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Soliton generation in optical fiber networks

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Abstract: It was shown in [1,2] that number of generated solitons for nonlinear Schrodinger equation is given by expression

$$N = \langle \frac{1}{2} + \frac{F}{\pi} \rangle \tag{1}$$

where $F = \int_{-\infty}^{\infty} |q(x,0)|dx$ is the area of the initial condition, and $\langle ... \rangle$ means the integer number smaller the the argument. In the past by several papers, special initial conditions are considered for computing soliton number.

In this work, we studied the problem of soliton generation in optical fiber networks using a model based on the well known [3] and Ablowitz-Musslimani nonlocal [4] nonlinear Schrodinger equations on metric graphs. Initial value (Cauchy) problem for the equations on metric graphs is solved for different graph topologies, such as star and tree graphs. For branched optical givers one can choose the initial pulse profile in different ways (e.g., at the vertex or branch, at given vertex or branch, with different shape sat different vertices). Therefore, unlike to linear (unbranched) fibers [1,2], soliton generation for optical fiber networks have richer dynamics and tools for manipulation by number of solitons. Analytic expression of finding the number of generated solitons is obtained in the form

$$N = \sum_{j=1}^{k} \langle \frac{1}{2} + \frac{F_j}{\pi} \rangle$$

where $F_j = \sqrt{\frac{\beta_j}{2}} \int_{-\infty}^{\infty} |q_j(x,0)|dx$ is the initial area for each bond of the metric graph, β_j is the nonlinearity coefficient and k is the number of bonds. The number of solitons is computed for different types initial conditions given at the vertex. And also in numerical calculations, radiation of the initial pulse is shown for the star graph when the sum rule is not fulfilled [3,5]. In Fig. 1 is described of six soliton generation for nonlocal nonlinear Schrodinger equation on the star graph with six bonds [5]. Different colors mean the different time moments, i.e., the red line at $t = 0$, green line $t = 0.02$ and blue line $t = 1$.

Method which is used in the research can be applied for different network topologies, provided a network has three and more semi-infinite outgoing branches. This allows to use our model for the problem of tunable soliton generation in optical fiber networks, which

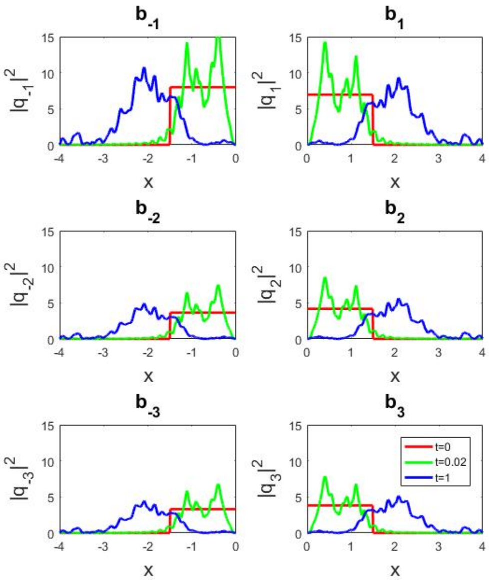

Figure 1: Soliton generation on the star graph with bonds for nonlocal nonlinear Schrodinger equation.

is of importance for practical applications in the areas, where optical fibers are used for information (signal) transfer.

In collaboration with D. Matrasulov (Turin Polytechnic University in Tashkent).

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Spectrum of Kagome and triangular networks with time-reversal non-invariant vertex coupling.

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Abstract: We investigate spectral properties of quantum graphs in the form of a kagome or a triangular lattice, assuming that wave functions at the vertices are matched through conditions which are non-invariant with respect to the time-reversal. We discuss, in particular, the high-energy behavior of such systems. The probability that a positive energy belongs to the spectral bands is obtained as two-thirds if the kagome lattice is equilateral or degenerate to the triangular one; if the lattice is asymmetric, the probability is different, however, we see that the Band-Berkolaiko universality holds if the edge lengths are incommensurate.

This is joint work with Prof. Pavel Exner based on the preprint [1] which has been recently accepted for publication in J. Math. Phys.

[1] M. Baradaran, and P. Exner: arXiv:2106.16019 [math-ph]

Summation of the An-harmonic part of the Propagator

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Abstract: We evaluated the time-dependent propagator for an-harmonic oscillator with quartic term in potential by conditional Wiener measure path integral. The result was presented in the form of the operator function in the spirit of the methods of the umbral calculus. In this talk, we present the summation of the infinite series representing the operator function in our previous article. We show, that the final function is finite.

In collaboration with P. Prešnajder (Comenius University, Bratislava, Slovakia, presnajder@fmph.uniba.sk)

Solving the One-Dimensional Time-Independent Schrdinger Equation with High Accuracy: The LagrangeMesh Mathematica Package

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Abstract: In order to find the spectrum associated with the one-dimensional Schrödinger equation, we discuss the Lagrange Mesh method (LMM) and its numerical implementation for bound states. After presenting a general overview of the theory behind the LMM, we introduce the LagrangeMesh package: the numerical implementation of the LMM in Mathematica. Using few lines of code, the package enables a quick home-computer computation of the spectrum and provides a practical tool to study a large class of systems in quantum mechanics. The main properties of the package are (i) the input is basically the potential function and the interval on which is defined; and (ii) the accuracy in calculations and final results is controllable by the user. As a consequence, a highly accurate spectrum of some relevant quantum systems can be obtained by employing the commands that the package offers.

Enhanced avionic sensing based on Wigners cusp anomalies

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Abstract: Typical sensors detect small perturbations by measuring their effects on a physical observable, using a linear response principle (LRP). It turns out that once LRP is abandoned, new opportunities emerge. A prominent example is resonant systems operating near Nth-order exceptional point degeneracies (EPDs) where a small perturbation $p << 1$ activates an inherent sublinear response $\sim p^{1/N} >> p$ in resonant splitting. Here, we propose an alternative sublinear optomechanical sensing scheme that is rooted in Wigners cusp anomalies (WCAs), first discussed in the framework of nuclear reactions: a frequency-dependent square-root singularity of the differential scattering cross section around the energy threshold of a newly opened channel, which we use to amplify small perturbations. WCA hypersensitivity can be applied in a variety of sensing applications, besides optomechanical accelerometry discussed in talk. Our WCA platforms are compact, do not require a judicious arrangement of active elements (unlike EPD platforms), and, if chosen, can be cavity free.

Work done in collaboration with Rodion Kononchuk, Joseph Knee & Tsampikos Kottos
Reference: Kononchuk et al., Sci. Adv. 2021; 7 : eabg8118

Aharonov–Casher theorem on a manifold with boundary

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Abstract: Aharonov and Casher proved in 1978 that the dimension of the kernel of the massless magnetic Dirac operator on $\mathbb{R}^2$ depends only on the flux Φ of the magnetic field. More precisely, they showed that it is the integer part of $\Phi/2\pi$ and that all the functions in the kernel have the same chirality, determined by the direction of the magnetic field.

I will present a result generalising the Aharonov–Casher theorem to manifolds with boundary. To determine the domain of the Dirac operator I use the Atiyah–Patodi–Singer (APS) boundary condition, which Atiyah, Patodi and Singer introduced in their famous paper from 1974 on the index theorem on manifolds with boundary.

The domains I consider are $\mathbb{R}^2$ with holes, a disc with holes and a sphere with holes. In the first case, the result is exactly the same as for the original case of $\mathbb{R}^2$. In the latter two cases, for certain fluxes we have an extra zero mode compared to the first case.

This talk is based on my PhD thesis, which was supervised by Jan Philip Solovej.

First-principle control of novel resonances in non-Hermitian photonic media

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Abstract: Photonic resonances play an essential role in the generation and propagation of light in optical and photonic devices, as well as in light-matter interaction including nonlinear optical responses. Previous studies in lasers and other open systems have shown exotic roles played by non-Hermiticity on modifying passive resonances, defined in the absence of optical gain and loss. Here we report a new type of resonances in non-Hermitian photonic systems that do not originate from a passive resonance, identified by analyzing a unique quantization condition in the non-Hermitian extension of the Wentzel-Kramers-Brillouin method. Termed active photonic resonances, these unique resonances are found in non-Hermitian systems with a spatially correlated complex dielectric function, which is related to supersymmetry quantum mechanics after a Wick rotation. Remarkably, such an active photonic resonance shifts continuously on the real frequency axis as optical gain increases, suggesting the possibility of a tunable on-chip laser that can span a wavelength range over $100nm$.

Non-self-adjoint relativistic point interaction in one dimension.

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Abstract: This talk will be primarily focused on presenting the recently published article [1]. We will discuss the one–dimensional Dirac operator with point interaction formally given by term $\mathbb{A} \otimes |\delta(x)\rangle\langle\delta(x)|$, where $\mathbb{A}$ is arbitrary complex 2×2 matrix and $\delta(x)$ stands for the delta function. This operator will be rigorously defined as closed, not necessarily self–adjoint operator. We will study its spectral properties and find its non–relativistic limit. We will also address the question of regular potential. In particularly, we will use the non–local potential in the form of a projection on a fixed scaled function and we will show that, contrary to the local potential, the renormalization of coupling constants does not occur. Moreover, we will discuss the non–relativistic limit of the studied non–local potential.

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Electron in triangular graphene dots

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Abstract: Two types of honeycomb lattice Fourier–Weyl transforms associated to the irreducible crystallographic root system A_2 are utilized to study electronic properties of triangular graphene quantum dots. The triangular dots with armchair and zigzag edges are represented by two fundamentally different geometric configurations of the honeycomb lattice inside the fundamental domain of the A_2 affine Weyl group. The Schrödinger equations produced by tight-binding models of electron propagation with the nearest and next-to-nearest couplings are exactly solved through armchair and zigzag honeycomb Fourier–Weyl transforms. The inclusion of boundary conditions in the tight-binding Hamiltonians provides four types of electronic stationary states expressed via the honeycomb Weyl orbit functions. The contrasting behavior of the armchair and zigzag electronic probability densities is demonstrated. This is a joint work with Lenka Motlochová.

Superintegrability in non-inertial reference frames with conic defects.

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Abstract: We analyze the superintegrability of geodesic and harmonically trapped systems on rigidly rotating backgrounds with conic defects. The problem is understood from different points of view since conic spaces with angular deficiency/excess admit a reinterpretation as cosmic string backgrounds with positive/negative mass density while rotating backgrounds connect with gravitoelectromagnetism. We use a local relationship of the background to the Euclidean plane for solving the classical equations of motion and the corresponding Schrdinger equation. We also construct integrals of motion that encode the classical path closure and the degeneracy of the spectrum after quantization. These objects only exist for particular values of the problem's parameters (conical defect and angular velocities).

New exactly solvable time-dependent two-state quantum models

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Abstract: We present several new exactly solvable quantum time-dependent two-state models describing level-crossing field configurations. Although the introduced field configurations belong to the Heun class of two-state models, the solution of the problem for these models can ultimately be written in terms of the ordinary hypergeometric functions. Nevertheless, these configurations can in no way be prescribed to any family of classical hypergeometric models. This is because of the fact that for all classical models each of the fundamental solutions involves only one Gauss hypergeometric function, while in our case each of the fundamental solutions necessarily includes two such functions the linear combination of which is generally irreducible.

Spectrum of the Laplacian on a domain perturbed by small resonators

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Abstract: It is known that the spectrum of the Dirichlet Laplacian is stable under small perturbations of a domain, while in the case of the Neumann or mixed boundary conditions the spectrum may abruptly change. In this talk we discuss an example of such a domain perturbation.

Let Ω be a (not necessary bounded) domain in $\mathbb{R}^n$. We perturb it to

$$\Omega_\varepsilon = \Omega \setminus \cup_{k=1}^m S_{k,\varepsilon},$$

where $\varepsilon > 0$ is small parameter, $S_{k,\varepsilon}$ are closed surfaces with small suitably scaled holes ("windows") through which the bounded domains enclosed by these surfaces ("resonators") are connected to the outer domain. When ε goes to zero, the resonators shrink to points.

We demonstrate that in the limit $\varepsilon \to 0$ the spectrum of the Laplacian on Ω_ε with the Neumann boundary conditions on $S_{k,\varepsilon}$ and the Dirichlet boundary conditions on the outer boundary converges to the union of the spectrum of the Dirichlet Laplacian on Ω and the numbers γ_k, $k = 1, \ldots, m$, being equal 1/4 times the limit of the ratio between the capacity of the kth window and the volume of the kth resonator. We obtain an estimate on the rate of this convergence with respect to the Hausdorff-type metrics. The proof is based the abstract results for studying the convergence of operators in varying Hilbert spaces developed in [2,3].

Also, an application of this result is presented: we construct an unbounded waveguide-like domain with inserted resonators such that the eigenvalues of the Laplacian on this domain lying below the essential spectrum threshold do coincide with prescribed numbers.

This is a joint work with G. Cardone (University of Naples) [1].

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Eigenvalues of a perturbed periodic differential system of arbitrary order

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Abstract: The talk concerns the discrete spectrum of the problem

$$(L_r + T_r)y(t) = \lambda y(t), \quad -\infty < t < \infty, \tag{2}$$

where L_r is a self-adjoint differential operator in $[L^2(\mathbf{R})]^m$ of an arbitrary order r with periodic $(m \times m)$ matrix coefficients and T_r is a symmetric perturbation of order $\leq r$, which does not change the essential spectrum $\sigma_{\text{ess}}(L)$ of L_r.

We obtain sufficient conditions guaranteeing that the prescribed edge of a spectral gap in $\sigma_{\text{ess}}(L)$ is an accumulation point for the eigenvalues of problem (2) lying in this gap. These sufficient conditions are close to be necessary ones. The obtained results are new even for perturbed systems with constant coefficients and for the scalar Hill equation.

Note that for a perturbed potential of the scalar Hill equation the criterion for the number of eigenvalues emerging in a spectral gap to be infinite was obtained in [1] (semi-infinite gap) and [2,3] (finite gap). In the case of a semi-bounded operator L_r with $r = 2$, the conditions for the number of eigenvalues emerging in a semi-infinite gap under a matrix perturbation T_0 to be infinite was obtained in [4].

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Quadratic integrable systems in magnetic fields: the generalized cylindrical and spherical cases.

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Abstract: Integrable systems are special due to their high amount of first integrals and the subsequent exact solvability. Despite its relevance e.g. in plasma physics or the physics of charged particle beams, the magnetic field has been so far mostly omitted from these systems. In its presence, the pairs of quadratic integrals can have a more complicated form [Marchesiello and Šnobl 2022 J Phys A]. Here we present the classification of integrable systems for 2 physically relevant nonstandard pairs, namely the generalized cylindrical and spherical ones, and consider a superintegrable system modeling the helical undulator in a solenoid.

On the Quantization of the Fermionic Vector Schwinger Model

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Abstract: In this talk, I would consider the fermionic vector Schwinger model in one space one time dimension. I would study the quantization of the theory using the standard quantization techniques.

Light-Front Maxwell Chern-Simons Higgs Theory in the Symmetry Phase

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Abstract: In this talk, I would study the quantization of the light-front Maxwell Chern-Simons Higgs theory in the symmetry phase" using the Hamiltonian and path integral formulations from the Old ones.

Effective quantum Hamiltonian in thin domains with non-homogeneity

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Abstract: We consider the Laplacian with a non-homogeneous metric in a tubular neighbourhood of a compact hypersurface in the Euclidean space of arbitrary dimension, subject to Neumann boundary conditions. In the limit of the width of the neighbourhood shrinking to zero, the operator converges in a generalised norm-resolvent sense to an effective Laplace-Beltrami-type operator on the hypersurface. In this way, we generalise and give an insight into the convergence of eigenvalues obtained for a piece-wise constant metric.

Solvable potentials from Heun type equation and their symmetries

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Abstract: The exact solutions of quantum mechanical potentials are typically obtained in terms of some special functions of mathematical physics. The most widely studied class is Natanzon potentials, in which case the solutions are expressed using (confluent) hypergeometric functions. For bound states they usually reduce to orthogonal polynomials. The Heun type differential equations offer a natural way to expand the range of solvable potentials. They contain more parameters, so, in principle, potentials with more flexible shapes can be generated from them. However, the solutions of the Heun type differential equations are much less known than the (confluent) hypergeometric functions. Here the confluent and bi-confluent Heun equations are disccussed, and the possibility of obtaining solvable potentials from them is analyzed. This procedure leads to some potentials that were obtained before from rather different approaches (e.g. certain quasi-exactly solvable (QES) potentials) [1]. Conditions under which polynomial solutions can be obtained are also discussed. Finally, the formalism of the confluent Heun equation is adapted to the requirements of of PT symmetry. It is demonstrated that although the explicit general solutions of this equation are not available, it is sometimes possible to identify which potentials can be obtained from this equation in PT-symmetric form and under which conditions they exhibit unbroken or broken PT symmetry [2].

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On Dunkl-Quantum mechanics

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Abstract: The Dunkl operators are given by combinations of differential and difference operators, associated with a finite reflection group. In this talk, first, we are going to introduce Dunkl quantum mechanics by employing the Dunkl operator instead of the ordinary differential operator, and then discuss several problems of relativistic and non-relativistic quantum mechanics.

A discrete quantum wave function for the inverted oscillator

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Abstract: While physical properties of the simple harmonic oscillator are well known, the study for the inverted oscillator is also necessary because it can be applied to many physical systems. The characteristics of the inverted oscillator is quite different from that of the simple harmonic oscillator. The wave packet in the inverted oscillator associated with the usual plane wave solutions is unbound, their eigenstates are not square-integrable. A general quantum mechanical properties of a mechanical system can be studied in terms of the invariant operators that are developed by Lewis and Riesenfeld. The invariant operator is useful for evaluating the eigenvalue problem of mechanical systems. The wave solutions that have discrete eigen spectrum are interesting. We focus in this talk on the quantum solutions with discrete eigen spectrum. The characteristics relevant to discrete eigen spectrum will be analyzed in detail. The difference between discrete eigen spectrum and the continuous one will be compared.

Robustness, sensitivity and pseudospectra around higher order exceptional points

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Abstract: One of the hallmarks of non-Hermitian photonics is the existence of unique degeneracies, the so called higher order exceptional points (HEPs). In the first part, we are going to present recent results regarding a systematic way of constructing infinite optical lattices that exhibit HEPs. In the second part of the talk, we are going to examine the interplay between robustness and sensitivity in non-Hermitian topological lattices. The extreme response and sensitivity is examined in the context of pseudospectra theory.

In the framework of non-Hermitian photonics, we present results [1-4] that are related to higher order exceptional points (HEP) in waveguide lattices. More specifically, in the first part of the talk, we propose a systematic methodology that allows us to construct one dimensional optical lattices that exhibit HEP. In the second part, we are going to focus on a new way of controlling the imaginary part of the refractive index, based on optical nonlinearity. Using such a scheme, we can tune the non-Hermitian and the topological characteristics of the prototypical non-Hermitian Su-Schrieffer-Heeger (NHSSH) lattice [1]. The interplay between topological robustness, sensitivity and extreme amplification is investigated by employing the pseudospectra theory [2]. Pseudospectra is a non-pertubative computational approach, that is widely used in fluid mechanics stability problems. Based on such a mathematical machinery, one can systematically investigate both the extreme power dynamics and the sensitivity around HEPs of photonic structures [2,3,4]. We believe that our results may provide an important connection of exceptional points physics to non-Hermitian topological photonics [1,3,4].

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Bifurcations of the detuned $2:4$ resonance

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Abstract: We consider families of Hamiltonian systems in two degrees of freedom with an equilibrium in 1 : 2 resonance. Under detuning, this Fermi resonance typically leads to normal modes losing their stability through period-doubling bifurcations. For cubic potentials, this concerns the short axial orbits, and in galactic dynamics, the resulting stable periodic orbits are called banana orbits. Galactic potentials are symmetric with respect to the coordinate planes whence the potentialand the normal formboth have no cubic terms. This $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry turns the 1 : 2 resonance into a higher order resonance, and therefore we also speaks of the 2 : 4 resonance. We study the 2 : 4 resonance in its own right, not restricted to natural Hamiltonian systems where $H = T + V$ would consist of kinetic and (positional) potential energy. The short axial orbit then turns out to be dynamically stable everywhere except at a simultaneous bifurcation of banana and anti-banana orbits, while it is now the long axial orbit that loses and regains stability through two successive period-doubling bifurcations. Joint work with H.Hanssmann and G. Pucacco.

Propagating-wave approximation and the transfer matrix in two-dimensional potential scattering

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Abstract: We introduce an approximation scheme for performing scattering calculations in two dimensions that involves neglecting the contribution of the evanescent waves to the scattering amplitude. This corresponds to replacing the interaction potential with an associated energy-dependent nonlocal potential that does not couple to the evanescent waves. We construct a transfer matrix for this class of nonlocal potentials and explore its representation in terms of the evolution operator for an effective non-unitary quantum system. The above approximation turns out to agree with the first Born approximation for weak potentials, and similarly to the semiclassical approximation, becomes valid at high energies. We identify an infinite class of complex potentials for which this approximation scheme is exact and discuss its appealing practical and mathematical aspects. These allow for a rigorous proof of the existence of the associated transfer matrix.

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Computing eigenvalues of SturmLiouville operators with PT-symmetric trigonometric polynomial potentials

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Abstract: We provide estimates for the periodic and antiperiodic eigenvalues of non-self-adjoint Sturm-Liouville operators with special potentials that are PT-symmetric trigonometric polynomials. We even approximate complex eigenvalues by the roots of some polynomials derived from some iteration formulas. Moreover, we give numerical examples with error analysis.

Entanglement of pseudo-Hermitian states

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Abstract: In a recent paper (A. Fring and T. Frith, Phys. Rev A 100, 101102 (2019)), Dyson's scheme to deal with density matrix of non- Hermitian Hamiltonians has been used to investigate the entanglement of states of a PT-symmetric bosonic system. They found that von Neumann entropy can show a different behavior in the broken and unbroken regime. We show that their results can be recast in terms of an abstract model of pseudo-Hermitian random matrices. It is found however that, although the formalism is practically the same, the entanglement is not of Fock states but of Bell states.

Supersymmetry and Shape Invariance of exceptional orthogonal polynomials

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Abstract: We discuss the exceptional Laguerre and the exceptional Jacobi orthogonal polynomials in the framework of the supersymmetric quantum mechanics (SUSYQM). We express the differential equations for the Jacobi and the Laguerre exceptional orthogonal polynomials (EOP) as the eigenvalue equations and make an analogy with the time independent Schr odinger equation to denfine "Hamiltonians" enables us to study the EOPs in the framework of the SUSYQM and to realize the underlying shape invariance associated with such systems. We show that the underlying shape invariance symmetry is responsible for the solubility of the differential equations associated with these polynomials.

Pairs of commuting quadratic elements in the universal enveloping algebra of Euclidean algebra and integrals of motion

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Abstract: Motivated by the consideration of integrable systems in three spatial dimensions in Euclidean space with integrals quadratic in the momenta we classify three-dimensional Abelian subalgebras of quadratic elements in the universal enveloping algebra of the Euclidean algebra under the assumption that the Casimir invariant $\vec{p} \cdot \vec{l}$ vanishes in the relevant representation. We show by means of an explicit example that in the presence of magnetic field, i.e. terms linear in the momenta in the Hamiltonian, this classification allows for pairs of commuting integrals whose leading order terms cannot be written in the famous classical form of [Makarov A A, Smorodinsky J A, Valiev K and Winternitz P Il Nuovo Cimento (1967) A 10 106184]. We consider limits simplifying the structure of the magnetic field in this example and corresponding reductions of integrals, demonstrating that singularities in the integrals may arise, forcing structural changes of the leading order terms.

Inspiration and need for this work came from numerous discussions with Pavel Winternitz over several years. Unfortunately, while he was alive we always found other more urgent research directions to follow together. Thus we dedicate it to his memory.

Spectral properties of some quantum systems with wires in $\mathbb{R}^3$

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Abstract: In this talk we consider two types of quantum systems governed by the Schrödinger operators with an attractive delta interaction supported on lines in $\mathbb{R}^3$. In the first class of models we analyze a star-shaped potential localized on segments joint at one point. We show that the configuration of star arms realizing the maximum of the ground state energy are related to the famous Thomson problem. In the second class of models we consider delta potential supported on infinite lines which do not admit a crossing point. We derive the asymptotics of the eigenvalues number if the lines tend to be parallel.

Meromorphic potentials and molecular physics

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Abstract: It will be derived meromorphic potentials of the special form, which describe the Born-Oppenheimer potential curve for the ground state of heteronuclear diatomic molecules with accuracy four figures.

On the Schrödinger Operator with a Periodic PT-symmetric Matrix Potential

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Abstract: We obtain asymptotic formulas for the Bloch eigenvalues of the operator L generated by a system of Schrödinger equations with periodic PT-symmetric complex-valued coefficients. Then using these formulas we classify the spectrum $\sigma(L)$ of L and find a condition on the coefficients for which $\sigma(L)$ contains all half line $[H, \infty)$ for some H.

Families of third-order shape-invariant Hamiltonians related to generalized Hermite and Okamoto polynomials

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Abstract: The classification of the most general construction of regular rational quantum potentials in terms of the generalized Hermite and Okamoto polynomials is established. This is achieved by exploiting the intrinsic relation between third-order shape-invariant Hamiltonians and the rational solutions of the fourth Painlevé equation. Such a relation unequivocally establishes the discrete spectrum structure, composed of the union of a finite- and infinite-dimensional sequence of equidistant eigenvalues separated by a gap for the $-1/x$ and $-2x$ hierarchies and three infinite sequences for the $-2x/3$ hierarchy.

Notably, the indices of the generalized Hermite and Okamoto polynomials define the dimension of the finite sequences and the spectral gaps. In all the cases, the eigensolutions are written as the product of polynomial times a weight function supported on the real line. These polynomials simultaneously fulfill a second-order differential equation and a three-term recurrence relation, similarly to classical orthogonal polynomials. This offers an advantage compared to other rational extensions reported in the literature, where recurrence relations of higher order are required.

Double-well potentials and the exact solvability of two types

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Abstract: For the displaced harmonic-oscillator potential $V \sim \min\left[(x+d)^2, (x-d)^2\right]$ the exact polynomial (a.k.a. quasi-exactly solvable, QES) bound states are obtained at certain real eigen-displacements $d = d_n$, $n = 1, 2, \ldots, N$. Schrödinger equation can still be declared non-polynomially exactly solvable (NES) since at any real d one can just match the known special-function halves of the wave function in the origin. Open problems emerge for a generalization of the model to complex d.

Reference:

MZ, Quantum Reports 4 (2022) 309 - 323