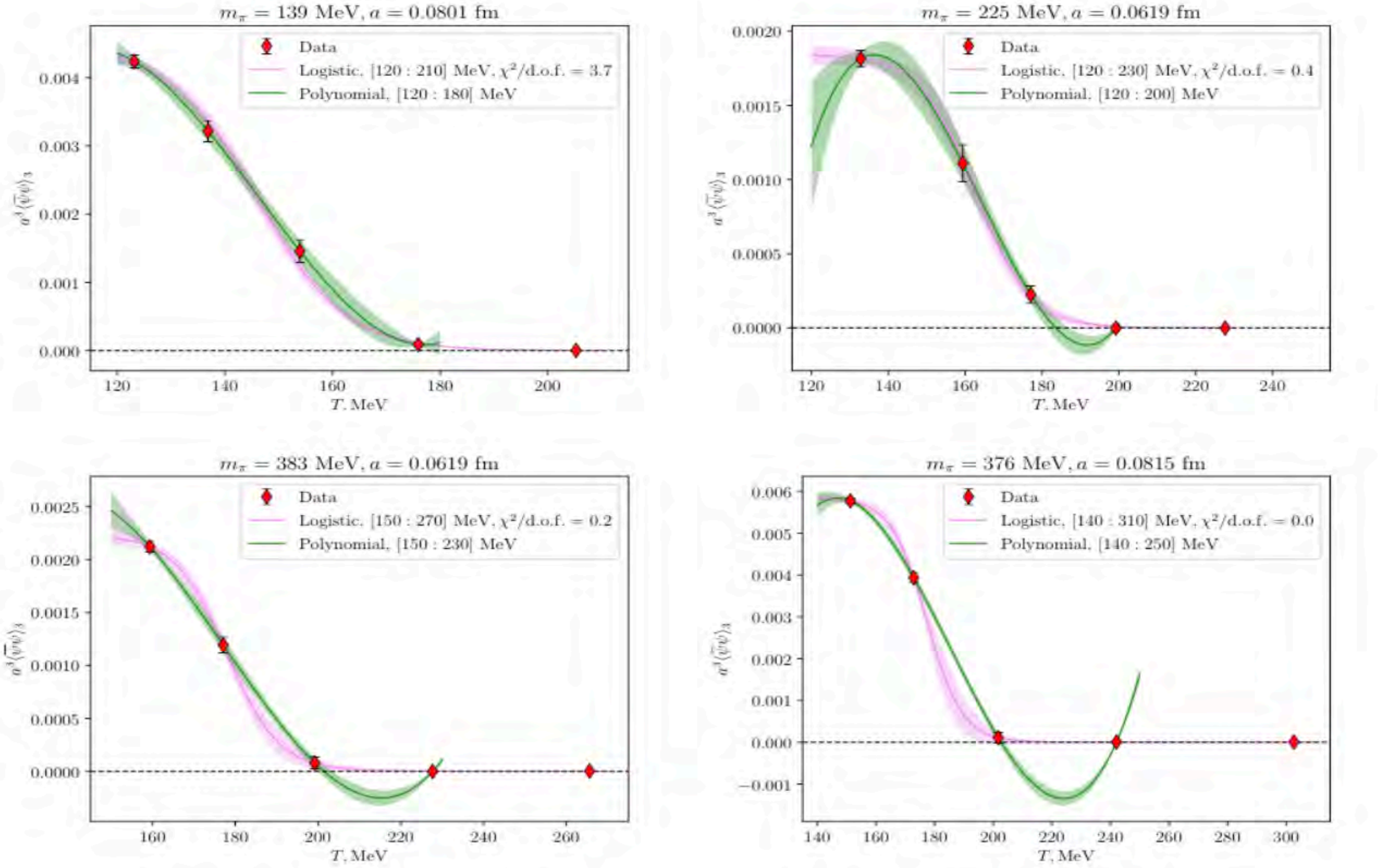


Within the scaling window of the transition one may identify two mass scales m1 and m2 with a quasi-conformal scaling of the order parameter

$$M \propto m^{1/\delta_{eff}}$$

$$m_1 < m < m_2$$

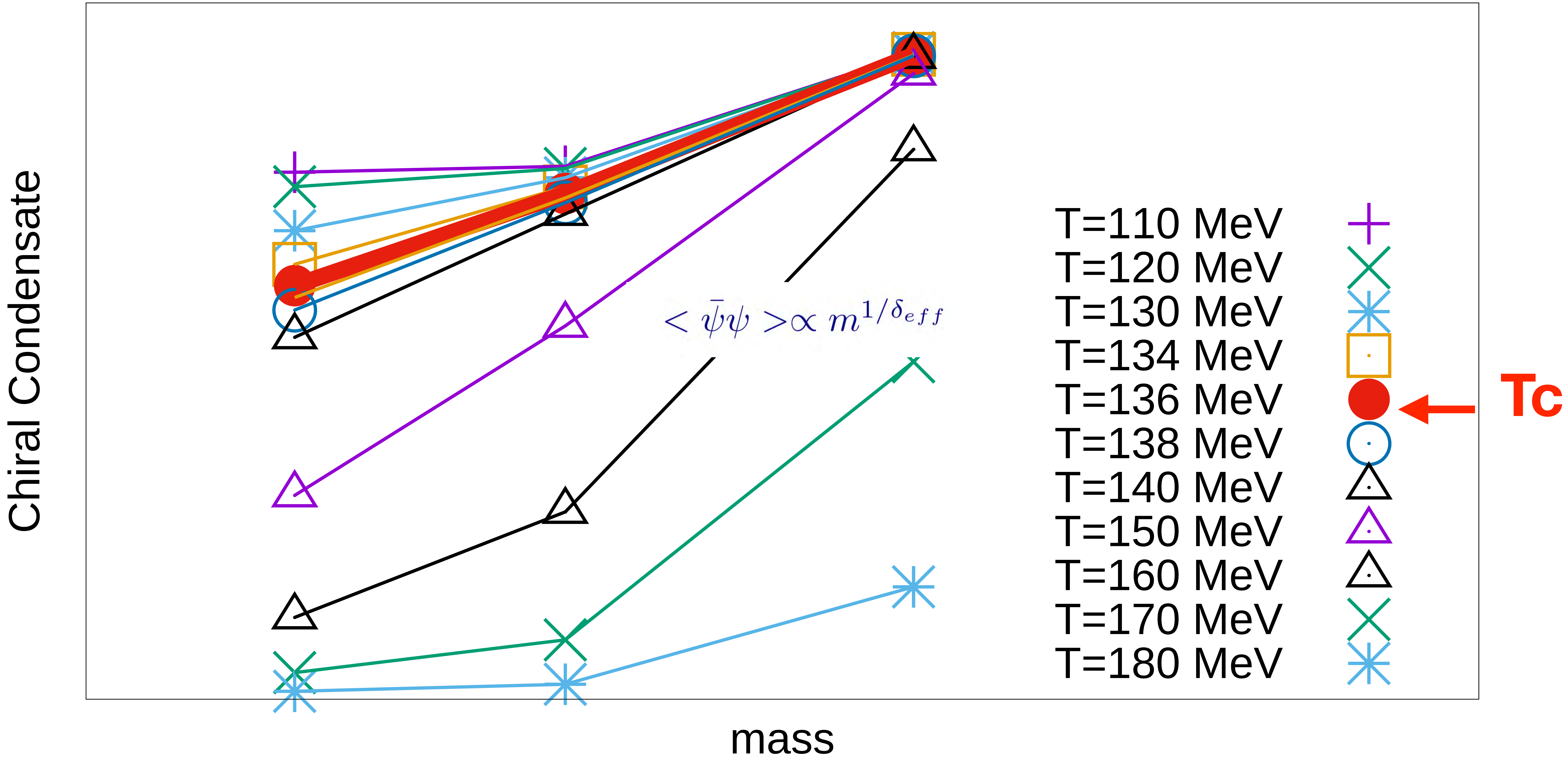
Fits to the chiral condensate



Kotov, Trunin, MpL

Log-Log plot: straight line at Tc, with slope $1/\delta$

(fits w logistic curves produce results for different temperatures)



Possible to identify an effective exponent in some mass window above Tc

$$\delta_{eff} : 1 < \delta_{eff} < \delta$$

Summary

The critical line in the T, Nf plane

The critical exponent δ is about 4.8 for Nf=2, approaches 1 at the onset of conformality and decreases smoothly till δ = 1/3 before loosing AF.

The coupling at the scale of the critical temperature increases with Nf and matches the IR fixed point at the onset of conformality

sQGP and conformality

For any Nf in the broken phase there is an Nf whose IRFP matches the coupling at Tc.

The critical exponents changes monotonously from Nf=2 till Nf=16, with δ in the broken phase always larger than the corresponding exponent in the conformal phase.

The scaling behaviour may be compatible with near-conformality in T-dependent mass intervals

$$m_{IR}(T) < m < m_{UV}(T)$$

but it is not immediate to relate this with the conformal window