Choreography in Physics

(living in motion, moving polymers, superintegrability etc)

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Abstract

By definition the choreography (dancing curve) is a closed trajectory on which n classical bodies move chasing each other without collisions. The first choreography (the Remarkable Figure Eight) at zero angular momentum was discovered unexpectedly by C Moore (Santa Fe Institute) at 1993 for 3 equal masses in R^3 Newtonian gravity numerically. At the moment about 6,000 choreographies in R^3 Newtonian gravity are found, all numerically for different $n > 2$. A number of 3-body choreographies is known in R^2 Newtonian gravity, for Lennard-Jones potential (hence, relevant for molecular physics), and for some other potentials, again numerically; it might be proved their existence for quarkonia potential.

Does exist (non)-Newtonian gravity for which dancing curve is known analytically? - Yes, a single example is known - it is algebraic lemniscate by Jacob Bernoulli (1694) - and it will be a concrete example of the talk. Astonishingly, R^3 Newtonian Figure Eight coincides with algebraic lemniscate with χ^2 deviation $\sim 10^{-7}$. Both choreographies admit any odd numbers of bodies on them. Both 3-body choreographies define maximally superintegrable trajectory with 7 constants of motion.

Talk will be accompanied by numerous animations.