Topology and dynamical quarks in hot QCD

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QCD topology with dynamical quarks

- partition function $Z = \int \mathscr{D}U \ \prod_f \det\left(D[U] + m_f\right) \cdot e^{-S_g[U]}$
- integer topological charge $Q[U]$
- index theorem $\rightarrow$ $D[U]$ has $|Q[U]|$ zero eigenvalues
- quark determinant suppresses topology
- to have exact zero evs. chiral D needed (overlap)
- quenched with reweighting

Reweighting

- only small eigenvalues expected to correlate with topology
  $\rightarrow$ det approximated with product of eigenvalues $|\lambda| < \lambda_{\text{cut}}$

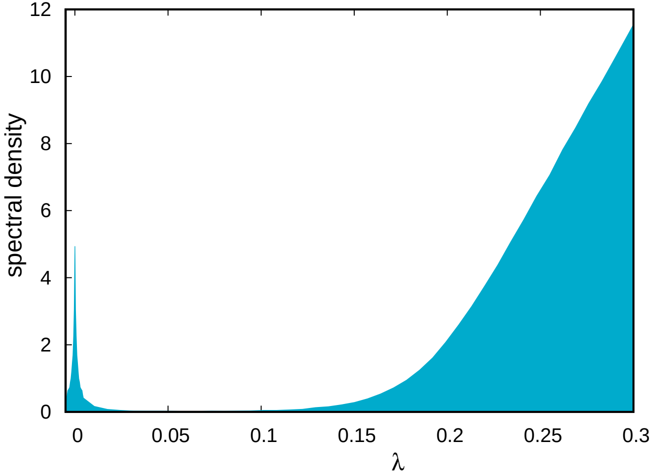
- $$\det \approx w = \prod_{\lambda < \lambda_{\text{cut}}} (\lambda + m)^{N_f}$$ for each configuration

- reweighted observables: $$\langle \mathcal{O} \rangle = \frac{\sum_i \mathcal{O}_i w_i}{\sum w_i}$$

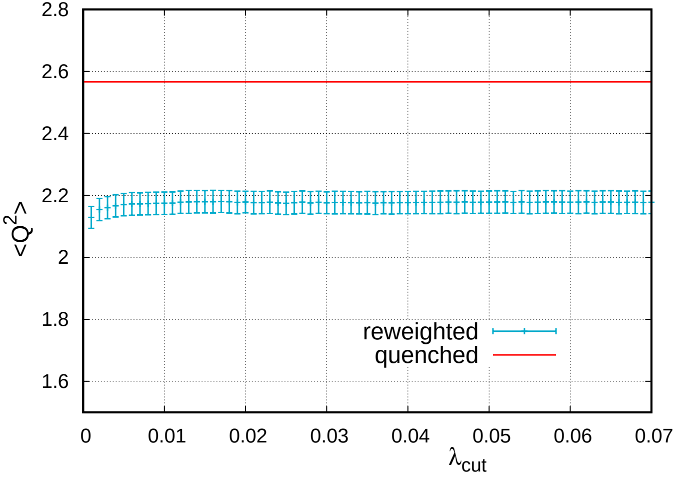
- works for topology if λ_{cut} small and/or m not too small

The spectral density of the overlap at $T = 1.1\,T_c$

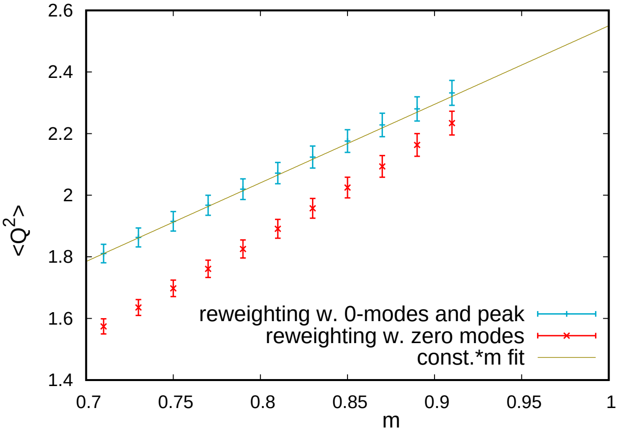
quenched, Wilson $\beta = 6.13$, $N_t = 8$, exact zero modes removed



Reweighting χ_{top} with 0-modes and the peak
($m = 0.85$)



Is it enough to reweight with the zero modes?



- eigenvalues in the peak correlated with topology
- slope$= \langle Q^2 \rangle_{\text{quenched}} \quad \rightarrow \quad \langle Q^2 \rangle_{\text{reweighted}} = m \cdot \langle Q^2 \rangle_{\text{quenched}}$

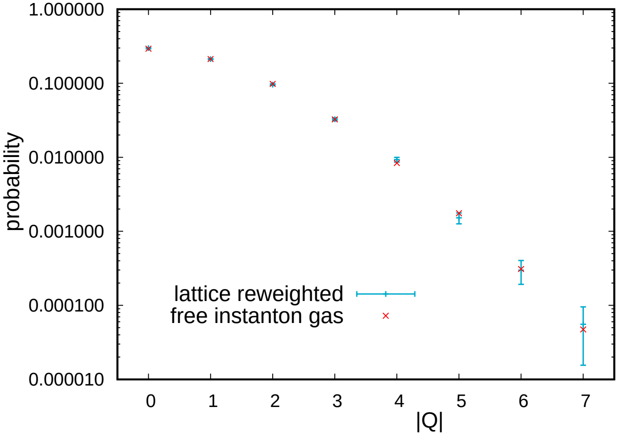
Free instanton gas?

- Assume noninteracting charge ± 1 topological objects
- n_i, n_a independent identical Poisson distributions
- If $\langle (n_i - n_a)^2 \rangle = \chi V,$ then $\langle n_i \rangle = \langle n_a \rangle = \chi V/2$
- Topological objects $\Leftrightarrow$ low modes
  - $|n_i - n_a|$ exact zero modes
  - The rest of the peak:
    $n_i + n_a - |n_i - n_a|$ near zero modes (zero mode zone $|\lambda| < \lambda_{zmz}$)
- ideal gas: $\Rightarrow \chi V = \langle n_+ + n_- \rangle \Rightarrow \lambda_{ZMZ}$
- $\chi \Longrightarrow \lambda_{ZMZ}$

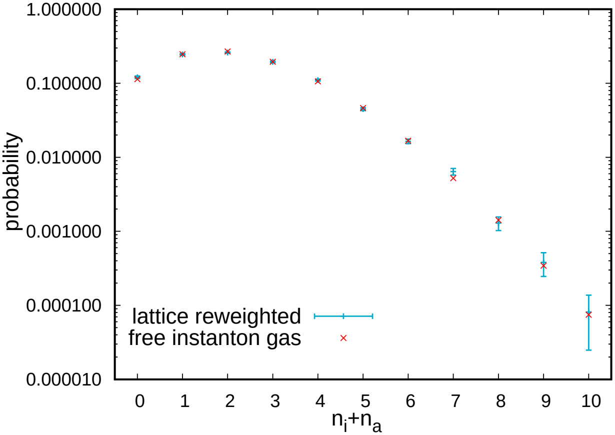
How to understand $\langle Q^2 \rangle = m \cdot \langle Q^2 \rangle_{\text{quenched}}$?

- Each topological object $\rightarrow$ $\lambda + m$ factor in determinant
- If $\lambda_{\text{zmz}} \ll m \rightarrow \lambda + m \approx m$
- Suppression: $P(n_i) \rightarrow m^{n_i}\, e^{-\frac{\chi V}{2}} \frac{(\chi V/2)^{n_i}}{n_i!}$
- Distribution still Poisson, but with $\chi \rightarrow m\chi$
- Suppression of susceptibility by ZMZ: $m\ \chi_{\text{quenched}}$

Distribution of $|Q|$ after reweighting with ZMZ

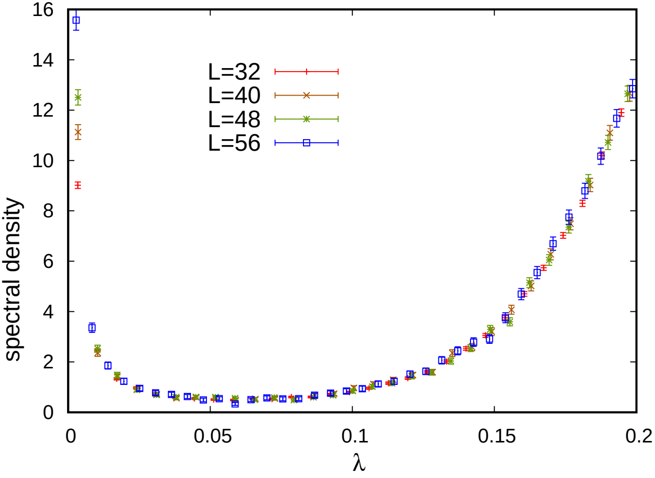


Distribution of $n_i + n_a$ after reweighting with ZMZ



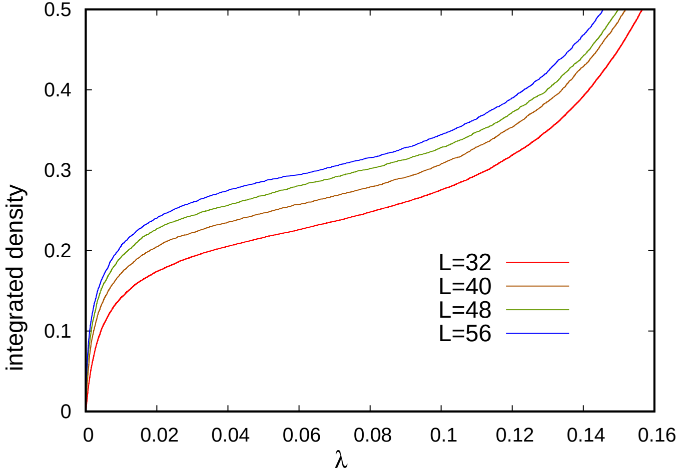
Volume dependence of the spectral density

Lattice $N_t = 8$, $T = 1.045\,T_c$



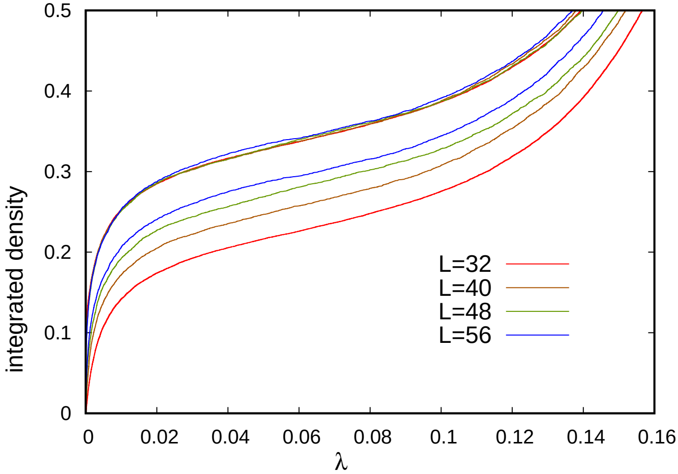
Integrated spectral density

Lattice $N_t = 8$, $T = 1.045\,T_c$

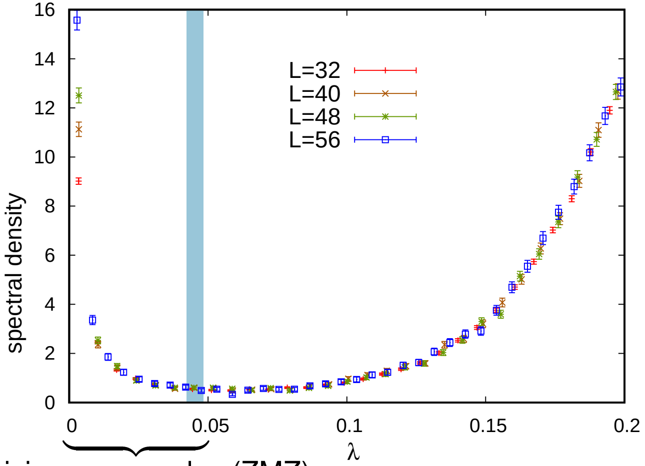


Integrated spectral density including zero modes

Lattice $N_t = 8$, $T = 1.045\,T_c$



The zero mode zone obtained from $\langle Q^2 \rangle = \langle n_+ + n_- \rangle$



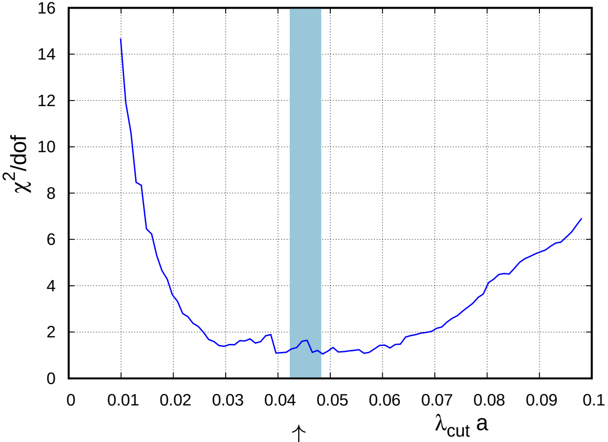
mixing zero modes (ZMZ)

Is λ_{zmz} a special point in the spectrum?

- Distribution of number of $|\lambda| < \lambda_{\text{zmz}}$ eigenvalues Poisson
- Is this only true for λ_{zmz} obtained from $\langle Q^2 \rangle$?

λ_{zmz} is indeed special

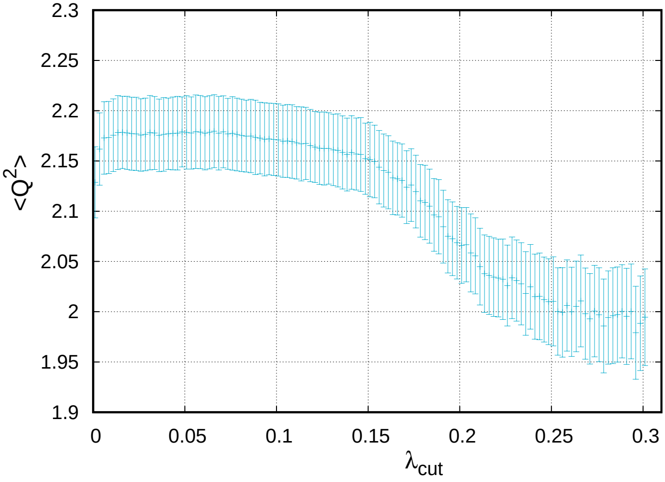
Fit distribution of the number of $|\lambda| < \lambda_{cut}$ eigenvalues with Poisson



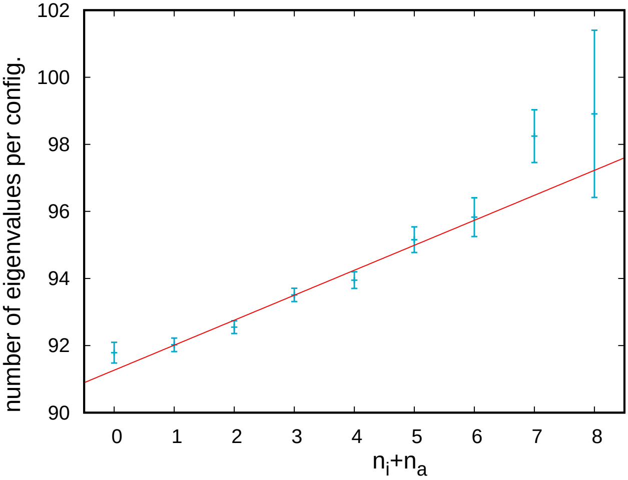
$\uparrow$

λ_{zmz} obtained from $\langle Q^2 \rangle = \langle n_+ + n_- \rangle$

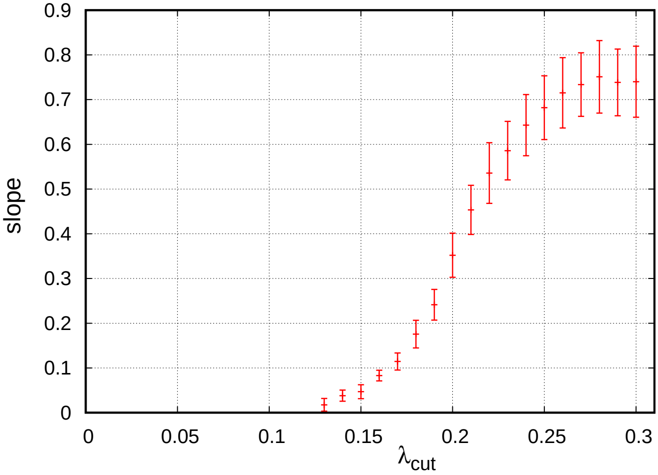
Reweighting beyond the zero mode zone $m = 0.85$



Number of eigenvalues in the ZMZ vs. in $[\lambda_{zmz}, 0.3]$



Slope of # of evs. in $[\lambda_{zmz}, \lambda_{cut}]$ vs. $n_i + n_a$

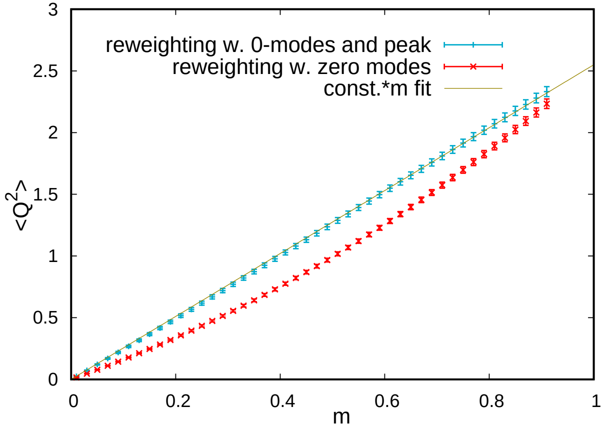


Conclusions

- Above T_c ideal gas of charge ± 1 topological objects
- Spectral peak: mixing instanton-antiinstanton 0-modes
- $V \to \infty$ area under peak $\to$ density of top. objects (χ_{top})
- Peak suppressed by the ZMZ proportionally to m^{N_f}
- Lower bulk of the spectrum: correlation with topology
  $\to$ further suppression of topological fluctuations

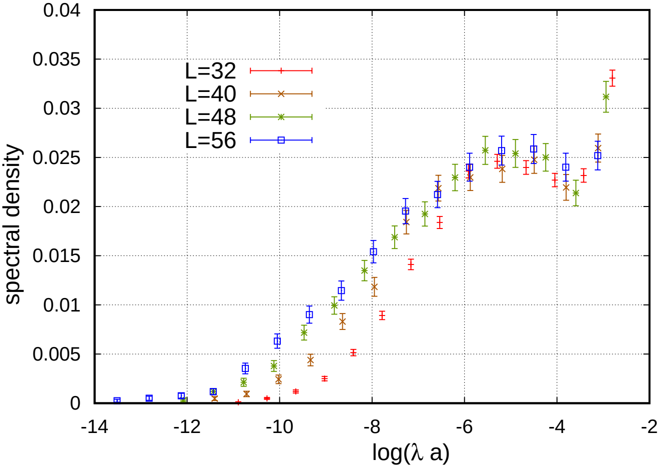
Backup slides

Reweighted susceptibility vs. quark mass



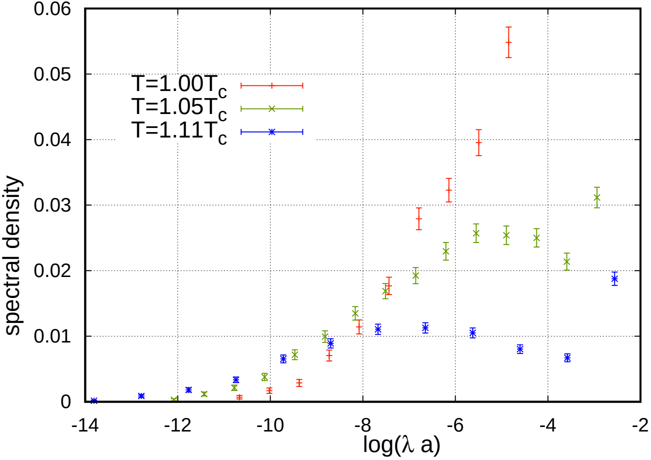
Spectral density at $T = 1.045\,T_c$
(density of $log\lambda$)

If $\rho(\lambda) \propto \lambda^{\alpha}$, then $\tilde{\rho}(y) \propto e^{(\alpha+1)y}$ $(y = \log\lambda)$



Spectral density at at different temperatures above T_c

(density of $log\lambda$)



Effective quark action vs. Polyakov loop

different fraction of the lowest eigenvalues included