Hadrons, Superconductor Vortices, and Cosmological Constant

- trace anomaly and confinement

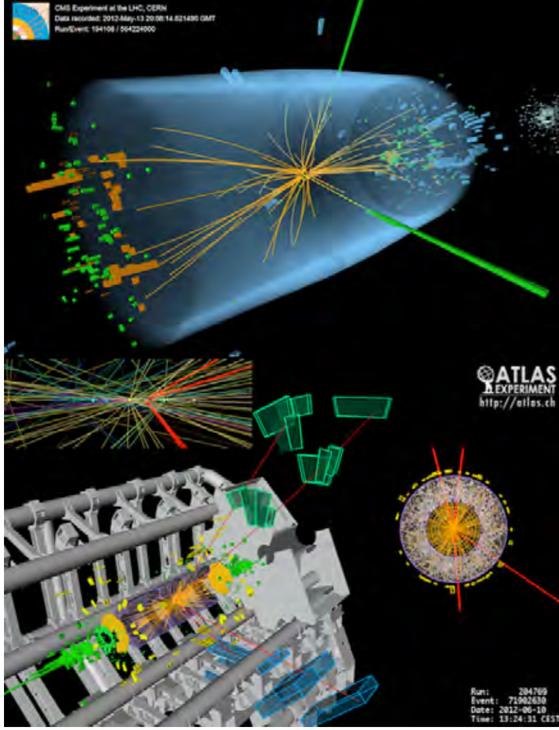
PRD104,076010 (2021)
[arXiv:2103.15768],

arXiv:2302.11600

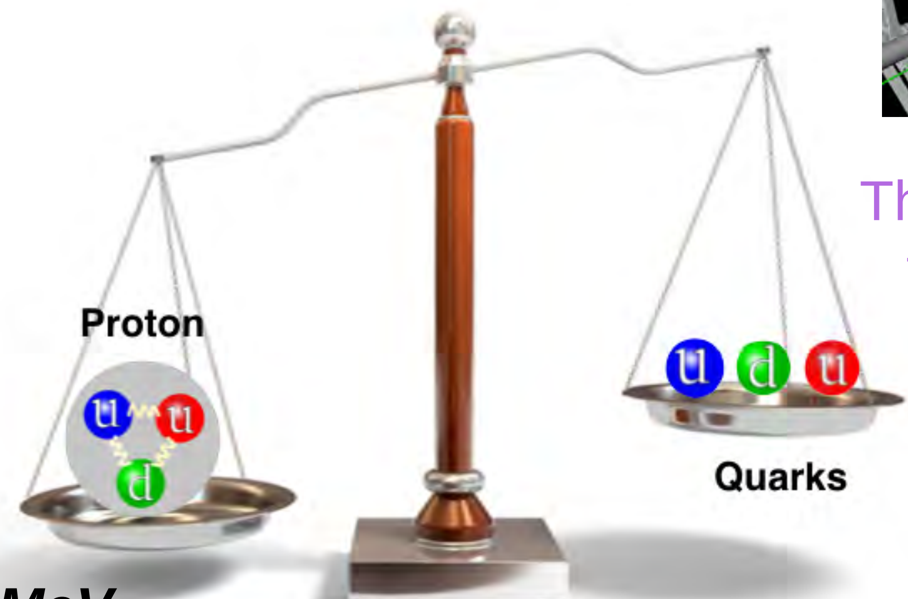
- Mass and rest energy – trace anomaly, Hamiltonian and gravitational form factors
- Pressure balance of hadrons and role of trace anomaly
- Equation of state – rest energy and equilibrium correspondence
- Vortices in type II superconductors
- Cosmological constant in Einstein's equation
- Pion mass and trace anomaly

Apr. 16, 2023
Prague

Motivation



Where does the proton mass come from, and how ?



But the mass of the proton is

938.272046(21) MeV.

~100 times of the sum of the quark masses!

The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$$m_u = 2.08(9)\ MeV$$
$$m_d = 4.73(12)\ MeV$$

Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010

Mass and Rest Energy

- $E = m c^2$ → m increases with E? converting mass to energy?
- $E_0 = m c^2$ (Einstein 1905, $m^2 = E^2 - p^2$)
- $e^+ e^- \to \gamma\gamma$ ($m_{\gamma\gamma} = 2m_e$)
- Mass is a scalar and E is a component of the four vector. E and p are additive, not mass
- In general relativity, the gravitational field is coupled to the EMT.
- In non-relativistic limit, Newton's law of force and universal gravitational involves E_0 or mass.
- Inertial mass and gravitational mass are the same mass.
- Relativistic mass in a misnomer, rest mass is redundant.
- -- L.B. Okun doi:10.1134/1.1358478

Mass of Hadrons – Energy Momentum Tensor

- Energy momentum tensor

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2$$

- Mass from trace of EMT – scalar, frame independent, scale invariant

$$\langle P|T^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

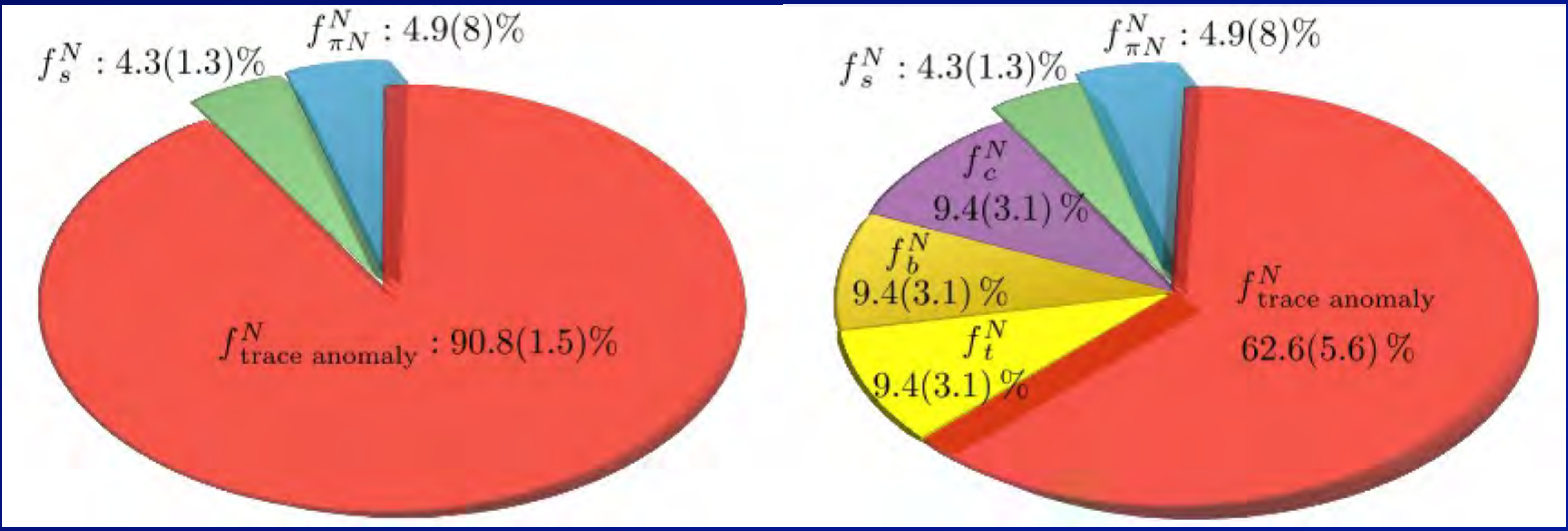
$$\frac{\langle P|\int d^3\vec{x}\,\gamma\, T^\mu_\mu(x)|P\rangle}{\langle P|P\rangle}| = M_N$$

$$T^\mu_\mu = \sum_f m_f\bar{\psi}_f\psi_f + \left[\frac{\beta(g)}{2g}G^{\alpha\beta}G_{\alpha\beta} + \sum_f m_f\gamma_m(g)\bar{\psi}_f\psi_f\right]$$

Chanowitz, Ellis, Crewther, Collin, Duncan, Joglekar

Mass from Trace of EMT

- Lattice calculation of of quark condensate
  - Y.B. Yang et al (χ QCD) [arXiv: 1511.15089]; M. Gong et al, (χ QCD) [arXiv:1304.1194]
  - Overlap fermion (Z_m Z_s =1), 3 lattices (one at physical m_π)



$$m_h\langle N|\bar{\psi}_h\psi_h|N\rangle \sim -\frac{n_f}{3}\frac{\alpha_s}{4\pi}\langle N|G^2|N\rangle + \mathcal{O}(1/m_h)$$

$$f_f^N = \frac{m_f\langle N|\bar{\psi}_f\psi_f|N\rangle}{M_N}$$

$$\beta_0 = 11 - \frac{2}{3}n_f$$

Shifman, et al., Phys.Lett. B 78, 443 (1978)

Decoupling theorem: $f_c^N + f_b^N + f_t^N + f_a^N \sim \sum_H \mathcal{O}_H(1/m_H)$

Rest Energy Decomposition from Hamiltonian

- Separate the EMT into traceless part and trace part (Ji, 1995)

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \frac{1}{4} g^{\mu\nu} (T^{\rho}_{\rho})$$

- Hamiltonian -- $H = \int d^3\vec{x}\, T^{00}(x)$

$$H_q(\mu) = \int d^3\vec{x} \left(\frac{i}{4} \sum_f \bar{\psi}_f \gamma^{\{0} \overleftrightarrow{D}^{0\}} \psi_f - \frac{1}{4} T^{\mu}_{q\,\mu} \right)_M ,$$

Quark momentum fraction (scale dependent)

$$H_g(\mu) = \int d^3\vec{x}\, \frac{1}{2} (B^2 + E^2)_M ,$$

Glue momentum fraction (scale dependent)

$$H_{tr} = \int d^3\vec{x}\, \frac{1}{4} (T^{\mu}_{\mu})_R .$$

Caracciolo:1989pt, Makino:2014taa, DallaBrida:2020gux

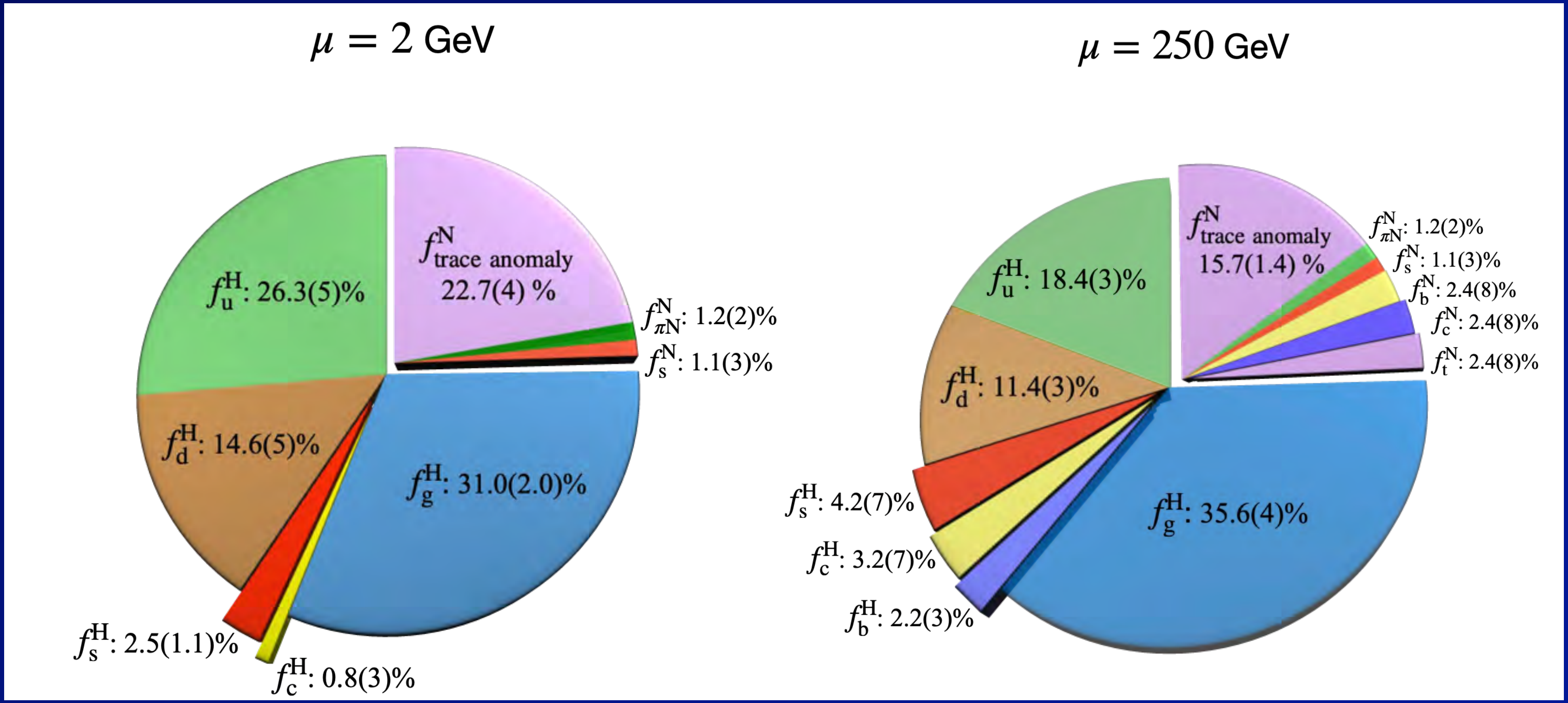
- Rest energy -- $E_0 = M = \langle H_{q_f}(\mu) \rangle + \langle H_g(\mu) \rangle + \langle H_{tr} \rangle ,$

$$\langle H_{q_f}(\mu) \rangle = \frac{3}{4} \sum_f \langle x \rangle_f (\mu) M, \qquad \langle H_g(\mu) \rangle = \frac{3}{4} \langle x \rangle_g (\mu) M,$$

$$\langle H_{tr} \rangle = \frac{1}{4} \langle T^{\mu}_{\mu} \rangle = \frac{1}{4} M.$$

<x> - momentum fraction experimentally measurable

Rest Energy Decomposition from Hamiltonian



$$f_f^H = \langle H_q \rangle / M = \frac{3}{4}\langle x \rangle_f(\mu), \qquad f_g^H = \langle H_g \rangle / M = \frac{3}{4}\langle x \rangle_g(\mu),$$

$$f_{\pi N}^N = \frac{1}{4}\frac{\sigma_{\pi N}}{M}, \qquad f_s^N = \frac{1}{4}\frac{\sigma_s}{M}, \qquad f_{\text{trace anomaly}}^N = \frac{1}{4}\frac{\langle H_{\text{ta}} \rangle}{M}$$

Y.B. Yang et al (χ QCD) [arXiv: 1511.15089]

Momentum fractions from CT18 (T.J. Hou et al, PRD, arXiv:1912.10053) at μ = 2 GeV and 250 GeV.

Rest Energy/Mass from Gravitational FF

- Gravitational Form factors from the EMT matrix elements

$$\langle P'|(T^{\mu\nu}_{q,g})(\mu)|P\rangle/2M_N = \bar{u}(P')[A_{q,g}(q^2,\mu)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(q^2,\mu)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha}{2M_N}$$
$$+ D_{q,g}(q^2,\mu)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{M_N} + \bar{C}_{q,g}(q^2,\mu)M_N\, g^{\mu\nu}]u(P)$$

  – A(0) and A(0)+ B(0) : momentum and angular momentum - [Ji]

  – D(0): D term (deformation of space = elastic property) - [Polyakov]

  – C-bar term: pressure-volume work - [Lorce, Liu]

$$E_0 = \langle P|T^{00}_{q+g}|P\rangle|_{\vec{P}=0}/2M_N = A_{q+g}(0)M_N + \bar{C}_{q+g}(0)M_N$$

$$M_N = \langle P|T^{\mu}_{\mu}|P\rangle|_{\vec{P}=0}/2M_N = A_{q+g}(0)M_N + 4\bar{C}_{q+g}(0)M_N$$

$$A_{q,g}(0,\mu) = \langle x\rangle_{q,g}(\mu)$$

Rest Energy/Mass from Gravitational FF

- Gravitational Form factors from the EMT matrix elements

$$3\bar{C}_{q,g}(0,\mu)M_N = -\langle P|(T^{ii}_{q,g})(\mu)|P\rangle|_{\vec{P}=0}/2M_N$$
$$= \langle P|(T_{q,g})^{\mu}_{\mu}(\mu) - (T^{00}_{q,g})(\mu)|P\rangle|_{\vec{P}=0}/2M_N$$

$$\bar{C}_q + \bar{C}_g = \frac{1}{4}(\sum_f f_f^N + f_a^N) - \frac{1}{4}(\langle x\rangle_q + \langle x\rangle_q) = 0$$

$$\partial_\nu T^{\mu\nu} = 0$$

$$E_0 = \langle P|T^{00}_{q+g}|P\rangle|_{\vec{P}=0}/2M_N = A_{q+g}(0)M_N + \bar{C}_{q+g}(0)M_N$$

$$= \frac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \frac{1}{4}(\sum_f f_f^N + f_a^N)M_N$$

Same as from Hamiltonian

$$M = \langle P|T^{\mu}_{\mu}|P\rangle|_{\vec{P}=0}/2M_N = A_{q+g}(0)M_N + 4\bar{C}_{q+g}(0)M_N$$

$$= (\sum_f f_f^N + f_a^N)M_N$$

Same as from trace of EMT

Role of Trace Anomaly --String tension in charmonium

- Heavy quarkonium is confined by a linear potential.
- Constant vacuum energy density and flux tube

$$V(r) = |\epsilon_{vac}|\, A\, r = \sigma\, r$$

- Infinitely heavy quark with Wilson loop

$$V(r) + r\frac{\partial V(r)}{\partial r} = \frac{\langle \frac{\beta}{2g}(\int d^3\vec{x}\, G^2)\, W_L(r,T)\rangle}{\langle W_L(r,T)\rangle}.$$

Dosch, Nachtmann, Rueter - 9503386; Rothe - 9504102

- For charmonium

$$2\sigma\langle r\rangle = \langle H_\beta\rangle_{\bar{c}c} = \frac{\langle \bar{c}c|\frac{\beta}{2g}\int d^3\vec{x}\, G^2|\bar{c}c\rangle}{\langle \bar{c}c|\bar{c}c\rangle}$$

$$\langle H_\beta\rangle_{\bar{c}c} = M_{\bar{c}c} - (1+\gamma_m)\langle H_m\rangle_{\bar{c}c}.$$

- Lattice calculation of charmonium (W. Sun et al.,2012.06228)

$$\langle H_\beta\rangle_{\bar{c}c} = 199\,\mathrm{MeV}$$ $\rightarrow$ σ = 0.153 GeV^2

KFL, 2103.15768

- Cornell potential fit of charmonium → σ = 0.164(11) GeV^2

Mateu:2018zym

Energy - Equilibrium Correspondence (EEC)

- Rest energy

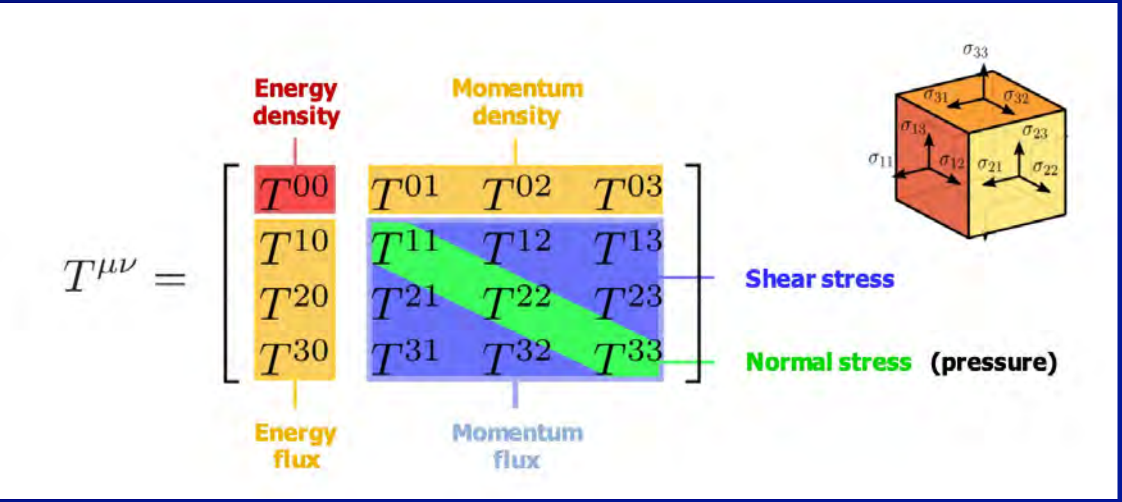
$$E_0 = \frac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \frac{1}{4}(\sum_f f_f^N + f_a^N)M_N$$

- Pressure balance – equilibrium

$$\langle P|(T^{ii}_{q,g})_{\rm M}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = -3\bar{C}_{q,g}(0,\mu)M_N$$

$$PV = -\frac{dE}{dV}V = -(\bar{C}_q + \bar{C}_g) = \frac{1}{4}(\langle x\rangle_q + \langle x\rangle_q) - \frac{1}{4}(\sum_f f_f^N + f_a^N) = 0$$

- Can one deduce the equation of state – E (V) ?



Generic bound state energy and equilibrium correspondence (EEC)

- Hadronic models:
  - MIT bag model (E_0 = BV+ $\Sigma_{q,g}$/R),

    pressure balance: $\partial E_0/\partial R = 0$
  - Skyrmion: Derrick's theorem ($r \to \lambda r$)

    $$\mathcal{L}_4 = \frac{1}{e^2} Tr\, [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 \longrightarrow 1/\lambda$$

    $$\mathcal{L}_2 = f_\pi^2\, Tr\, [\partial_\mu U^\dagger \partial^\mu U] \longrightarrow \lambda$$
  - Light front holographic QCD: potential from the ADS_5 action.
- Atomic and nuclear systems with r^n potential:
  - $E_0 = \langle H \rangle = \langle T \rangle + \langle V \rangle$
  - Equation of state: $E_0 = \epsilon_K / R^2 + \epsilon_V\, R^n$
  - Equilibrium: $F R = -R\, \partial E_0/\partial R = 2\langle T \rangle - n\langle V \rangle = 0$
  - Same as Virial theorem

Energy - Equilibrium Correspondence (EEC)

- Rest energy

$$E_0 = \frac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \frac{1}{4}(\sum_f f_f^N + f_a^N)M_N$$

- Pressure balance

$$PV = -\frac{dE}{dV}V = -(\bar{C}_q + \bar{C}_g) = \frac{1}{4}(\langle x\rangle_q + \langle x\rangle_q) - \frac{1}{4}(\sum_f f_f^N + f_a^N) = 0$$

- Inferred volume dependence $E = V^x, \;\; V\,dE/dV = dE/d(\log V) = x$

$$E_0 = E_T + E_S,$$

$$E_S = \frac{1}{4}[\langle H_m\rangle + \langle H_a\rangle] \propto V,$$

$$E_T = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle \propto V^{-1/3}$$

$$PV = -\frac{d\,E_0}{d\,V}V = -E_S + \frac{1}{3}E_T = 0$$

Trace Anomaly and Gluon Condensate

- Equation of state $$E_0 = \epsilon V + \epsilon_K V^{-1/3},$$

  where $\epsilon = \frac{E_S}{V}, \quad \epsilon_K = E_T V^{1/3}$ are constants

- Picture: Nucleon is a bubble in the sea of gluon condensate, where

$$\epsilon = -\epsilon_{\rm vac}$$

N.B. $\langle OG_2 \rangle_{\rm correlated} = \langle OG_2 \rangle - \langle O \rangle \langle G_2 \rangle$

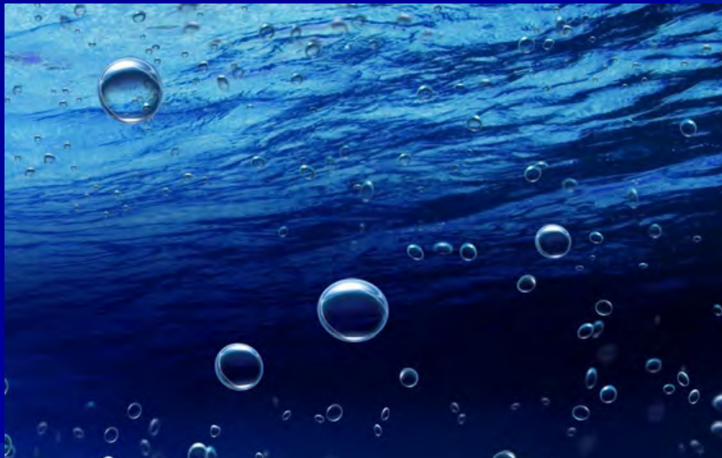
$$\epsilon_{vac} = \frac{\beta(g)}{2g} \langle 0 | F^{\alpha\beta} F_{\alpha\beta} | 0 \rangle < 0$$

$$V = \frac{E_S}{|\epsilon_{\rm vac}|}$$

- Trace anomaly gives a negative constant pressure → confinement

Same as in charmonium $$V(r) = |\epsilon_{vac}| \, A \, r = \sigma \, r$$

Bali ('97), Baker ('18)



- Many facets of color confinement
  - Dual superconductor
  - Magnetic monopole
  - Center vortices

Energy - Equilibrium Correspondence (EEC) Principle

- For bound states, the decomposition of their rest energies with relevant and meaningful physical contents are those having EEC.
  - Gravitational FF without $\bar{C}$

$$E_0 = (A_q(0) + A_g(0))\, M_N = (\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))\, M_N$$

No potential energy, no relation to pressure

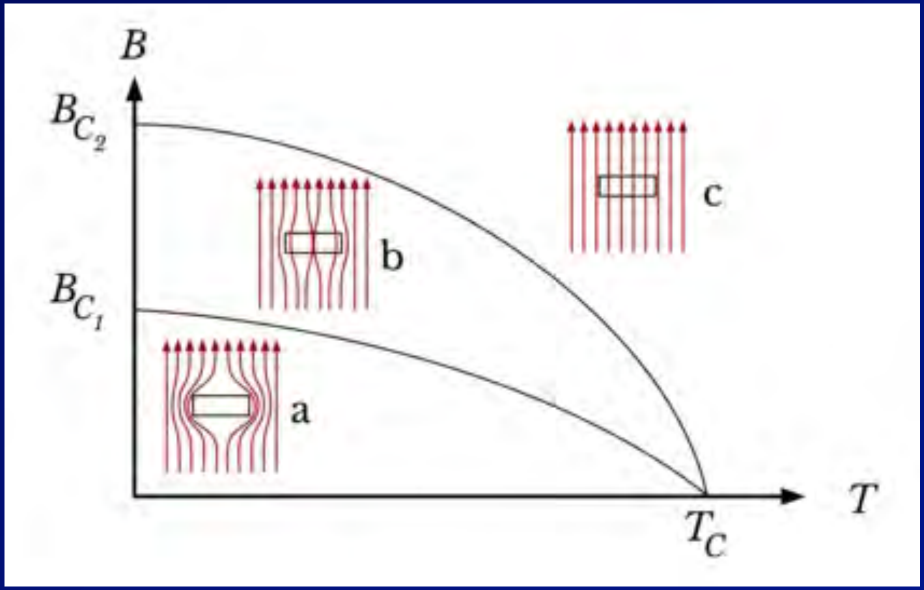
  - Trace of EMC FF

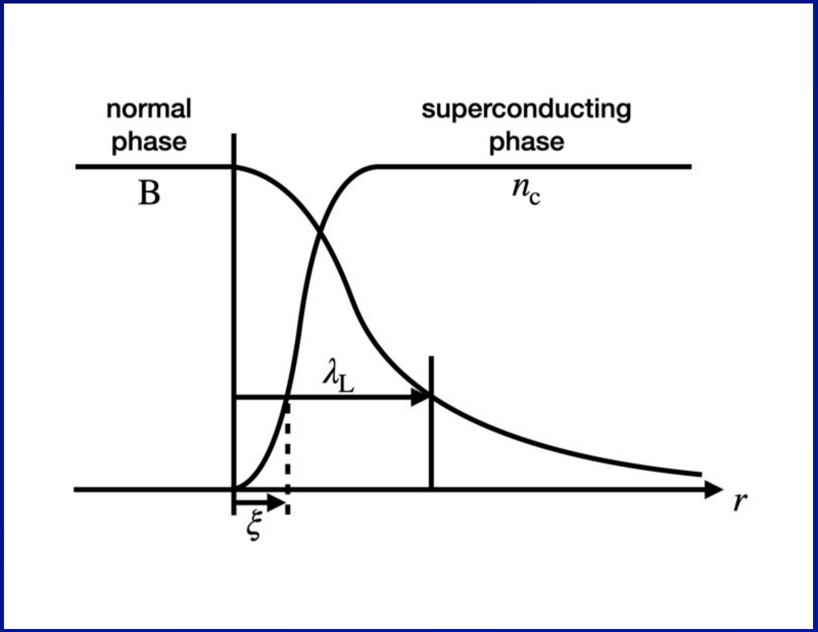
$$M = \langle T^\mu_\mu \rangle = (A_q(0) + A_g(0))\, M_N = (\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))\, M_N$$

Does not have trace anomaly

- Virial Theorem
  - Coulomb potential: H = T + V, 2<T> = - <V>
  - E_0 = - <T>, E_0 = ½ <V> are not good physics explanation of the decomposition of the binding energy.

Type II Superconductor





Ginzburg–Landau equations

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar\nabla - 2e\mathbf{A})^2\psi = 0$$

$$\nabla \times \mathbf{B} = \mu_0\mathbf{j} \;\; ; \;\; \mathbf{j} = \frac{2e}{m}\,\mathrm{Re}\{\psi^*\,(-i\hbar\nabla - 2e\mathbf{A})\,\psi\}$$

$$|\psi|^2 = n_s$$

London penetration depth

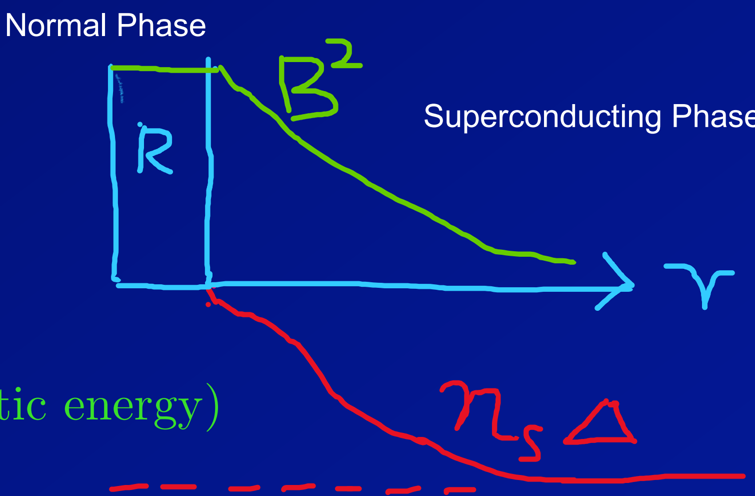
$$\lambda_L = \sqrt{\frac{m}{4\mu_0 e^2\psi_0^2}}$$

Coherent length ξ

Type II: $\kappa = \lambda_L/\xi > 1/\sqrt{2}$

Energetics and Equilibrium

- Type II superconductor



$$F = F_s + F_B + F_{sc}$$

$$F_s = \text{ cost of condensation energy}$$

$$F_B = \int dv\, B^2/2\mu_0 \ \ (\text{magnetic energy})$$

$$F_{sc} = 1/2 \int dv\, \lambda_L^2 J_s \cdot J_s \ \ (\text{supercurrent kinetic energy})$$

- Variational model (J.R. Clem, Jour. Low Temp. Phys. 18, 5/6 (1975))

$$\frac{|\psi|^2}{n_0} = \frac{n_s}{n_0} = \frac{\rho^2}{\rho^2 + R^2} \underset{\rho \to \infty}{\longrightarrow} 1$$

$$\frac{1}{\sqrt{2}H_c}\frac{E}{l} = \phi_0 H_c'/4\pi \ \text{ where } \phi_0 = hc/2e, \ \sqrt{2}H_c = \kappa\phi_0/2\pi\lambda_L^2$$

Equation of state

$$F'/l = \underbrace{\kappa R'^2/8}_{F_s} + \underbrace{1/8\kappa + K_0(R')/2\kappa R'\, K_1(R')}_{F_B + F_{sc}}, \ \text{ where } \ R' = R/\lambda_L$$

EEC $\quad -\dfrac{dF'/l}{dA} A = 0$

Trace Anomaly and Cosmological Constant

- A constant vacuum energy density is analogous to the cosmological constant in the $g^{\mu\nu}$ term as Einstein introduced for a static universe.

$$R_{\mu\nu} + \frac{1}{2} R\, g_{\mu\nu} - \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \Lambda = 4\pi G \rho$$

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \frac{1}{4} g^{\mu\nu} (T^{\rho}_{\rho})$$ in QCD

- Friedmann equation of Friedmann-Robertson-Walker scale parameter for the accelerating expansion of the Universe

$$\rho_{\rm vac} = \frac{\Lambda}{8\pi G}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \rho_{\rm vac} + 3(P + P_{\rm vac}))$$

$$P_{\rm vac} = -\frac{\Lambda}{8\pi G}$$

- Trace anomaly behaves like a `hadron cosmological constant’.

SC, Hadrons, Cosmos

- Type II Superconductor

$$P_s = -\frac{\partial F_s}{\partial V} < 0, \quad P_{B+sc} = -\frac{\partial F_B + F_{sc}}{\partial V} > 0$$

- Hadrons

$$P_{tr} = -\frac{\partial E_S}{\partial V} < 0, \quad P_{q+g} = -\frac{\partial E_T}{\partial V} > 0$$

- The common theme is the existence of a condensate
- Hadrons: condensates from breaking of conformal and chiral symmetries. SC: Cooper pair condensate from gauge symmetry breaking.
- Cosmos – cosmological constant is vacuum energy itself

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \rho_{\text{vac}} + 3(P + P_{\text{vac}})) \qquad P_{\text{vac}} < 0$$

anti-gravitates

- Suppose: Outside of our Universe, there is a condensate with negative Λ, so that it is positive in the Universe. This scenario would be similar to hadrons and vortices.

Cf. -- S. K. Blau. E. I. Guendelman. and A. H. Guth. Phys. Rev. 035. 1747 (1987)

Pion Mass Puzzle (?)

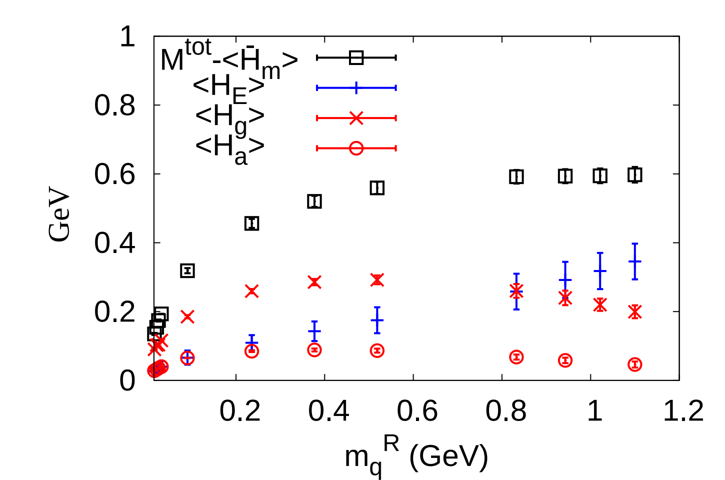
- Pion mass in terms of trace of EMT

$$m_\pi = m\langle\pi|\bar{\psi}\psi|\pi\rangle + \langle\pi|\frac{\beta}{2g}G_{\mu\nu}^2 + m\gamma_m\bar{\psi}\psi|\pi\rangle$$

- Gellmann-Oakes-Renner relation and Feynman-Hellman relation

$$m_\pi^2 = -2m\langle\bar{\psi}\psi\rangle/f_\pi^2, \quad \sum_f m_f\frac{\partial m_\pi}{\partial m_f} = \sum_f m_f\langle\pi|\bar{\psi}\psi|\pi\rangle_f$$

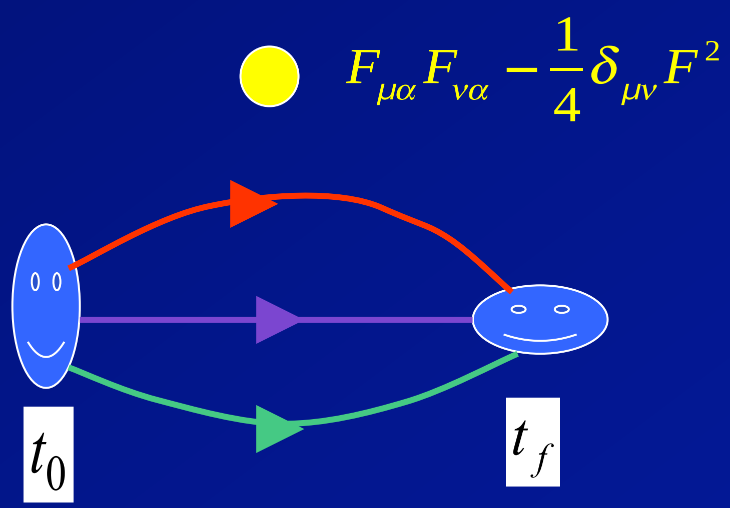
- $m\langle\pi|\bar{\psi}\psi|\pi\rangle = m_\pi/2$ But, why should the trace anomaly be proportional to $\sqrt{m}$? V → 0 ?



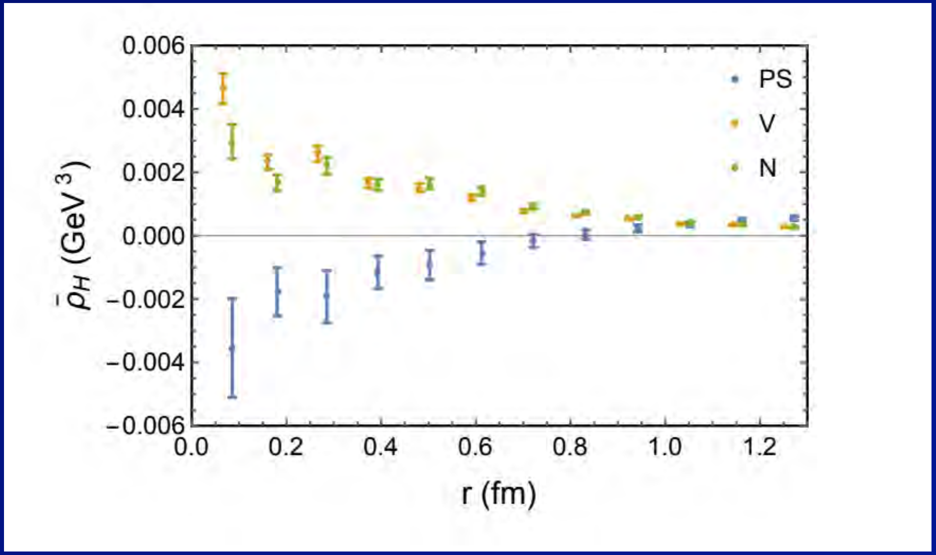
Y.B. Yang et al. (χQCD), PRD (2015); 1405.4440

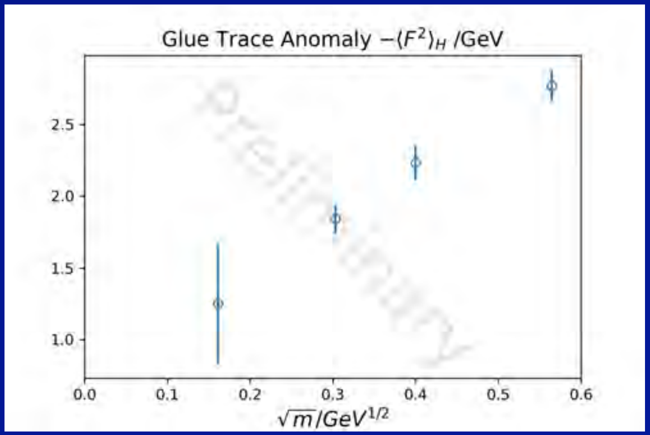
Trace anomaly Distribution

- Distribution as a function of the relative distance between the glue operator and the sink positions.



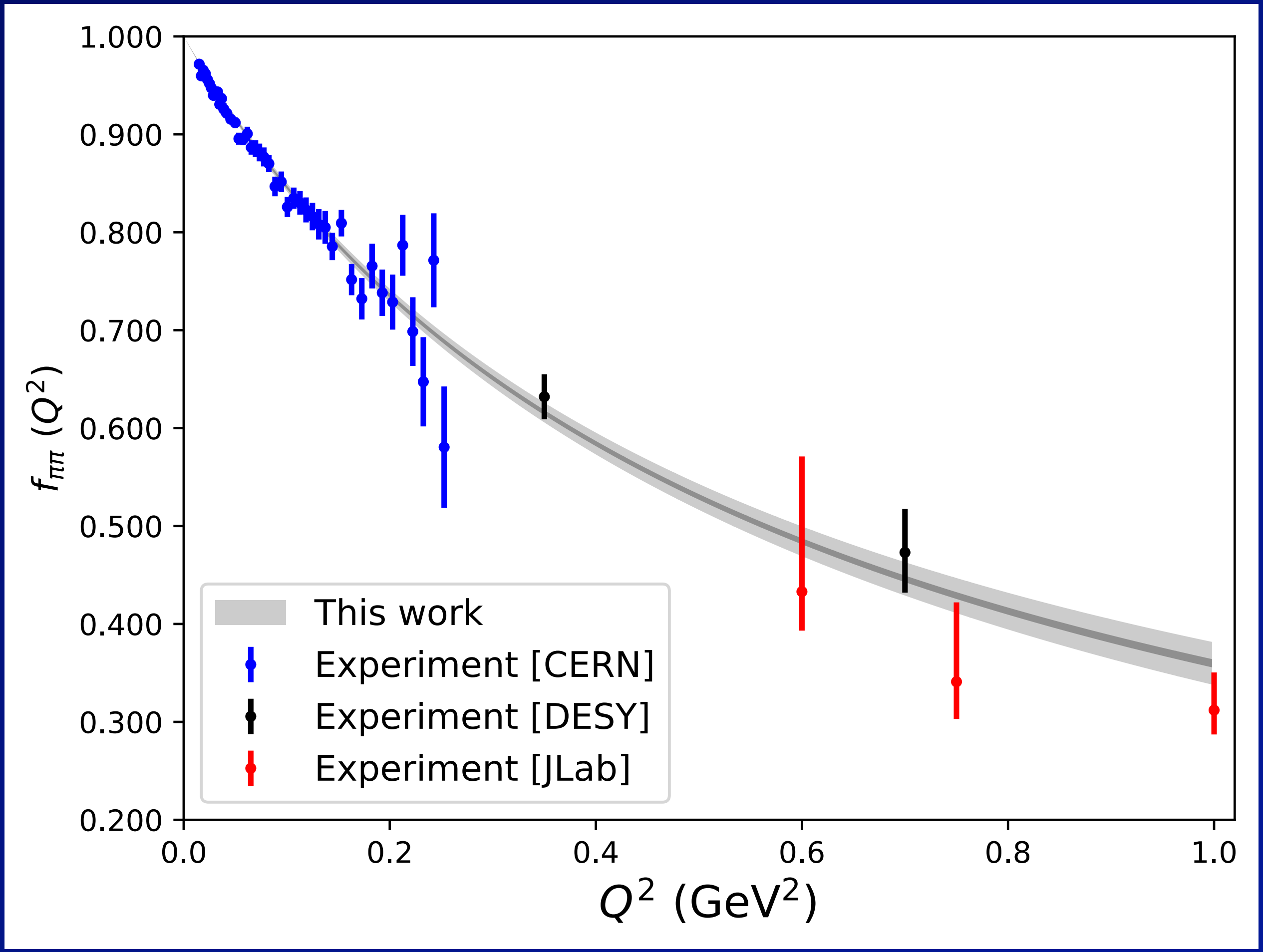
- F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)





- It changes sign in pion so that the integral approaches zero at the chiral limit.

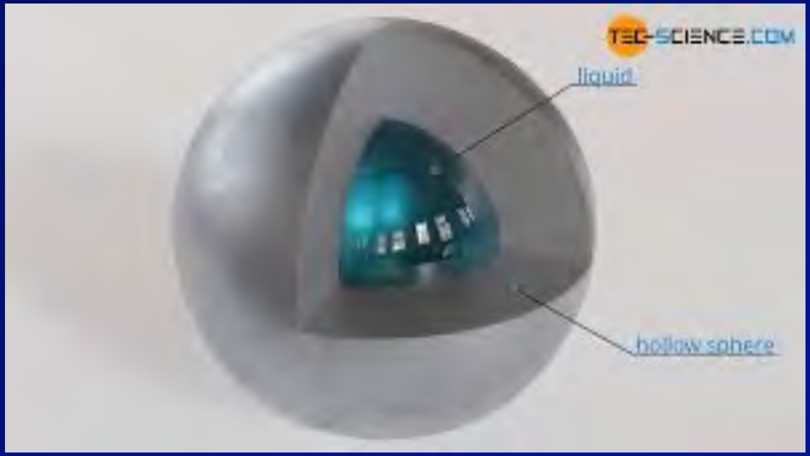
Pion Electric Form Factor

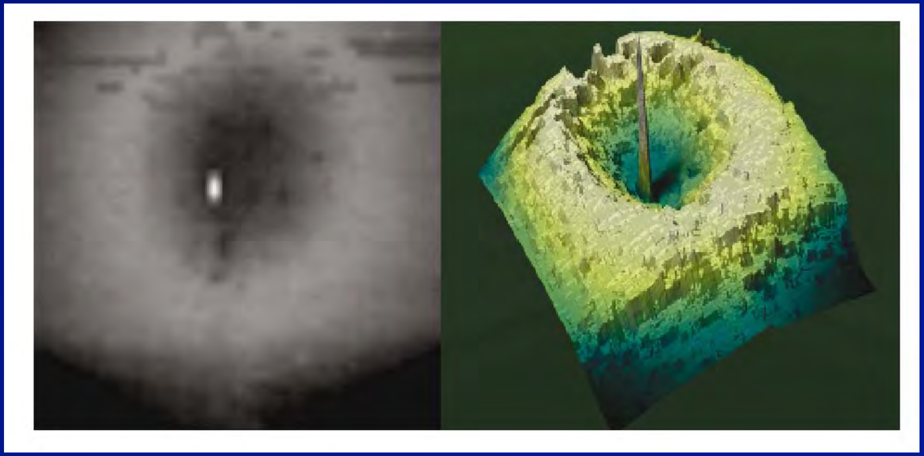


G.~Wang et al. [chiQCD], PRD, 2006.05431

Ring-shaped Type II Superconductor

A. Groeger et al., PRL 90, 237004 (2003)



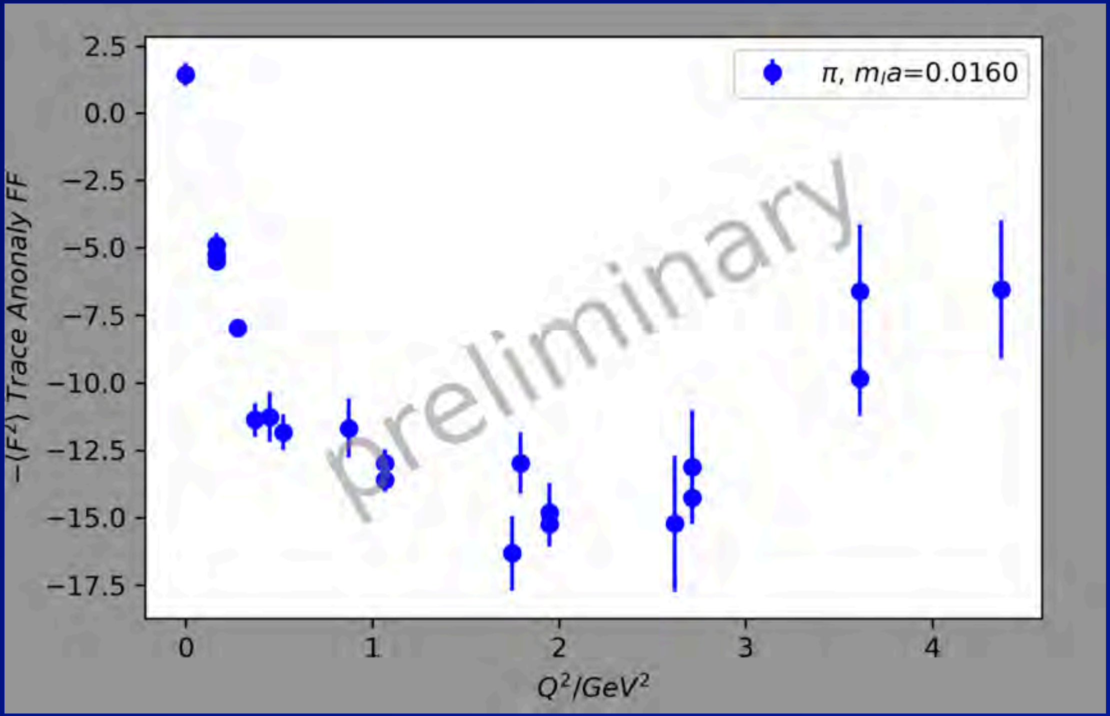


Pion with shell of positive trace anomaly and an inner core with negative trace anomaly.

Niobium, normal conducting vortex ring around a superconducting region,

Pion trace anomaly FF

m_π = 340 MeV



Photoproduction of J/Ψ at threshold
and Drell-Yan process

X.B. Tong, J.P. Ma and F. Yuan
arXiv:2203.13493

Summary and Challenges

- It seems that from femto-scale to micro-scale to that of the cosmos, Nature adopts the same mechanism (condensates) for confinement . Similar mechanism (vacuum energy) for acceleration.
- Glue part of trace anomaly responsible for confinement. Thus, glue condensate should an order parameter for confinement – deconfinement transition.
- m_q ← Higgs mechanism
- Quark condensate ← chiral symmetry breaking (restoration at T and μ)
- Trace anomaly (confinement) ← conformal symmetry breaking (conformal phases with multi-flavors and SU(N); finite $T > T_c$)
- Chiral symmetry breaking and conformal symmetry breaking are apparently linked in the case of the pion trace anomaly distribution → needs an explanation.
- Challenges for EIC and COMPASS is to measure the trace anomaly form factors for the proton and, particularly, the pion. Photoproduction of J/Ψ at threshold is a possibility.

Back up slides

Pion Mass Puzzle

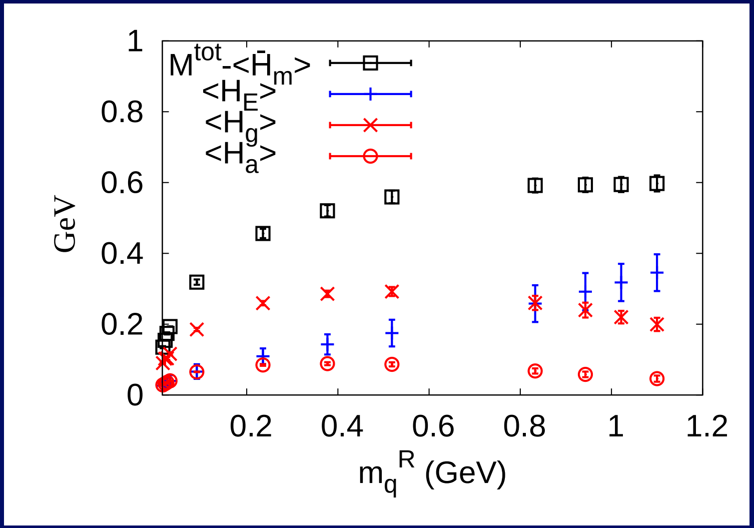
- Pion mass in terms of trace of EMT

$$m_\pi = m\langle\pi|\bar{\psi}\psi|\pi\rangle + \langle\pi|\frac{\beta}{2g}G^2_{\mu\nu} + m\gamma_m\bar{\psi}\psi|\pi\rangle$$

- Gellmann-Oakes-Renner relation

$$m_\pi^2 = -2m\langle\bar{\psi}\psi\rangle/f_\pi^2, \quad m_\pi^2 \propto m$$

- $\langle\pi|\bar{\psi}\psi|\pi\rangle \propto 1/\sqrt{m}$ But, why should the trace anomaly be proportional to $\sqrt{m}$? V → 0 ?



Y.B. Yang et al. (χQCD), PRD (2015); 1405.4440

Virial Theorem

- D dimension
- Mass

$$M = E_T + E_S, \quad E_T = \langle \overline{T}^{00} \rangle = \frac{D-1}{D} M, \ E_S = \frac{1}{D} \langle T^{\mu}_{\mu} \rangle = \frac{1}{D} M$$

- Pressure

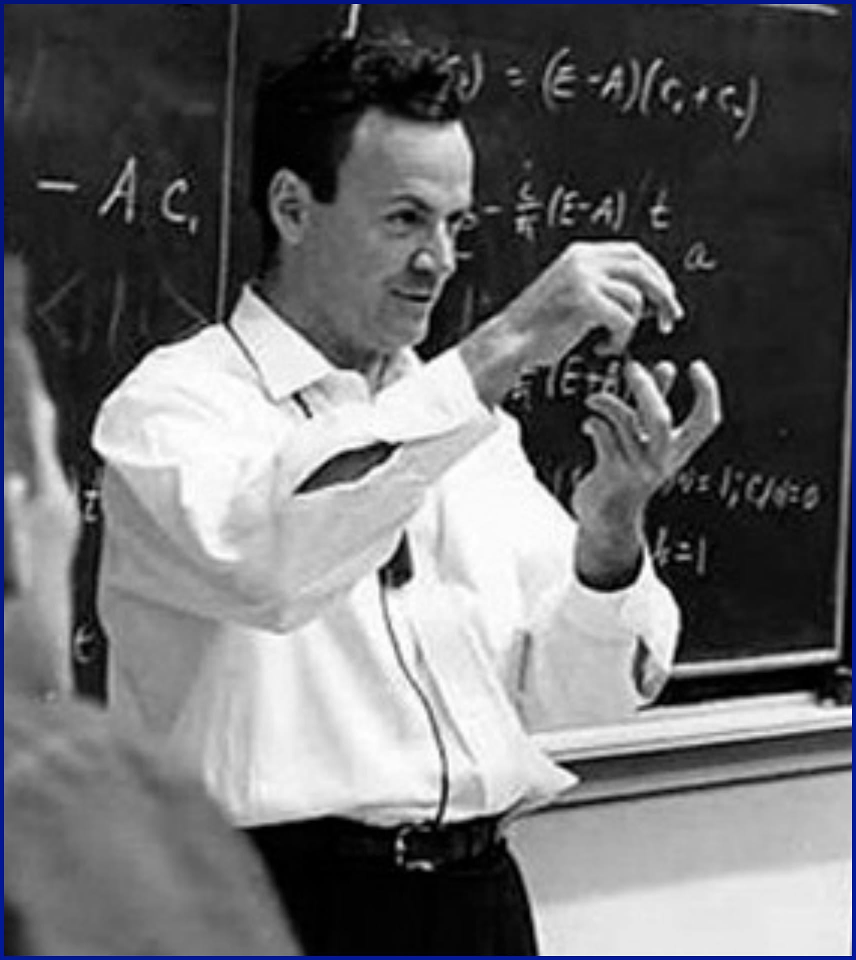
$$PV = \frac{1}{D-1} \langle T^{ii} \rangle = -\frac{1}{D-1} [\langle T^{\mu}_{\mu} \rangle - \langle T^{00} \rangle]$$

$$= -\frac{1}{D-1} [D E_s - E_T - E_s] = -E_s + \frac{1}{D-1} E_T = 0$$

Feynman's quote

From the very beginning of his first-ever lecture comes this timeless gem (mentioned in Daniel Bor's excellent *The Ravenous Brain*) that set the tone for both Feynman's academic contribution and his broader cultural legacy:

If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words?



[David Gross]

I believe it is the atomic hypothesis that all things are made of atoms — little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.