Generalization of the Coleman-Mandula theorem and ring paradigm

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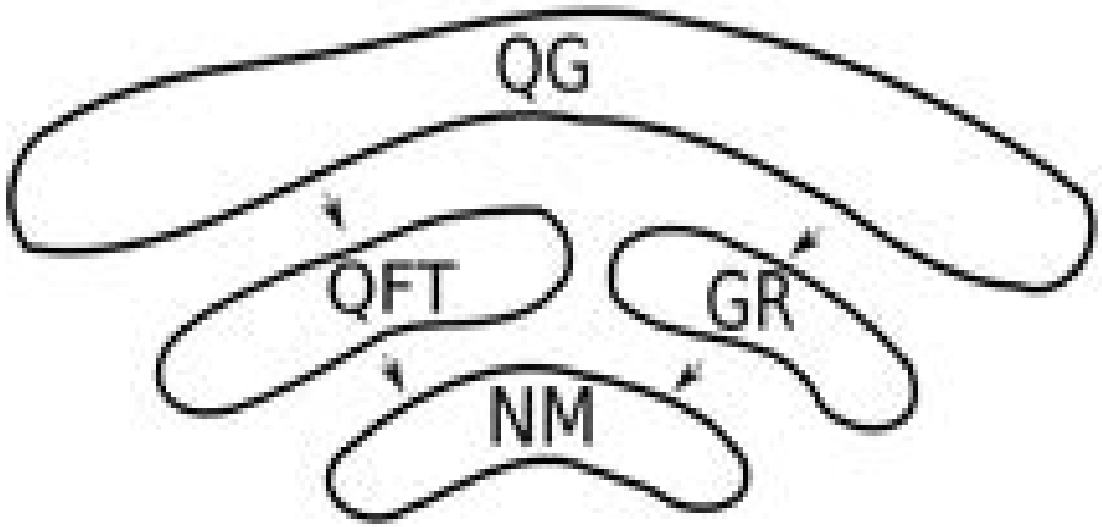
Albert Schweitzer (1875-1965)

"In the hopes of reaching the moon men fail to see the flowers that blossom at their feet."

Stephen Hawking (1942 - 2018)

"Remember to look up at the stars and not down at your feet."

Shaped by the past creating the future

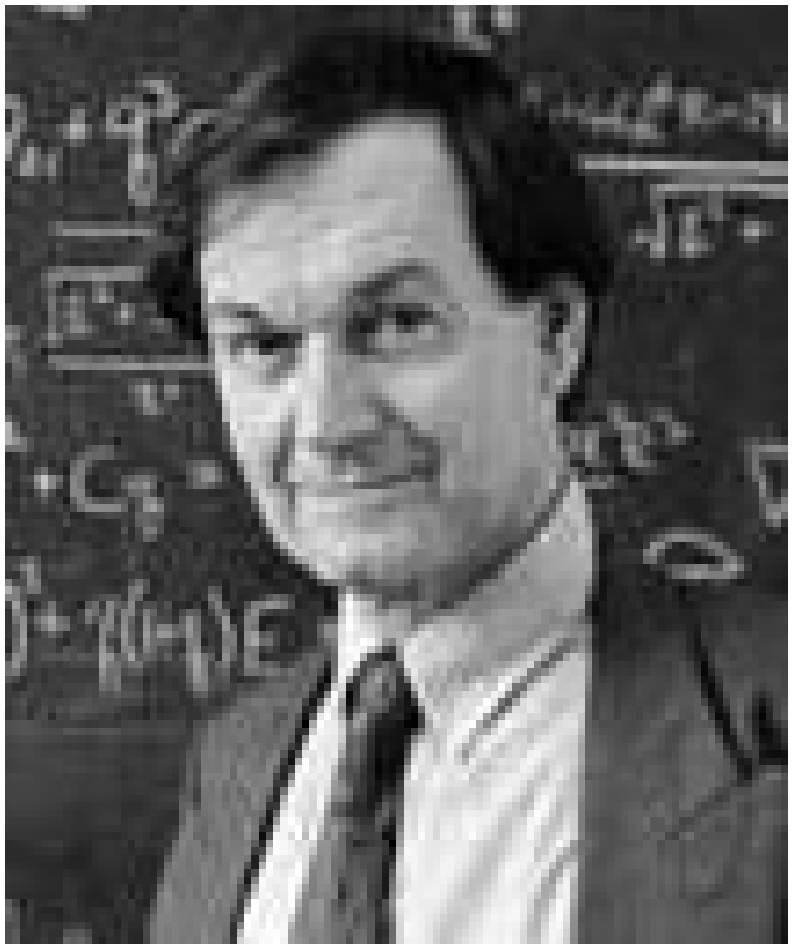


Approaches to quantum gravity

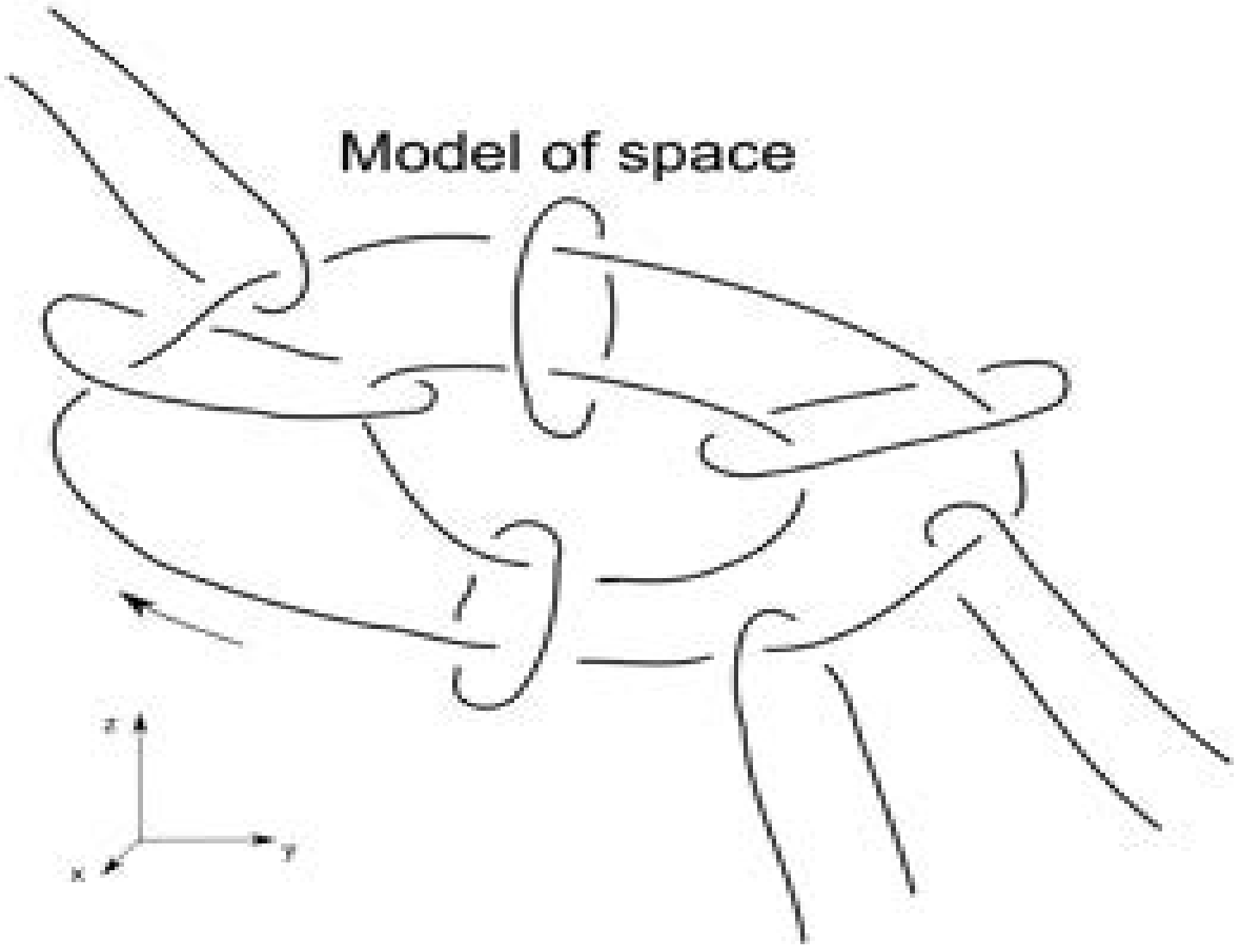
1. string theory
2. loop quantum gravity
3. causal set approach
4. causal dynamical triangulation
5. asymptotic safety
6. non-commutative geometry
7. ...

Dark energy

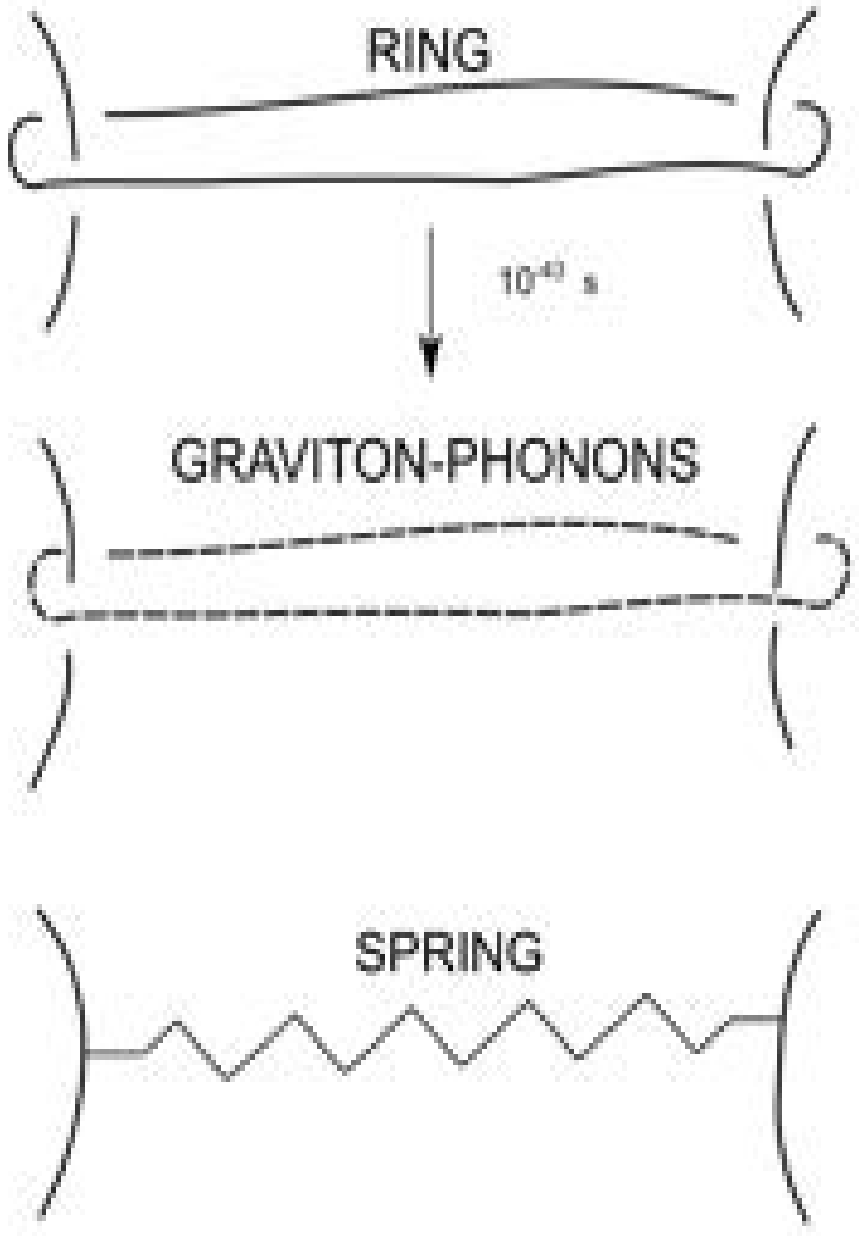
"It is not dark and it is not energy!"



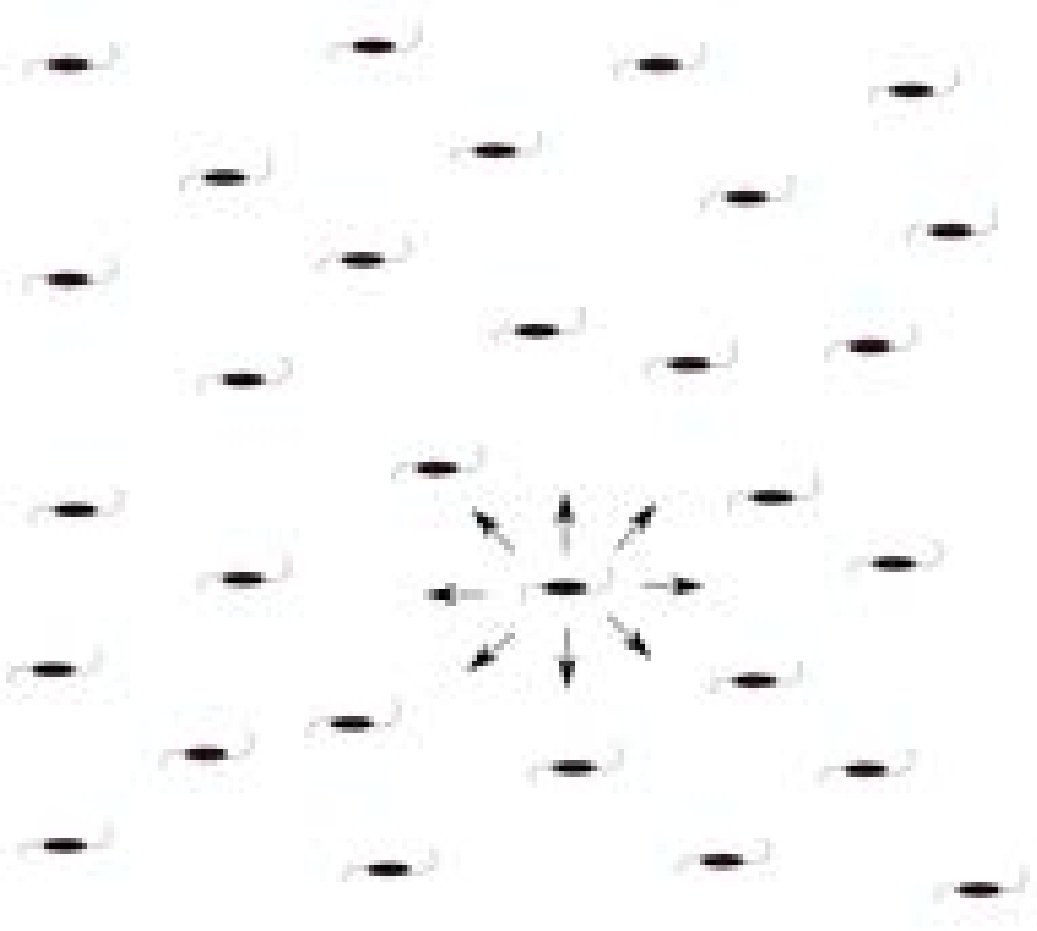
Ring paradigm



We substitute the picture of the gravitons carrying the initial impulse by the creation of a gravitational ring, which tightens the objects in Planck time.

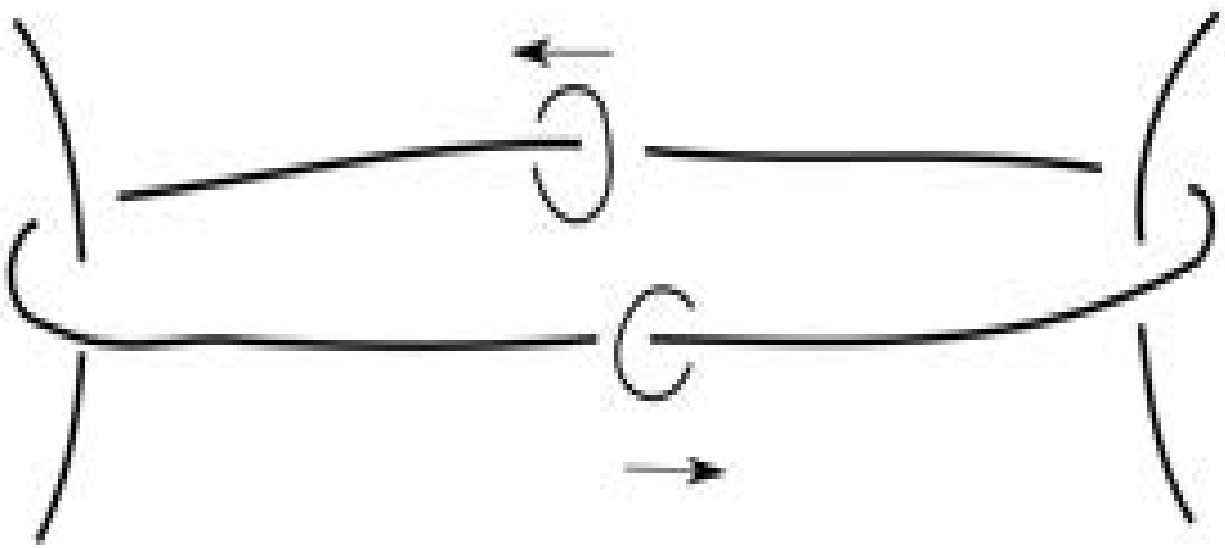

RING
10^{-43} s
GRAVITON-PHONONS
SPRING

Symmetries



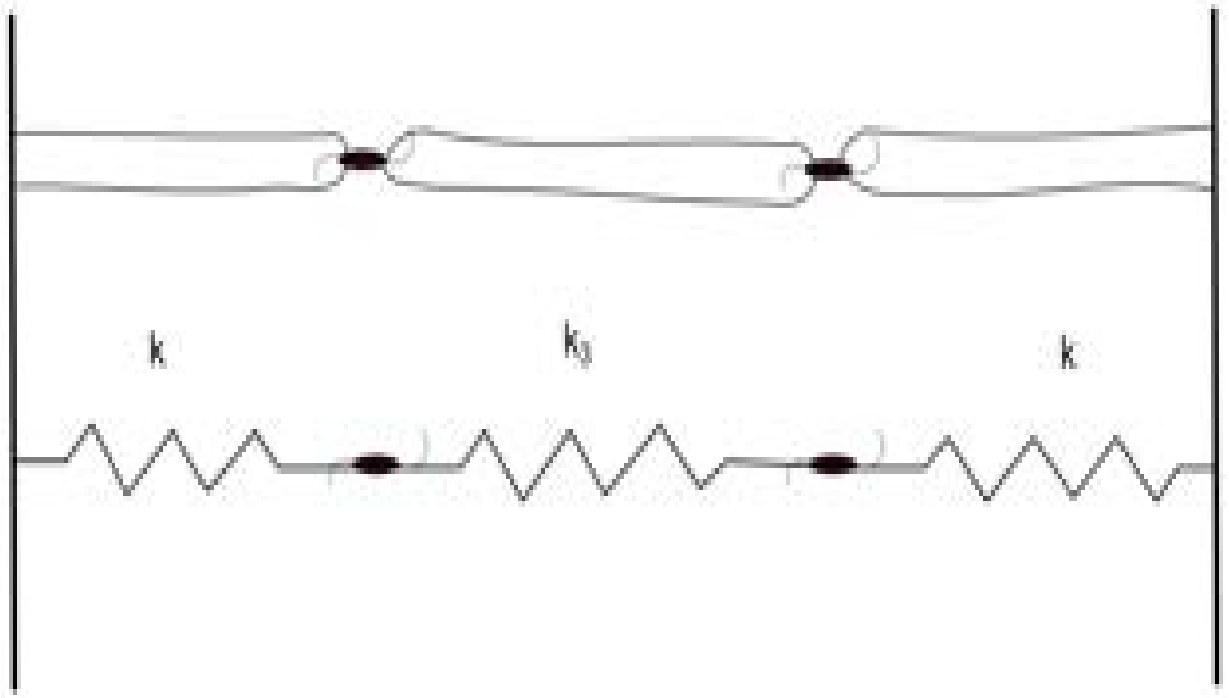
Unchanged particle sector

The elementary particles of the standard model could move only around gravitational rings.



Graviton as a phonon

The creation of rings in Planck time effectively gives rise to springs between the galaxies. We quantize their longitudinal vibrations and obtain the graviton-phonons, which mediate the Newtonian force.



$$H = \sum_{i=1}^{2} \frac{1}{2m} P_i^2 + \sum_{i,j=1}^{2} V_{ij} Q_i Q_j,$$

where

$$V = \begin{pmatrix} \frac{1}{2}k + \frac{1}{2}k_3 & -\frac{1}{2}k_3 \\ -\frac{1}{2}k_3 & \frac{1}{2}k + \frac{1}{2}k_3 \end{pmatrix},$$

$k, k_3 > 0$.

$$L(\psi,\dot{\psi}) = \frac{1}{2}\int \left[\dot{\psi}(x)\right]^2 dx - \frac{1}{2}\iint K(x-x')\psi(x)\psi(x')\,dxdx', \quad (1)$$

where $K(x-x') = K(x'-x)$ denotes a potential, and we keep only one dimension. It is obtained directly from the Euler-Lagrange equations that

$$0 = \frac{\partial}{\partial t}\frac{\delta L}{\delta \dot{\psi}(x)} - \frac{\delta L}{\delta \psi(x)} = \ddot{\psi}(x) + \int K(x-x')\psi(x')\,dx'. \quad (2)$$

A similarity with the equations in the finite-dimensional case can be observed:

$$L = \frac{1}{2}\sum_i \dot{q}_i^2 + \frac{1}{2}\sum_{i,j} U_{ij} q_i q_j,$$

$$0 = \ddot{q}_i + \sum_j U_{ij} q_j, \quad (3)$$

the identification is $x \leftrightarrow i$ and $\psi \leftrightarrow q$.

Let's do an important case. We put $K(x-x') = -c^2 \frac{\partial^2}{\partial x^2}\delta(x-x')$ and the Lagrangian is according to the one of previous equations the following:

$$L = \frac{1}{2}\int [\dot{\psi}(x)]^2 - c^2 \left[\frac{d}{dx}\psi(x)\right]^2 dx \tag{4}$$

Therefore we get a normal wave equation with the velocity of propagation c (it will be later identified with the velocity of light in vacuum):

$$\frac{d^2}{dx^2}\psi(x) - \frac{1}{c^2}\ddot{\psi}(x) = 0 \tag{5}$$

We construct the standard Hamiltonian and so the conjugate momentum $\Pi(x) = \frac{\delta L}{\delta \dot{\psi}(x)} = \dot{\psi}(x)$ is needed.

We construct the standard Hamiltonian and so the conjugate momentum $\Pi(x) = \frac{\delta L}{\delta \dot{\psi}(x)} = \dot{\psi}(x)$ is needed. Further step is to introduce the annihilation and creation operators, a and a^+, and we use the following definition of momentum:

$$P = \frac{1}{2\pi} \int \hbar k a^+(k) a(k)\, dk$$

Then $\left[P, a^+(k)\right] = \hbar k a^+(k)$, $\left[P, a(k)\right] = -\hbar k a(k)$ and so

$$P|k_1, k_2, ...\rangle = (\hbar k_1 + \hbar k_2 + ...)|k_1, k_2, ...\rangle. \tag{6}$$

We finished the preparation for the main computation. Once again, we need to reproduce the Newtonian force with the bouncing of graviton-phonons. Therefore it is cardinal to compute $\frac{dP}{dt}$ in the Heisenberg representation (we trivially obtain 0 in the Schrödinger representation) and compare it with the formula for the **Newtonian force**.

$$\frac{dP}{dt} = \frac{1}{i}[P, H]$$

Graviton-phonon

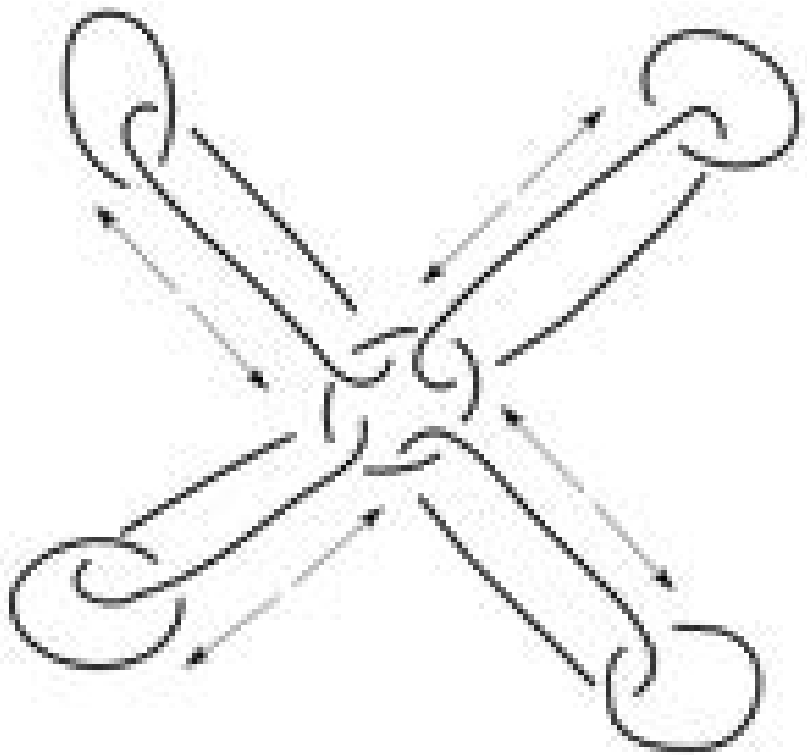
$$graviton \leftrightarrow phonon \leftrightarrow photon$$

The most important observation is that photons are the field particles for electromagnetic interaction, which we describe by Maxwell equations containing the Coulomb law, $\vec{F}_{Q_1 Q_2} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{L^2}$. It has exactly the similar form to the formula for the Newtonian force, $F_{m_1 m_2} = G\frac{m_1 m_2}{L^2}$.
Graviton-phonons are the correct "mediators"of the **Newtonian force**!

Let's stress the result: RP could give an explanation for two facts in classical physics. We know from experiments that gravitational waves are traveling by velocity c_{gw}, which is very close to the velocity of light c with an amazing precision, $|c - c_{gw}| < 10^{-15}$. But c_{gw} should be exactly c according to RP (it was supposed in GR that $c_{gw} = c$, but there had been no reason for that). Further, the similarity of the formulas for the Coulomb and Newton laws is not accidental, but it is necessary for building **the parallelism between photons and graviton-phonons**. As we will see later, these are only the first little surprises that RP brings us.

Accelerated expansion of the Universe

The classical description is that the gravitational rings are effectively made from some material, which has an inner dependence on the deformation due to the stress. The "gravitational"material breaks at *Mpc* distances, which causes accelerated expansion in the Universe.



Modification of gravity

$$\mathscr{R}_{\mu\nu} - \frac{1}{2}\mathscr{R}\mathscr{G}_{\mu\nu} + \Lambda_r \mathscr{G}_{\mu\nu} = \frac{8\pi G \mathscr{T}_{\mu\nu}}{c_g^4}, \tag{7}$$

where $\mathscr{G}_{\mu\nu}$ is the metric and also all the other quantities have an analogous meaning as in GR. The cosmological constant Λ_r could be computed from QFT. We neglect the RHS with respect to the LHS, so

$$\mathscr{R}_{\mu\nu} - \frac{1}{2}\mathscr{R}\mathscr{G}_{\mu\nu} + \Lambda_r \mathscr{G}_{\mu\nu} = 0. \tag{8}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{9}$$

A new cosmological constant term Λ appeared approximately 8 billion years after Big Bang due to the QG phenomenon:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{10}$$

Application of the paradigm

1. singularity theorems
2. cyclic universes
3. black hole information paradox
4. dimensional reduction
5. curvature of the universe
6. determinism of physical theories

Generalization of transformations

$$t' = \frac{t - \frac{x}{v}\frac{v^2}{c^2}\epsilon - \frac{x}{v}\frac{v^2}{c_g^2}}{\sqrt{1 - \frac{v^2}{c^2}\epsilon - \frac{v^2}{c_g^2}}},$$

$$x' = \frac{x - tv}{\sqrt{1 - \frac{v^2}{c^2}\epsilon - \frac{v^2}{c_g^2}}},$$

where $\epsilon = \epsilon(v)$ denotes some step function defined by the prescription

$$\epsilon(v) = \begin{cases} 1 & \text{for } v \leq c, \\ 0 & \text{for } v > c. \end{cases}$$

Coleman-Mandula theorem

Let E be a connected symmetry group of the S matrix, and let the following five conditions hold:

1. E contains a subgroup locally isomorphic to the inhomogeneous Lorentz (Poincaré) group L
2. all particle types correspond to positive-energy representations of L, and, for any finite mass M, there are only a finite number of particle types with mass less than M
3. elastic-scattering amplitudes are analytic functions of the center of mass energy and of the momentum transfer in some neighbourhood of the physical region
4. at almost all energies, any two plane waves scatter
5. the generators of E are representable as integral operators in momentum space, with kernels that are distributions.

Then E is locally isomorphic to $L \times T$, the direct product of the Poincaré group and the internal symmetry group.

McGlinn theorem

Let L be the Lie algebra of the inhomogeneous Lorentz group, M and P the homogeneous and translation parts of L, respectively, and T any semisimple internal symmetry algebra. If E is a Lie algebra whose basis consists of the basis of L and the basis of T, and if $[T, M] = 0$ (i.e. the internal symmetry is Lorentz invariant) then $[T, P] = 0$. Hence $E = L \otimes T$.

O'Raifeartaigh (LOR) theorem

Let L be the Lie algebra of the inhomogeneous Lorentz group, consisting of the homogeneous part M and the translation part P. Let E be any Lie algebra of finite order, with radical R and Levi factor G. If L is a subalgebra of E, then only the following four cases occur:

1. $R = P$.
2. R abelian but larger than, and containing P.
3. R solvable but not abelian, and containing P.
4. $R \cap P = 0$.

In all cases, $M \cap R = 0$.

Open problems

- The proofs of these theorems rely heavily on the use of mathematical tools that are peculiar to flat space. It is not immediately obvious how to generalize it to other situations.
- For RP will be important that the Lorentz group is a non-compact group. And we need to ask what type of theorems we will obtain for the case of compact groups.
- We wish to see, how the gravitational ring (quasi-field, meta-field) is mathematically represented, because it could not be classified according to an irreducible representation of the Poincaré group.

Review article with Umi Mahnuna Hanung

Content (approaximately 120 pp):

1. Riesz-Schauder theory of compact operators
2. Banach algebras
3. Gelfand tranform
4. Spectral theory in Hilbert spaces
5. Functional calculus
6. Further classes of operators
7. Unbounded operators
8. Mathematical foundations of quantum field theory
9. Remarks about quantum gravity

Conference about quantum gravity in preparation

Two sections:

- Section of mathematical physics: noncommutative geometry, Banach algebras, group theory, spinors, twistors
- Section of quantum gravity phenomenology: dark energy, higher dimensions, generalized uncertainity principle, superluminal signalling

Book Two faces of Johny Newman

Introduction of artificial language KAHCG at the end...

Al torre de pornhuut [AL TÓRÉ DE PORNAT]

El orthel dhe natuura laama pharsii [EL ORTÉL DE NATÚRA LÁMA FARSÍ],
ho como andree tarathe [NO KÓMO ANDRÉ TARATÉ].
Aroh thems dhe pluhna talsie [ARÓ TÉM DE PLÚNA TALSÍ],
dhes ewhomo proh changeh plavathe [DEZE WÓMO PRO ŠANŽÉ PLAVATÉ].
El phuto the perehn pelh leeben pandee [EL PÚTO TE PERÉN PEL LÉBEN PANDÉ]?

Car dareh my geebene hatn [KAR DARÉ MY GÉBN HATN],
la paldho proh loyte paramahn [LA PALDO PRO LOJTE PARAMÁN],
ma phaldoo proh natuura tarzahne. [MA FALDÓ PRO NATÚRA TARZÁN],
Dan smee poreetene phor amiegho paten [DAN SMÉ PORÉTN FOR AMÍGO PATN],
cara dozohn lo phunte lavandeh[KARA DOZÓN LO FUNTE LAVANDÉ]!!

The gate of intimacy

This is a mathematical universe, governed by the laws of Nature, we could not change it.
The arrow of time goes in one direction, there's nothing we can do about it.
Is there something to hold on to in this cold world? The source from which we draw, what gives us strength?
...
When we understand what a gift of life we have been given, and when we realize that the real meaning of life is in helping others and in finding harmony with the surrounding Nature.
Then we are ready to find a real life friend who will continue to accompany us,
then we have hope of finding true love.

Thank You for paying attention!
Vielen Dank für Ihre Aufmerksamkeit!
Merci de Votre attention!
¡Gracias por prestar atención!
Grazie per aver prestato attenzione!

(Some pictures were taken from the web and some were created by myself...)

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