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→ Book of Abstracts ←
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A simple model of the EPR phenomenon

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We study the evolution of a nonrelativistic system made of two quantum particles and a spin, constrained to move on a line. The two particles are initially in a maximally entangled state. One of them interacts with the spin, which is located at a fixed point on the line. We prove that there is a correlation between the state of the spin and the state of the particle that does not interact with it. More precisely, we show that, in a suitable regime, if the spin flips then such a particle possesses a defined momentum in the direction opposite to the spin. We stress the importance of choosing a suitable regime for the EPR effect to occur.

Dirac operators with critical shell interaction in a finite box

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We explore examples of Dirac operators on bounded domains exhibiting an interval of essential spectrum. In particular, we consider three-dimensional Dirac operators on Lipschitz domains with critical electrostatic and Lorentz scalar shell interactions supported on a compact smooth surface. Unlike typical bounded-domain settings where the spectrum is purely discrete, we show that the criticality of these interactions can generate a nontrivial essential spectrum interval, whose position and length are explicitly controlled by the coupling constants and surface curvatures. Based on joint work with J. Behrndt (TU Graz), M. Holzmann (TU Graz), and K. Pankrashkin (Univ. Oldenburg).

Single State Variational Calculations with applications to Large and Small Magnetic Fields

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Variational techniques to calculate estimates of bound-state energies and wave functions for single states are applied to simple electronic systems in magnetic fields. Explicit constraints to

other state functions are not required. Our numerical energies compare favourably with results of other calculations, and the speed of our calculations is greatly enhanced by using a separable basis set and analytical integration.

On the C^*-algebraic rigidity of Heisenberg groups

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We plan to discuss to what extent a Lie group can be distinguished from the other Lie groups in terms of their Hilbert space representations, as encoded in the group C*-algebras. That property is called C*-rigidity. Since a direct problem in representation theory would require the description of the representations of a group in terms of its structure, we address a kind of inverse problem in representation theory of Lie groups. That problem actually has an easy negative answer for several Lie groups, in the sense that there exist nonisomorphic groups that share the same space of unitary representations. Nevertheless, we highlight the special place held by the Heisenberg groups among the other connected and simply connected Lie groups Lie groups, by proving that they are C*-rigid. The main step of the proof is a characterization of the nilpotent Lie groups among the solvable Lie groups solely in terms of topological properties of their coadjoint orbits.This presentation is based on joint work with Ingrid Beltita.

Towards stationary solutions for the cubic NLS with linear band crossing of Dirac type

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It is a well-established fact that 1d Schrödinger equations with suitable periodic potentials admit linear crossing in their dispersion bands. Considering equations with such linear part, linear or nonlinear Dirac equations appear as effective model for the dynamics associated to suitable initial data, on certain time scales. To our knowledge there are no such results for stationary solutions for the NLS and the corresponding effective stationary NLD. In this talk I will first review known results on the limit equations. Then, I will describe some advances in the case of the cubic NLS, based on an ongoing work with W. Borrelli (Politecnico di Milano), S. Dovetta and L. Tentarelli (Politecnico di Torino).

Absence of real resonances of Dirac operators

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The purpose of this talk is to introduce the resonances of Dirac operators by continuing mero-morphically the resolvent and to establish a result about their localization: a kind of Rellich

Theorem. We first consider the case of the Dirac operator in an external field which is essentially bounded and compactly supported. Secondly, we consider the case of the MIT bag model outside a smooth and bounded obstacle.

A connection between quantum dot Dirac operators and $\overline{\partial}$-Robin Laplacians

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This talk addresses Faber-Krahn type inequalities for quantum dot Dirac operators with non-negative mass on domains in $\mathbb{R}^2$. We show that this family of inequalities is equivalent to a family of Faber-Krahn type inequalities for $\overline{\partial}$-Robin Laplacians. Thanks to this, we prove them in the case of simply connected domains for quantum dot boundary conditions asymptotically close to zigzag boundary conditions.

Magnetic quantum transport

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We discuss transport in quantum systems exposed to a homogeneous magnetic field due to various translationally invariant perturbations. The latter can be of a different nature, either a potential, regular or singular, added to the two-dimensional Landau Hamiltonian, or coming from geometric perturbations of Dirichlet layers. In particular, we will demonstrate situations in which such a magnetic transport has no classical counterpart.

Solitons in a Lifshitz-type deformation of the Gross-Neveu model.

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We consider a Lifshitz-type deformation of the 3+1 dimensional Gross-Neveu model in the large-N limit. Like its relativistically invariant counterpart, this field theory bears topological and non-topological solitons. We construct these solitons and analyze their properties.

Eigenvalue bounds in the linear stability of the Ekman spiral

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Originating in fluid dynamics, the stability analysis of Ekman boundary layers leads to a spectral problem for a family of non-selfadjoint linear operator matrices. We present new enclosures for the point spectrum (leading to the solution of an open problem posed by L. Greenberg and M. Marletta in 2004) and investigate the number of eigenvalues. Our analysis is based on a Birman–Schwinger type argument which exploits underlying similarities to Schrödinger operators. Based on joint work with O. Ibrogimov and P. Siegl.

Two-dimensional Schrödinger operators with non-local singular interactions: Self-adjointness and the non-relativistic limit

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We investigate new self-adjoint realizations of the Schrödinger operator in $L^2(\mathbb{R}^2)$ with a novel type of transmission condition along a closed Lipschitz curve Σ, as well as self-adjoint realizations of the Schrödinger operator in $L^2(\Omega_\pm)$ with a new class of boundary conditions. These conditions make use of the Dirichlet traces of the functions belonging to the operator domains, together with the Dirichlet traces of their Wirtinger derivatives. Here, $\Omega_+ \subset \mathbb{R}^2$ is an open domain bounded by Σ, and $\Omega_- = \mathbb{R}^2 \setminus \overline{\Omega_+}$. Using a boundary triple framework, we prove self-adjointness and reveal how these models naturally connect to self-adjoint Dirac operators through the relativistic limit. This talk is based on [1] and [2] and introduces new results extending these works.

References

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Green's Function for the One-Dimensional Dirac Operator with a Regular Potential

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The study of eigenvalues and eigenfunctions of quantum mechanical operators provides key insights into the dynamics of quantum phenomena. This work focuses on analyzing the resolvent operator $\mathfrak{R}_\lambda$ and the resolvent set ρ_λ of the one-dimensional Dirac operator—specifically, the set of values $\lambda \in \mathbb{C}$ for which Dirac unidimensional, es decir el conjunto $\lambda \in \mathbb{C}$ para los cuales el operador $\mathfrak{R}_\lambda$ exists. This analysis is carried out using the Green's function and its connection to the Spectral Parameter Power Series method (SPPS).

Four-parametric generalization of the second Demkov-Kunike two-state model

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We introduce a new time-dependent level-crossing model that describes a quantum two-state system subjected to a constant-amplitude laser field. In this configuration, which presents a four-parametric generalization of the three-parametric second Demkov-Kunike level-crossing model, the frequency detuning changes within a finite range and the resonance crossing generally occurs asymmetrically in time with respect to the crossing point. The general solution to this problem can be written as a linear combination with arbitrary constant coefficients of two independent generalized hypergeometric functions of Clausen, each of which can be presented as a linear combination of two ordinary hypergeometric functions. We analyze the asymptotes of the solution in terms of the quasi-energies and compute the final transition probability for the scenario where the system starts in the first quasi-energy state.

Comparison Principles for Non-local Operators

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The well known subharmonic comparison principle due to Agmon [1] is a useful tool for analysis of solutions of the Schrödinger equation. It relays point values between the super-solutions and sub-solutions of the given equation. We present a few recent results generalizing Agmon's original result. One small extension was done in [2] where we translated the original result to the quadratic form setting. This allows for functions with lower regularity compared to the original to be used. Furthermore, we present the subharmonic comparison principle for discrete Schrödinger operators [3]. The main result of the talk will show a comparison principle in the setting of a non-local operator described by an integral kernel. Joint work with Dirk Hundertmark, Markus Lange, and František Štampach.

References

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Bound States in the Continuum as Missing States in Darboux Deformed Free Particle Systems

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In 1929, von Newmann and Winger proposed the existence of a quantum system that admitted a square integrable state embedded at an energy within the continuous spectrum. This system challenged the apparent separation between discrete and continuous spectra. Despite the significance, their result did not gain widespread recognition at the time, partly due to the incomplete theoretical description and the perception that the interactions involved were merely toy models -unlikely to manifest in real physical systems. Today, however, bound states in the continuum (BICs) are recognized as a general wave phenomenon, observed in electromagnetic and mechanical waves. Their unique properties hold considerable potential for technological applications. In this work, we employ a confluent version of the Darboux transformation to deform a free-particle system, constructing families of analytical von Newmann-Wigner-type potentials that support a BIC. We emphasize how this peculiar state can be interpreted as a missing state - analogous to the case of the ordinary Darboux transformations, where certain states are not explicitly provided by the transformation yet are required by the underlying theory.

Emergent Dimensions: Geometry and Topology from Parameter Spaces

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Recent studies have shown that the Schrödinger equation can be realized geometrically. In this talk, we begin with a brief review of some basic ideas in differential geometry and explore their similarities and connections with quantum physics. We then demonstrate how hidden dimensions can emerge naturally from parameter spaces and derive the evolution equations for arbitrary quantum states along these emergent dimensions, as well as the equations governing the fiber metric of the associated Hilbert space bundle. Next, we show how the Berry connections relate to the state evolution equations. Finally, we show that exceptional points act as topological defects in the Hilbert space bundle.

On the Path Integral Approach to Wigner-Dunkl Quantum Mechanics

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Both, the formal Feynman path-integral approach as well as the properly defined Feynman-Kac formula are discussed within the framework of the Wigner-Dunkl deformed quantum mechanics (WDQM). We first revisit both formulations for the standard Heisenberg-Schrödinger quantum dynamics (HSQM) and then apply them to the WDQM. Whereas in HSQM the Euclidean path integral is based on the Wiener measure associated with the Wiener process of Brownian motion, in WDQM the Feynman-Kac formula is based on the Dunkl process, a càdlàg modification of a Wiener process with jumps. The Dunkl process can be viewed as a combination of two Bessel

processes being continuous Markov processes.

Asymptotics for Magnetic Steklov Eigenvalues

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This talk explores the behavior of eigenvalues for the magnetic Steklov problem on smooth domains. For strong magnetic fields, I'll present curvature-dependent spectral asymptotics in 2D domains, including the splitting of closely spaced eigenvalues. The 3D case reveals a dependence on the angle between the magnetic field and the boundary. Weak constant fields reveal a connection to the domain's geometry through a standard vector potential. Joint work with B. Helffer and F. Nicoleau, Nantes University.

Approximation of non-local interface and boundary conditions via homogenization

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We discuss an approximation of the Laplace operator subject to non-local interface conditions of the δ'-type by Neumann Laplacians defined on a family of Riemannian manifolds exhibiting a sieve-like geometry. We prove a form of resolvent convergence for these operators, which subsequently ensures the convergence of their spectra and eigenspaces, as well as the associated semigroups. Additionally, we construct a concrete example of a manifold capable of reproducing any given integral kernel arising in such interface conditions. Finally, we extend the discussion to similar approximations for the Laplacian with non-local Robin-type boundary conditions.

Resonances in various quantum waveguides systems

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We study various two- and three-dimensional non-relativistic quantum systems with asymptotically straight soft waveguides. In the first part, we focus on locally deformed symmetric waveguides and show that embedded eigenvalues become resonances under weak perturbations. We analyze the asymptotic behavior of the corresponding complex poles as a function of the deformation.

In the second part, we consider a two-dimensional waveguide with a transversally invariant profile and a Kato-class perturbation located at distance ρ. Under suitable assumptions, the resolvent exhibits a second-sheet pole at $z(\rho)$, whose imaginary part satisfies Fermi's golden rule, which we explicitly derive.

Finally, we analyze Dirichlet waveguides in $\mathbb{R}^n$ ($n = 2, 3$) with an attached cavity. If the cavity has a small opening of size ε, embedded eigenvalues turn into resonances. In two dimensions, the resonance behaves as $\mathcal{O}(\varepsilon^2)$; in three dimensions, for an opening of volume $\mathrm{vol}_\varepsilon \sim \varepsilon^2$, it scales as $\mathcal{O}(\varepsilon^4)$. This shows that the resonance lifetime scales as

$$\mathcal{O}\left((\mathrm{vol}_\varepsilon)^{-2}\right),$$

highlighting the dependence on the aperture size.

References

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Magnetic Dirac operator in strips subjected to strong magnetic fields

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We consider the magnetic Dirac operator on a curved strip whose boundary carries the infinite mass boundary condition. When the magnetic field is large, we provide the reader with accurate estimates of the essential and discrete spectra. In particular, we give sufficient conditions ensuring that the discrete spectrum is non-empty. This is a joint work with Julien Royer from Toulouse III University and Nicolas Raymond from Angers University.

Dunkl Algebra and Time-Dependent Quantum Systems

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This contribution presents recent developments in the application of Dunkl algebra to time-dependent quantum systems with symmetry deformation. Dunkl operators, which generalize classical derivatives by incorporating reflection symmetries, enable the exact treatment of parity-sensitive quantum dynamics. Analytical solutions to time-dependent Schrödinger equations are obtained for harmonic oscillators with variable mass and angular-dependent potentials using the Lewis–Riesenfeld invariant method in the Dunkl framework. Additionally, the path integral formalism is extended to Wigner–Dunkl quantum mechanics, allowing the construction of exact propagators for systems such as the Dunkl–Caldirola–Kanai oscillator and models with strongly pulsating mass. These results demonstrate the strength of algebraic techniques in modeling non-stationary and symmetry-deformed quantum systems, with potential applications in quantum optics, dis- sipative dynamics, and non-Hermitian frameworks.

Effective dynamics for interacting magnetic fermions

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Many-body quantum theory is plagued by the "curse of dimensionality": although the Schrödinger dynamics is described mathematically by a linear PDE, the presence of a large number of interacting particles makes the concrete computation of properties of its solutions practically inaccessible. One looks therefore for effective, non-linear descriptions of many-body quantum dynamics, which retain only partial (but still relevant) information on the full wave function, but on the other hand have the advantage of depending on fewer degrees of freedom. Following this general principle, we derive magnetic Hartree-Fock equations from the many-body Schrödinger evolution of interacting fermions subject to an external uniform magnetic field, in the mean-field regime. I'll then discuss the effectiveness of the Hartree-Fock equations, both in regard to time and the number of particles. The talk is based on joint works with Margherita Ferrero and with Niels Benedikter, Chiara Boccato, Nhi Ngoc Nguyen.

Reciprocity in potential scattering and anti-pseudo-unitarity of the fundamental transfer matrix

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Stationary potential scattering admits a formulation in terms of the quantum dynamics generated by a non-Hermitian effective Hamiltonian operator $\widehat{\mathbf{H}}$. The evolution operator for $\widehat{\mathbf{H}}$ determines the standard transfer matrix $\mathbf{M}$ of scattering theory in one dimension, and an integral operator $\widehat{\mathbf{M}}$, called the fundamental transfer matrix, in two and three dimensions, which similarly to $\mathbf{M}$ stores all the information about the scattering properties of the potential. We establish a particular operator identity satisfied by $\widehat{\mathbf{M}}$ and use it to give a proof of the reciprocity theorem in two and three dimensions that does not rely on the properties of the scattering operator, Green's functions, or Green's identities. We reveal an analog of the identity: $\det \mathbf{M} = 1$, that is satisfied by $\widehat{\mathbf{M}}$, and establish a generic anti-pseudo-Hermiticity of the scattering operator.

Generalization of the Haag-Lopuszanski-Sohnius theorem and the ring paradigm

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We formulate an approach to quantum gravity, called the ring paradigm. Gravity is mediated superluminally, and the graviton is described as a phonon on the grid of matter in the Universe. This theory has very interesting applications to cosmology and would ultimately solve the old problem of the cosmological constant. It further gives new impulses to the scalar field theories because the gravitational ring decays to some phantom field. As is obvious, we radically break

the Lorentz invariance, which means that some generalization of the Haag-Lopuszanski-Sohnius theorem in quantum field theory is possible.

Explicit Estimates for the Energy Bands and Forbidden Gaps of the One-dimensional Schrödinger Operator and the Kronig-Penney Mode

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We we consider the small and large eigenvalues of the one-dimensional Schrödinger operator $L(q)$ with a periodic, real and locally integrable potential q. First, we explicitly write out the first and second terms of the asymptotic formulas for the large periodic and antiperiodic eigenvalues and illustrate these formulas for the Kronig-Penney model. Then we give estimates for the small periodic and antiperiodic eigenvalues and for the length of the first gaps in the case of the Kronig-Penney model. Moreover, we give error estimations and present a numerical example.

References

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The thermodynamic energy density of a mixture of dilute Bose gases

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We consider the many body Hamiltonian for a quantum system composed of two species of bosons in 3D interacting through positive, compactly supported intra-species and inter-species potentials. We use suitable variational techniques to estimate the bottom of the spectrum of such Hamiltonian. In the case of integrable potentials, we derive a second order expansion for the thermodynamic energy density in dilute regime. If the potentials are soft, it is possible to refine the estimates and derive also the right constant for the second order. The expansion is coherent with the known formula from Physics and, if the density and scattering length of one species tend to zero, it gives the famous Lee-Huang-Yang formula for one species of bosons.

Discrete complex scale invariance in few-body problems

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In scale-invariant many-body problems of nonrelativistic quantum mechanics, the problems are often reduced to the inverse-square-potential problem described by the following effective Hamiltonian:

$$H_{\text{eff}} = -\frac{d^2}{dr^2} + \frac{\lambda}{r^2}, \tag{1}$$

where r stands for the radial coordinate of many-body configuration space and λ is some dimensionless real determined by spatial dimension and many-body interactions. As is well known in the context of self-adjoint extensions, there exist two critical values in λ: the upper critical value $\lambda_{**} = 3/4$ and the lower critical value $\lambda_* = -1/4$. If $\lambda > \lambda_{**}$, the system enjoys invariance under continuous scale transformation $r \to e^t r$ for any t. As a consequence, H_{eff} admits only continuum spectrum. If $\lambda < \lambda_{**}$, on the other hand, continuous scale invariance can be broken to invariance under discrete scale transformation $r \to e^{n\pi/\sqrt{\lambda_* - \lambda}} r$, where n is an arbitrary integer. As a consequence, H_{eff} admits the following discrete spectrum $\{E_n\}$ that forms a geometric sequence:

$$E_n = E_0 \exp\left(-\frac{2n\pi}{\sqrt{\lambda_* - \lambda}}\right), \tag{2}$$

where E_0 is some energy scale associated with the breakdown of continuous scale invariance. A typical many-body example that exhibits this geometric sequence of bound-state energies is the celebrated Efimov effect of three particles.

In this talk, I revisit the case of intermediate window $\lambda \in (\lambda_*, \lambda_{**})$ and show that, in this case, continuous scale invariance can be broken to the invariance under *discrete phase transformation* $r \to e^{in\pi/\sqrt{\lambda - \lambda_*}} r$ — a complexified version of discrete scale invariance. As a consequence, it can be shown that the scattering matrix admits circularly distributed simple poles at the following positions in the complex E-plane:

$$E_n = E_0 \exp\left(\frac{i2n\pi}{\sqrt{\lambda - \lambda_*}}\right). \tag{3}$$

I also discuss several few-body examples that exhibit this discrete phase invariance.

On the spectrum of electromagnetic waveguides

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In this presentation, we focus on the Maxwell operator with perfect conductor boundary conditions, defined in a tubular neighborhood of a smooth curve in the three-dimensional Euclidean space whose curvature tends to zero at infinity. The aim of this work is to understand the influence of the curvature and the so-called "twist" on the spectrum of this operator. First, we will show that the essential spectrum is the union of two half-lines and the singleton 0. Then, we will provide a sufficient condition for the existence of discrete eigenvalues. This is a joint work with P. Briet, M. Cassier & M. Zaccaron.

Perturbative approach to the time evolution of periodic quantum systems

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In this work, we present a perturbative approach for the description of the time evolution of a quantum system driven by a periodic external field in the Floquet scheme. The cases of systems with non-degenerate as well as degenerate states are considered. We include some examples to illustrate our results.

Hexagonal lattice with time reversal non-invariant vertex coupling

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We analyze the band spectra of a periodic hexagonal lattice with the vertex condition that violates the time reversal invariance of the system, the so-called $-R$ coupling. We give its complete spectral picture and compare it to earlier results for the R coupling. Later we show that the spectral character remains the same even for the dilated lattice, with the exception of flat bands, which are absent if the lengths of the elementary cell are incommensurate.

Convergence of first order operators on thick graphs

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In this talk we discuss the convergence of first order operators on a thickened graph (a graph-like space) towards a similar operator on the underlying metric graph. On the graph-like space, the first order operator is of the form exterior derivative (the gradient) on functions and its adjoint (the negative divergence) on closed 1-forms (irrotational vector fields). Under the assumption that each cross section of the tubular edge neighbourhood is convex, that each vertex neighbourhood is simply connected and under suitable uniformity assumptions (which hold in particular, if the spaces are compact) we establish generalised norm resolvent convergence of the first order operator on the graph-like space towards the one on the metric graph. The square of the first order operator is of Laplace type; on the metric graph, the function (0-form) component is the usual standard (Kirchhoff) Laplacian. A key ingredient in the proof is a uniform Gaffney estimate: such an estimate follows from an equality relating here the divergence operator with all (weak) partial derivatives and a curvature term, together with a (localised) Sobolev trace estimate. Joint work with Pavel Exner

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Magnetic ground states and the conformal structure of a surface

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We present some recent results on the first eigenvalue of the magnetic Laplacian associated with closed potential 1-forms on compact Riemannian surfaces. We introduce some (conformal) spectral invariants associated with the first eigenvalue, which can be estimated by the geometry of the Jacobian torus of the surface, and we present explicit computations in genus 1, providing a simple geometric interpretation of the invariants. Then we define the notion of ground state spectrum and a corresponding notion of "ground state isospectrality", and we prove that if two Riemannian metrics are ground state isospectral, then they are conformal and have the same volume. Joint work with Bruno Colbois (Université de Neuchâtel) and Alessandro Savo ("Sapienza" University of Rome).

Planar Dirac equation with radial contact potentials

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We considered the analysis of the planar Dirac equation with the most general time independent contact interaction supported on a circumference of radius R. Where the interaction is given in terms of four physical parameters, namely the strengths corresponding to a scalar and the three components of a singular Lorentz vector potential in $(2 + 1)$ dimensions. We investigate the bound, scattering and resonant solutions for several choices of the physical parameters, and discussed their confining properties and applications on topological materials.

Making sense of Green's function integrals and use them for superoscillations

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Often in quantum mechanics one is interested in solving the time dependent Schrödinger equation

$$i\frac{\partial}{\partial t}\Psi(t,x) = \Big(- \frac{\partial^2}{\partial x^2} + V(t,x)\Big)\Psi(t,x), \qquad t > 0, \, x \in \mathbb{R},$$
$$\Psi(0,x) = F(x), \qquad\qquad\qquad x \in \mathbb{R}.$$

One way of doing so is to use the integral

$$\Psi(t,x) = \int_{\mathbb{R}} G(t,x,y)F(y)dy, \tag{4}$$

where the integral kernel $G : (0,\infty) \times \mathbb{R} \times \mathbb{R} \to \mathbb{C}$ is the so called Green's function, a function which is given by the potential $v(t,x)$, and is in particular independent of the initial condition F.

Classically, Green's functions of the Schrödinger equation are bounded functions which do not admit any decay at infinity. This means, in situations where the initial condition $F \notin L^1(\mathbb{R})$ is not integrable, it is unclear how to interpret the integral (4).

In this talk I will present several ways how to give rigorous meaning to (4). Also the application to the time evolution of superoscillations will be discussed, an example where the initial condition $F(x) = e^{ikx}$ is a plane wave.

Local form subordination without a power decay and the Riesz property of spectral projections

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We revisit the local form subordination condition on the perturbation of a self-adjoint operators with compact resolvent. This condition relates the size of gaps between the unperturbed eigenvalues with the strength of perturbation and it is used to establish the Riesz basis property of the eigensystem of the perturbed operator. Our new approach allows for a slow and non-monotone decay in the subordination condition as well as for a general behavior of unperturbed eigenvalues. The abstract results are applied in Schrödinger operators with possibly unbounded or singular complex potential perturbations. The talk is based on joint works with B. Mityagin (OSU, USA).

On the incompleteness of the classification of quadratically integrable Hamiltonian systems in the three-dimensional Euclidean space

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We present an example of an integrable Hamiltonian system with scalar potential in the three-dimensional Euclidean space whose integrals of motion are quadratic polynomials in the momenta, yet its Hamilton-Jacobi / Schrodinger equation cannot separate in any orthogonal coordinate system. This demonstrates a loophole in the derivation of the list of quadratically integrable Hamiltonian systems in [Makarov et al., A systematic search for nonrelativistic systems with dynamical symmetries. Nuovo Cimento A Series 10, 52:1061-1084, 1967] where only separable systems were found, and the need for its revision.

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Local perturbations of potential well arrays

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We consider an equidistant array of disjoint potential wells in $\mathbb{R}^\nu, \nu \geq 2$, built over a straight line, and show that, under a restriction on the potential support aspect ratio, a perturbation consisting of longitudinal shifts of a finite number of them preserving the disjointness gives rise to a nonempty discrete spectrum below the threshold of the lowest spectral band.

Orthogonality of Weyl orbit functions on lattices and the central splitting mechanism

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Abstrakt: Each simple Lie algebra corresponds to a certain root system which gives rise to a particular Weyl group. Each Weyl group permits a certain class of invariant functions, called Weyl orbit functions. The Weyl group can be expanded by adding shifts from an invariant lattice, resulting in a generalized affine Weyl group. Studying invariance properties of orbit functions by the extended group yields a basis of a Hilbert space of functions restricted to lattice points inside the fundamental domain. Discrete Fourier-like transforms can be obtained in the standard manner. The central splitting mechanism describes how the Fourier coefficients are related for different choices of lattices with the potential to speed up their calculation.

Some Novel Aspects of Singular Interactions

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Singular interactions is a subject that is fairly well-understood both from a mathematical and from a physical point of view. Nevertheless, there are some new interesting observations which can help us to better appreciate and understand the underlying structures. We plan to present some ideas on the spectral behavior of singular finite-rank perturbations of the Laplacian on a compact manifold (a subject we revisited recently in our publication with F. Erman). In the same spirit, we investigate the orthogonality and completeness relations of the resulting eigenfunctions, which is a fundamental property of any quantized system. Suprisingly, this gave us some insight into the structure of the renormalized Hamiltonian, as well as alternative calculation methods for perturbations of these Hamiltonians.

Fluctuations of δ-moments for Schrödinger and Helmholtz equations

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I will present recent work done with J. Canto, S. Kumar, F. Ponce-Vanegas, L. Roncal, and N. Schiavone. In the first part of the talk we study the process of dispersion of low-regularity solutions to the free Schrödinger equation using fractional weights. We give another proof of the uncertainty principle for fractional weights and use it to get a lower bound for the

concentration of mass. We consider also the evolution when the initial datum is the Dirac comb in the real line. In this case we find fluctuations that concentrate at rational times and that resemble a realization of a Lévy process. Furthermore, the evolution exhibits intermittency and multifractality. In the second part we will show how these results can be extended to solutions of the Helmholtz equation.

Spectral properties of Schrödinger Operators with δ'-interactions via Robin-Laplacians

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We discuss various spectral properties of two-dimensional Schrödinger operators with δ'-potentials supported on star graphs. In particular, we review some elementary proof ideas and demonstrate how Robin Laplacians can assist in determining the spectral properties of Schrödinger operators with δ'-potentials. The main observation is that a rather detailed spectral picture in the δ'-case can be obtained by comparison with both Robin boundary conditions and δ-interactions. We also discuss possibilities for generalization beyond star graphs.

Eigenvalues of operator functions: A commutativity result with an application to 1D Dirac operators

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We introduce the notion of eigenvalues of finite type for analytic operator functions $W(\cdot) : \Omega \to \mathcal{L}(\mathcal{H}, \mathcal{G})$, where $\mathcal{H}$ and $\mathcal{G}$ are Hilbert spaces and $\Omega \subset \mathbb{C}$ is open and connected. Given two analytic operator functions $A(\cdot) : \Omega \to \mathcal{L}(\mathcal{H}, \mathcal{G})$ and $B(\cdot) : \Omega \to \mathcal{L}(\mathcal{G}, \mathcal{H})$, we then show that the operator families

$$I - A(\cdot)B(\cdot) : \Omega \to \mathcal{L}(\mathcal{G})$$
$$I - B(\cdot)A(\cdot) : \Omega \to \mathcal{L}(\mathcal{H})$$

have the same eigenvalues of finite type, with coinciding algebraic multiplicities. Finally, the usefulness of the above commutativity result is demonstrated, by applying it to the Birman-Schwinger family of a weakly coupled one-dimensional Dirac operator H_ε, that is given for $\varepsilon > 0$ and $m \geq 0$ in $L^2(\mathbb{R}; \mathbb{C}^2)$ by

$$H_\varepsilon = -i\partial_x \sigma_1 + m\sigma_3 + \varepsilon V, \quad \mathrm{dom}\, H_\varepsilon = H^1(\mathbb{R}, \mathbb{C}^2),$$

where $V : \mathbb{R} \to \mathbb{C}^{2\times 2}$ is a sufficiently regular and decaying potential, to show that the bound states of H_ε in the weak coupling limit, that were obtained in [1], must be simple. This talk is based on joint work with J. Behrndt and P. Siegl.

References

[1] J.-C. Cuenin, P. Siegl, *Eigenvalues of one-dimensional non-self-adjoint Dirac operators and applications*, Lett. Math. Phys., **108** (2018), 1757-1778.

Construction of maximally non-Hermitian potentials under unbroken PT-symmetry constraint

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A family of discrete Schroedinger equations with imaginary potentials is considered. Attention is paid to the maximally non-Hermitian dynamical regime occurring near the Kato's exceptional points of non-Hermitian spectral degeneracy. The symbolic-manipulation localization of these points is sampled. Some global features of the shape of the boundary of the domain of parameters supporting the spontaneously unbroken PT-symmetry of the whole quantum system are finally deduced.