Chiral spin symmetry of QCD and its implications for hadrons in vacuum and for hot QCD

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Symmetries of electric and magnetic interactions in electrodynamics

$$
\begin{aligned}
div\mathbf{E} &= 4\pi\rho \\
rot\mathbf{B} - \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} &= \frac{4\pi}{c}\mathbf{j} \\
rot\mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} &= 0 \\
div\mathbf{B} &= 0
\end{aligned}
$$

How do we define **E** and **B** in a given Lorentz frame?

$$\mathbf{F} = q\mathbf{E} + q\frac{\mathbf{v}}{c} \times \mathbf{B}$$

Possible to measure directly **F** in electrodynamics but not possible in quantum chromodynamics. Is there another method to distinguish **E** and **B**? Yes!

Consider charges to be particles with $s = 1/2$. Two states: $\uparrow$ (spin up) and $\downarrow$ (spin down):

$$\begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}$$

Symmetries of electric and magnetic interactions in electrodynamics

Instead of spin-up and spin-down consider helicities (chiralities for massless particles):

$$R: \ \mathbf{s} \cdot \mathbf{p} > 0$$
$$L: \ \mathbf{s} \cdot \mathbf{p} < 0$$

$$\begin{pmatrix} R \\ L \end{pmatrix}$$

Consider a $SU(2)$ chiral spin transformation that mixes R and L:

$$\begin{pmatrix} R \\ L \end{pmatrix} \rightarrow \begin{pmatrix} R' \\ L' \end{pmatrix} = \exp\left(i\frac{\varepsilon^n \sigma^n}{2}\right)\begin{pmatrix} R \\ L \end{pmatrix}$$

What happens with the charge density ρ?

$$R'^\dagger R' + L'^\dagger L' = R^\dagger R + L^\dagger L$$

i.e.

$$\rho' = \rho$$

Charge density is invariant under the chiral spin transformation.

Symmetries of electric and magnetic interactions in electrodynamics

What happens with the current density $\mathbf{j} = \rho \mathbf{v}$?

Upon the chiral spin transformation **v** and **j** change.

$$\mathbf{F}_E = q\mathbf{E}$$

$$\mathbf{F}_B = \sim \mathbf{j} \times \mathbf{B}$$

Interaction of a charge with the electric field is invariant under the chiral spin transformation, while interaction of a current with the magnetic field is not.

We can distinguish the electric and magnetic fields by the chiral spin symmetry!

The electric part of the EM theory is more symmetric than the magnetic part!

$$\mathcal{L} = \mathcal{L}(\mathbf{E}, \mathbf{B}) - \rho\phi + \mathbf{j} \cdot \mathbf{A} + matter\ \ part$$

Symmetries of quantum chromodynamics

$$\mathcal{L} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m)q + g\bar{q}\gamma^{\mu}\mathbf{T}q \cdot \mathbf{A}_{\mu} - \frac{1}{4}(\mathbf{F}_{\mu\nu} \cdot \mathbf{F}^{\mu\nu})$$

In the chiral limit the Lagrangian can be separated into the right- and left handed parts of quarks $q = R + L$:

$$\mathcal{L} = \bar{R}i\gamma^{\mu}\partial_{\mu}R + \bar{L}i\gamma^{\mu}\partial_{\mu}L + g\bar{R}\gamma^{\mu}\mathbf{T}R \cdot \mathbf{A}_{\mu} + g\bar{L}\gamma^{\mu}\mathbf{T}L \cdot \mathbf{A}_{\mu} - \frac{1}{4}(\mathbf{F}_{\mu\nu} \cdot \mathbf{F}^{\mu\nu})$$

This Lagrangian is invariant under the $U(1)_A$ transformation:

$$q \longrightarrow e^{i\theta\gamma_5} q$$

It is also invariant under $SU(2)_R \times SU(2)_L$ chiral transformation:

$$\begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow \begin{pmatrix} u'_R \\ d'_R \end{pmatrix} = \exp\left(i\frac{\varepsilon^n\sigma^n}{2}\right)\begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow \begin{pmatrix} u'_L \\ d'_L \end{pmatrix} = \exp\left(i\frac{\varepsilon^n\sigma^n}{2}\right)\begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

Symmetries of quantum chromodynamics

The chromoelectric field is defined via interaction with the color charge

$$\mathbf{F} = Q^a \mathbf{E}^a; \quad Q^a = \int d^3x \; q^\dagger(x) T^a q(x), \quad a = 1, ..., 8$$

It is invariant under $SU(2)_{CS}$: $\quad q \to q' = \exp\left(i \frac{\varepsilon^n \Sigma^n}{2}\right) q, \quad \Sigma = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$

$$\overline{q}(x)\gamma^\mu T^a q(x) \; A^a_\mu = q^\dagger(x) T^a q(x) \; A^a_0 + \overline{q}(x)\gamma^i T^a q(x) \; A^a_i$$

In a given Lorentz frame interaction of quarks with the electric part of the gluonic field is chiral spin invariant like in electrodynamics.

$SU(2)_{CS} \times SU(N_F) \subset SU(2N_F)$; $SU(2N_F)$ is also a symmetry of the color charge.

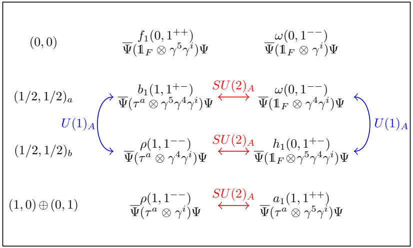
The color charge and its interaction with the chromoelectric field have a $SU(2N_F)$ symmetry that is larger than the chiral symmetry of the QCD Lagrangian as a whole.

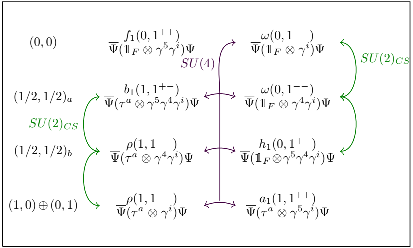
The fundamental vector of $SU(2N_F)$ at $N_F = 2$

$$\Psi = \begin{pmatrix} u_{\rm R} \\ u_{\rm L} \\ d_{\rm R} \\ d_{\rm L} \end{pmatrix}.$$

L.Ya.G., EPJA, 2015; L.Ya.G., M.Pak,PRD, 2015

CS and $SU(4)$ multiplets





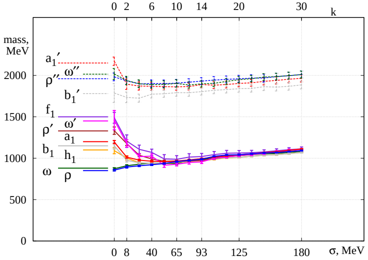
Observation of the chiral spin symmetry at T=0

Banks-Casher:

$$i\gamma_\mu D_\mu \psi_n(x) = \lambda_n \psi_n(x), \qquad < \bar{q}q >= -\pi\rho(0).$$

Low mode truncation, M.Denissenya, L.Ya.G., C.B.Lang, 2014-2015:

$$S = S_{Full} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|.$$



$SU(2)_{CS}$ and $SU(4)$ symmetries.
The magnetic interaction is located predominantly in the near zero modes while the confining electric interaction is distributed among all modes.

Confinement and chiral symmetry breaking are not directly related.

Observation of the chiral spin symmetry at T=0

Minkowski QCD Hamiltonian in Coulomb gauge:

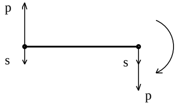
$$H_{QCD} = H_E + H_B + \int d^3x \Psi^\dagger(\mathbf{x})[-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla}]\Psi(\mathbf{x}) + H_T + H_C,$$

with the transverse and instantaneous "Coulombic" interactions to be:

$$H_T = -g \int d^3x \, \Psi^\dagger(\mathbf{x})\boldsymbol{\alpha} \cdot t^a \mathbf{A}^a(\mathbf{x}) \, \Psi(\mathbf{x}) \ ,$$

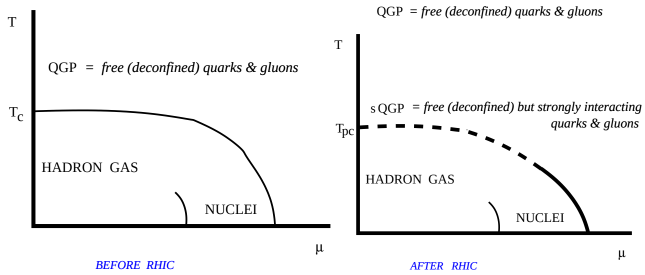
$$H_C = \frac{g^2}{2} \int d^3x \, d^3y \, J^{-1} \ \rho^a(\mathbf{x}) F^{ab}(\mathbf{x}, \mathbf{y}) \, J \, \rho^b(\mathbf{y}) \ .$$

The confining "Coulombic" part is $SU(2N_F) \times SU(2N_F)$-symmetric.



Before and after RHIC

What happens with hadrons in the medium upon increasing T?



The chiral restoration crossover is observed at $T = 120 - 180$ MeV with the pseudocritical temperature at $T_{ch} \sim 155$ MeV.

We need objective information about degrees of freedom. Can be obtained in computer simulations on the lattice.

Before and after RHIC

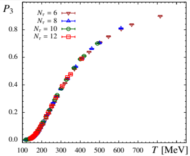


Figure : P. Petreczky and H.-P. Schadler, 2015.

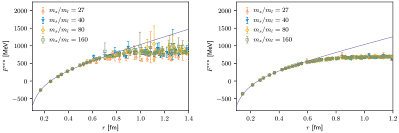


Figure : Left: $T = 141$ MeV; right: $T = 166$ MeV.
D. A. Clarke, O. Kaczmarek, F. Karsch, A. Lahiri, 2019.

There are no evidences that above T_{ch} the degrees of freedom are (quasi)quarks and (quasi)gluons!

Correlation functions

The most detailed information about QCD is encoded in correlation functions

$$C_\Gamma(t,x,y,z) = \langle O_\Gamma(t,x,y,z)\, O_\Gamma(0,\mathbf{0})^\dagger \rangle \, .$$

They carry the full spectral information $\rho_\Gamma(\omega,\mathbf{p})$

$$C_\Gamma(t,\mathbf{p}) = \int_0^\infty \frac{d\omega}{2\pi}\, K(t,\omega)\rho_\Gamma(\omega,\mathbf{p}), \quad K(t,\omega) = \frac{\cosh(\omega(t-1/2T))}{\sinh(\omega/2T)} \, .$$

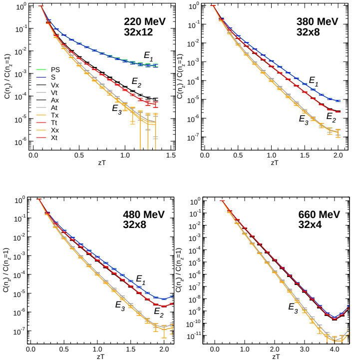
The spatial and temporal correlators are defined as

$$C_\Gamma^s(z) = \sum_{x,y,t} C_\Gamma(t,x,y,z) \, ,$$

$$C_\Gamma^t(t) = \sum_{x,y,z} C_\Gamma(t,x,y,z) \, .$$

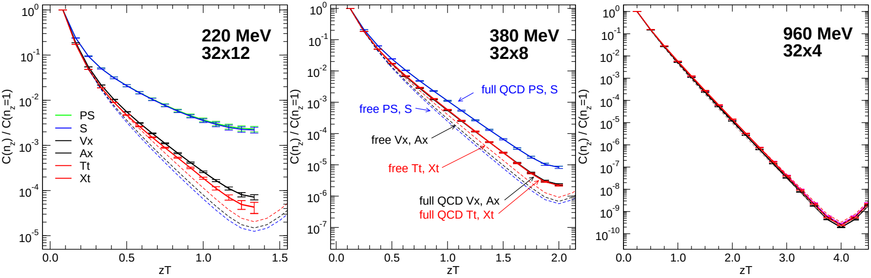
Spatial correlators above T_{ch}

C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, C. Gattringer, L.Ya.G., S. Hashimoto, C.B. Lang, S. Prelovsek , 2017 - 2019
$N_f = 2$ QCD with the chirally symmetric Dirac operator.



$E1$ - $U(1)_A$ symmetry; $E2$ - chiral spin and $SU(4)$ symmetries; $E3$ consistent with both chiral symmetry and chiral spin ($SU(4)$) symmetry.
$SU(2)_{CS}$ and $SU(4)$ symmetries persist up to $T \sim 500$ MeV.

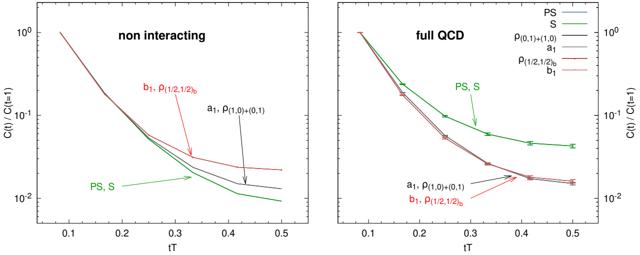
Spatial correlators above T_{ch}



Temporal correlators above T_{ch}

C. Rohrhofer, Y. Aoki, L.Ya.G., S. Hashimoto, 2020

$N_F = 2$ QCD at $T = 220$ MeV

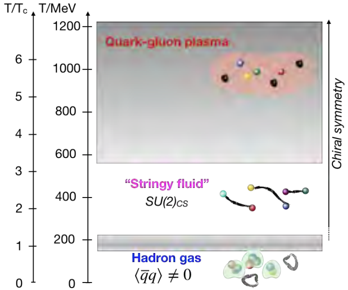


Free quarks: $SU(2)_L \times SU(2)_R$ and $U(1)_A$ multiplets.

Full QCD at $T = 220$ MeV: $U(1)_A$, $SU(2)_L \times SU(2)_R$, $SU(2)_{CS}$ and $SU(2N_F)$ multiplets.

Above T_{ch} QCD is approximately $SU(2)_{CS}$ and $SU(4)$ symmetric.

Three regimes of QCD



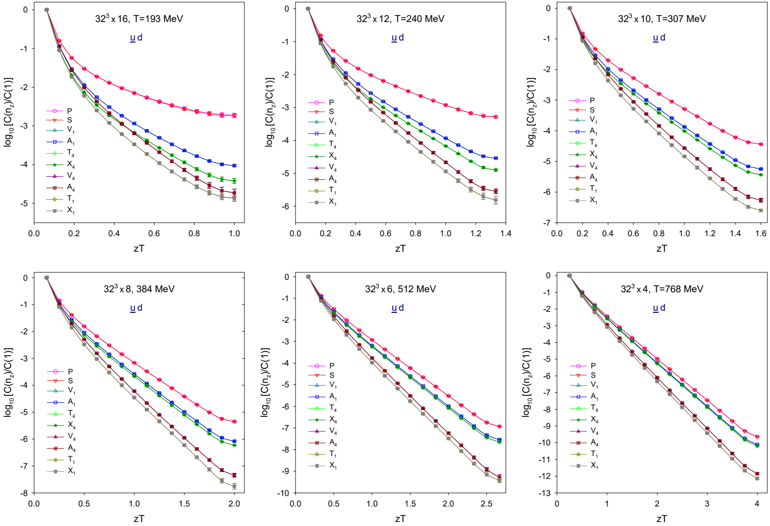
$0 - T_{ch}$ - Hadron Gas (broken chiral symmetry);

$T_{ch} - 3T_{ch}$ - Stringy Fluid (chiral, $SU(2)_{CS}$ and $SU(4)$ symmetries; electric confinement)

Stringy fluid is mostly populated with $J = 0, 1$ states. It is a densely packed system of mesons that interact strongly.

$T > 3T_{ch}$ - a smooth approach to QGP (chiral symmetry)

Correlators in 2+1+1 QCD with domain walls. T.-W. Chiu, 2302.06073



Parity doublets and CS symmetry

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix},$$

$$\Psi_R = \frac{1}{\sqrt{2}}(\Psi_+ + \Psi_-); \quad \Psi_L = \frac{1}{\sqrt{2}}(\Psi_+ - \Psi_-).$$

$$\mathcal{L} = i\bar{\Psi}_+\gamma^\mu\partial_\mu\Psi_+ + i\bar{\Psi}_-\gamma^\mu\partial_\mu\Psi_- - m\bar{\Psi}_+\Psi_+ - m\bar{\Psi}_-\Psi_-$$

or

$$\mathcal{L} = i\bar{\Psi}_L\gamma^\mu\partial_\mu\Psi_L + i\bar{\Psi}_R\gamma^\mu\partial_\mu\Psi_R - m\bar{\Psi}_L\Psi_L - m\bar{\Psi}_R\Psi_R.$$

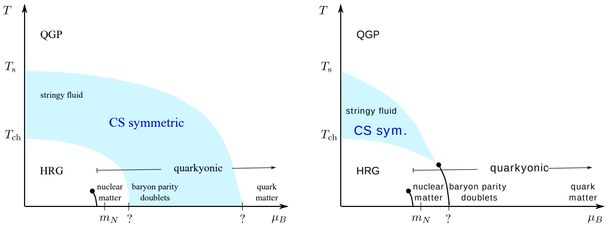
$$\tilde{\Psi} = \begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix}.$$

$$\begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix} \to \exp\left(i\frac{\varepsilon^n\sigma^n}{2}\right)\begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix}.$$

The parity doublet Lagrangian is not only chirally invariant, but also $SU(4)$-invariant with the generators of $SU(4)$ being

$$\{(\tau^a \otimes \mathbb{1}), (\mathbb{1} \otimes \sigma^n), (\tau^a \otimes \sigma^n)\}.$$

Chiral spin symmetric band, L.Ya.G., O. Philipsen, R. Pisarski, 2022

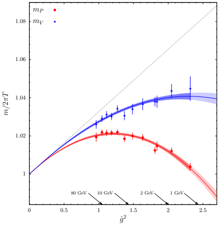


Screening masses and stringy fluid, L.Ya.G., O. Philipsen, R. Pisarski, 2022

$$
\begin{aligned}
e^{pV/T} = Z &= \mathrm{Tr}(e^{-aHN_\tau}) \\
&= \mathrm{Tr}(e^{-aH_z N_z}) = \sum_{n_z} e^{-E_{n_z} N_z} ,
\end{aligned}
$$

QGP - (quasi)parton dynamics drives observables.

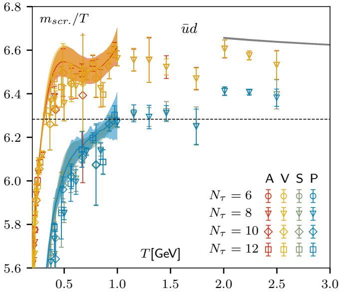
Lattice at $T \sim 1$ - 160 GeV (M.D. Brida et al, 2022) :



$$
\begin{aligned}
\frac{m_{PS}}{2\pi T} &= 1 + p_2\,\hat{g}^2(T) + p_3\,\hat{g}^3(T) + p_4\,\hat{g}^4(T) , \\
\frac{m_V}{2\pi T} &= \frac{m_{PS}}{2\pi T} + s_4\,\hat{g}^4(T) ,
\end{aligned}
$$

Screening masses and stringy fluid, L.Ya.G., O. Philipsen, R. Pisarski, 2022

From A. Bazavov et al, 2019:



An independent demonstration of the existence of a temperature window $T_{pc} < T < 3T_{pc}$, in which chiral symmetry is restored but the dynamics is inconsistent with a (quasi)partonic description.

Screening masses and stringy fluid, L.Ya.G., O. Philipsen, R. Pisarski, 2022

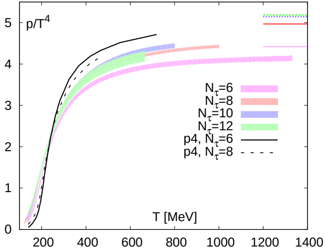
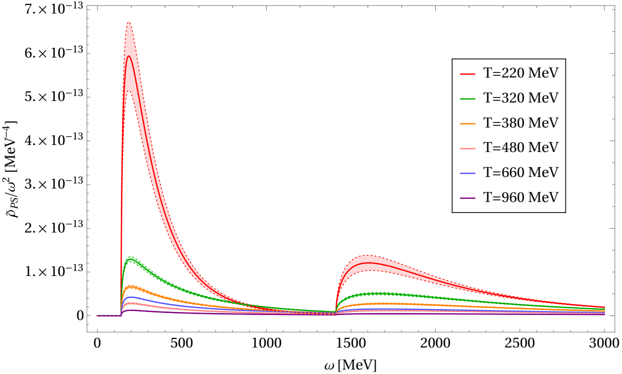


Figure : The pressure calculated with HISQ action for $N_F = 2 + 1$ QCD. From A. Bazavov et al, 2018.

π spectral function

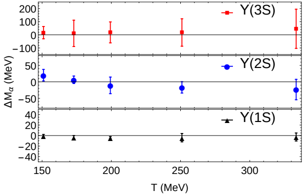
From spatial correlators via generalised Lehmann representation to spectral functions; P. Lowdon and O. Philipsen, 2022

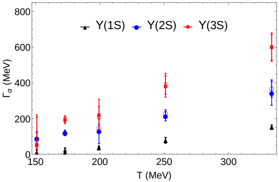


Existence of a pion state and its first radial excitations above T_{ch}. A clear demonstration that above T_{ch} the degrees of freedom are hadron-like.

Bottomonium above T_{ch}. R. Larsen et al, 2020

Mass shifts with respect to zero temperature mass and widths:





A clear demonstration that above T_{ch} the degrees of freedom are hadron-like.