Absence of real resonances of Dirac operators

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Let $\Omega \subset \mathbb{R}^3$ be open and $V \in \mathcal{C}_c^\infty(\Omega)$ with $V \geq 0$.
We wish to describe $f : \mathbb{R} \times \Omega \to \mathbb{C}$ such that

$$\begin{cases} \partial_t^2 f(t,x) = -\Delta_x f(t,x) + V(x) f(t,x) & \text{for } (t,x) \in \mathbb{R} \times \Omega \\ f(t,x) = 0 & \text{for } (t,x) \in \mathbb{R} \times \partial\Omega \\ f(0,x) = 0 & \text{for } x \in \Omega \\ \partial_t f(0,x) = u(x) & \text{for } x \in \Omega \end{cases},$$

$u \in \mathcal{C}_c^\infty(\Omega)$ being a prescribed data.

Need of the resonances

- If Ω is bounded then for $(t, x) \in \mathbb{R} \times \Omega$

$$f(t, x) = \sum_{n=0}^{+\infty} \frac{\langle u_n, u \rangle_{\mathrm{L}^2(\Omega)}}{\sqrt{\lambda_n}} \sin\left(\sqrt{\lambda_n} t\right) u_n(x),$$

  $(u_n)_{n \geq 0} \subset \mathrm{H}_0^1(\Omega) \cap \mathrm{H}^2(\Omega)$ being an orthonormal basis of eigenvectors of $-\Delta + V$ associated with the eigenvalues $(\lambda_n)_{n \geq 0} \subset (0, +\infty)$.

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Resonances describe time decay

- There exists $R : \mathbb{C} \to \mathcal{L}(\mathsf{L}^2_{\mathrm{comp}}(\mathbb{R}^3), \mathsf{H}^2_{\mathrm{loc}}(\mathbb{R}^3))$ meromorphic such that if $\Im\omega > 0$ then

$$R(\omega) = (-\Delta + V - \omega^2) \text{ on } \mathsf{L}^2_{\mathrm{comp}}(\mathbb{R}^3).$$

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- For some $A > 0$ and all x in a compact set,

$$f(t,x) = \sum_{\substack{\kappa \text{ resonance} \\ \Im\kappa > -A}} e^{-i\kappa t} f_\kappa(t,x) + O_{t \to +\infty}(e^{-tA})$$

  where $t \mapsto f_\kappa(t,x)$ is polynomial and $x \mapsto f_\kappa(t,x) \in \mathsf{H}^2_{\mathrm{loc}}(\mathbb{R}^3)$.

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- The study of the resonances close to $\mathbb{R}$ is crucial!

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Construct "discrete spectral data" for Dirac hamiltonians and prove that it is not real.

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- Kungsmann '17 - Cheng 21' (using Sjöstrand-Zworski 91' approach)

The Dirac differential operator

- The Dirac matrices $\beta, \alpha_1, \alpha_2, \alpha_3$ are 4×4 hermitian matrices satisfying

$$\beta^2 = I_4 \ , \ \alpha_j \beta + \beta \alpha_j = 0_4 \ , \ \alpha_j \alpha_k + \alpha_k \alpha_j = 2\delta_{jk} I_4.$$

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- The dimensionless Dirac operator associated with the energy of a free massive particle living in $\mathbb{R}^3$ is the differential operator

$$\mathsf{D} = \beta - \imath \alpha \cdot \nabla.$$

It acts on four components distributions.

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- Set $H_V := \mathsf{D} + V$ with domain $\mathsf{H}^1 := \mathsf{H}^1(\mathbb{R}^3)^4$ in the Hilbert space $\mathsf{L}^2 := \mathsf{L}^2(\mathbb{R}^3)^4$.

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- H_V is closed and

$$\mathrm{Sp}_{\mathrm{ess}}(H_V) = (-\infty, -1] \cup [1, +\infty).$$

Integral kernel of the free resolvent

For $z \notin \mathrm{Sp}(H_0) = (-\infty, -1] \cup [1, +\infty)$ and $\phi \in \mathsf{L}^2$ and

$$(H_0 - z)^{-1}\,\phi(x) = \int_{\mathbb{R}^3} K(x-y, z)\phi(y)dy$$

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where

$$K(x, z) := \frac{e^{\imath\omega(z)|x|}}{4\pi |x|} \left(zI_4 + \beta + (\imath + \omega(z) |x|)\alpha \cdot \frac{x}{|x|^2} \right)$$

$$\omega(z) := (z^2 - 1)^{1/2}$$

with $\Im(s^{1/2}) > 0$ for $s \notin [0, +\infty)$.

Appropriate setup

We introduce the Riemann surface

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$$L(x, z, \omega) := \frac{e^{\imath\omega|x|}}{4\pi|x|}\left(zI_4 + \beta + (\imath + \omega|x|)\alpha \cdot \frac{x}{|x|^2}\right)$$

in such a way that if $\Im\omega > 0$ then $\omega = \omega(z)$ and

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For $\phi \in \mathsf{L}^2_{\mathrm{comp}}$ we set

$$R_0(z, \omega)\phi(x) := \int_{\mathbb{R}^3} L(x - y, z, \omega)\phi(y)dy.$$

Holomorphic continuation of the free resolvent

Proposition

- If $\Im\omega > 0$ then

$$R_0(z,\omega) = (H_0 - z)^{-1} \text{ on } \mathsf{L}^2_{\mathrm{comp}}.$$

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- R_0 is holomorphic from $\mathcal{M}$ to $\mathcal{L}(\mathsf{L}^2_{\mathrm{comp}}, \mathsf{H}^1_{\mathrm{loc}})$.
- Uniformly in $s \in \mathbb{S}^2$, as $r \to +\infty$

$$R_0(z,\omega)\phi(rs) = \frac{e^{\imath\omega r}}{4\pi r}\left(\left(zI_4 + \beta + \omega\alpha\cdot s\right)\hat{\phi}(\omega s) + O\left(\frac{1}{r}\right)\right).$$

Resonances of H_V - Rellich Theorem

Theorem (D. 24')

- There exists $R_V : \mathcal{M} \to \mathcal{L}(\mathsf{L}^2_{\mathrm{comp}}, \mathsf{H}^1_{\mathrm{loc}})$ meromorphic such that if $\Im\omega > 0$ then

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- $(\lambda, \kappa) \in \mathcal{M}$ is resonance $\iff \exists \psi \in \mathsf{H}^1_{\mathrm{loc}} \setminus \{0\}$, $\exists \phi \in \mathsf{L}^2_{\mathrm{comp}}$

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- If $V^* = V$ and $\kappa \in \mathbb{R} \setminus \{0\}$ then (λ,κ) is not resonance.

Perturbation by an obstacle

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- Set $H_\Omega := \mathrm{D}$ with domain

$$\mathfrak{D} := \left\{ \psi \in \mathrm{H}^1(\Omega)^4 \,|(-\imath\beta\alpha \cdot n)\psi_{|\,\partial\Omega} = \psi_{|\,\partial\Omega} \right\}.$$

in the Hilbert space $\mathrm{L}^2 := \mathrm{L}^2(\Omega)^4$.

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- H_Ω is self-adjoint (Vega-Ourmières 16') and

$$\mathrm{Sp}_{\mathrm{ess}}(H_\Omega) = (-\infty, -1] \cup [1, +\infty).$$

Resonances of H_Ω - Rellich Theorem

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- There exists $R_\Omega : \mathcal{M} \to \mathcal{L}(\mathsf{L}^2_{\mathrm{comp}}, \mathfrak{D}_{\mathrm{loc}})$ meromorphic such that if $\Im\omega > 0$ then

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- If Ω is connected and $\kappa \in \mathbb{R}$ then (λ, κ) is not resonance.

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Thank you for your attention!