Non-critical particle number fluctuations

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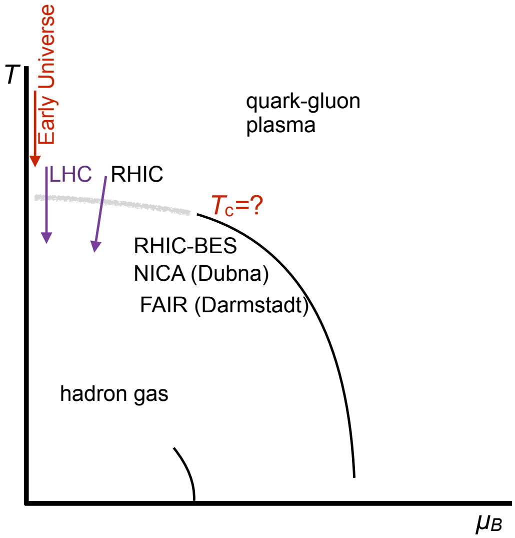
New Trends in Thermal Phases of QCD
Praha, Vila Lanna

15.4.2023

The big goal: phase diagram of the QCD



Fluctuations of a conserved charge I

$$\langle N \rangle = \sum_i N_i P_i = \frac{\sum_i N_i w_i}{\sum_i w_i} = \frac{\sum_i N_i \exp\left(-\frac{E_i - \mu N_i}{T}\right)}{\sum_i \exp\left(-\frac{E_i - \mu N_i}{T}\right)} = \frac{\frac{\partial Z}{\partial \frac{\mu}{T}}}{Z} = \frac{\partial \ln Z}{\partial \frac{\mu}{T}}$$

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Relativistic system:

- creation and annihilation of particle-antiparticle pairs
- study charges which are **conserved in microscopic interactions**
- fluctuations by exchange with the heatbath

Fluctuations of a conserved charge I

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Relativistic system:

- creation and annihilation of particle-antiparticle pairs
- study charges which are **conserved in microscopic interactions**
- fluctuations by exchange with the heatbath

mean baryon number

$$\langle B \rangle = \frac{\partial \ln Z}{\partial \frac{\mu_B}{T}}$$

Fluctuations of a conserved charge II

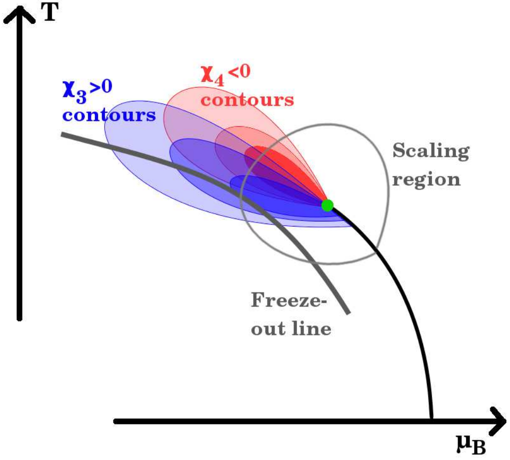
Cumulants of the net-baryon number distribution from derivatives of log Z

$$
\begin{aligned}
\frac{\partial \ln Z}{\partial \frac{\mu_B}{T}} &= \langle B \rangle = \mu_1 = \kappa_1 = VT^3 \chi_1 \\
\frac{\partial^2 \ln Z}{\partial \left(\frac{\mu_B}{T}\right)^2} &= \langle B^2 \rangle - \langle B \rangle^2 = \mu_2 = \kappa_2 = \sigma^2 = VT^3 \chi_2 \\
\frac{\partial^3 \ln Z}{\partial \left(\frac{\mu_B}{T}\right)^3} &= \langle B^3 \rangle - 3\langle B^2 \rangle \langle B \rangle + 2\langle B \rangle^3 = \mu_3 = \kappa_3 = VT^3 \chi_3 \\
\frac{\partial^4 \ln Z}{\partial \left(\frac{\mu_B}{T}\right)^4} &= \langle B^4 \rangle - 4\langle B^3 \rangle \langle B \rangle - 3\langle B^2 \rangle^2 + 12\langle B^2 \rangle \langle B \rangle^2 - 6\langle B \rangle^4 = \mu_4 - 3\mu_2^2 = \kappa_4 = VT^3 \chi_4 \\
\frac{\partial^5 \ln Z}{\partial \left(\frac{\mu_B}{T}\right)^5} &= \kappa_5 = VT^3 \chi_5 \,, \qquad \frac{\partial^6 \ln Z}{\partial \left(\frac{\mu_B}{T}\right)^6} = \kappa_6 = VT^3 \chi_6
\end{aligned}
$$

central moments μ_i, cumulants κ_i, susceptibilities χ_i

Susceptibilities and the phase diagram

Susceptibilities in the Ising model (same universality class)



[J.W. Chen et al.: Phys. Rev. D 95 (2017) 014038]

Combinations of cumulants

variance, skewness, kurtosis, hyperskewness, hyperkurtosis

$$\sigma^2 = \kappa_2\,, \quad S = \frac{\kappa_3}{\kappa_2^{3/2}}\,, \quad \kappa = \frac{\kappa_4}{\kappa_2^2}, \quad S^H = \frac{\kappa_5}{\kappa_2^{5/2}}\,, \quad \kappa^H = \frac{\kappa_6}{\kappa_2^3},$$

These cumulants, moments, and their combinations still depend on volume
$\Rightarrow$ construct volume-independent combinations

$$\frac{\chi_2}{\chi_1} = \frac{\kappa_2}{\kappa_1} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = \frac{\kappa_3}{\kappa_2} = S\sigma \qquad \frac{\chi_4}{\chi_2} = \frac{\kappa_4}{\kappa_2} = \kappa\sigma^2$$

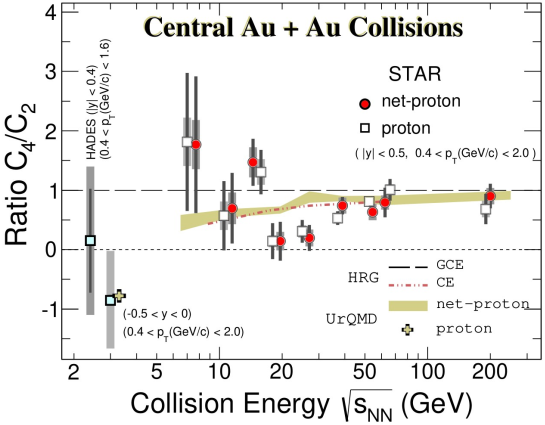
$$\frac{\chi_5}{\chi_1} = \frac{\kappa_5}{\kappa_1} = \frac{S^H\sigma^5}{M} \qquad \frac{\chi_5}{\chi_2} = \frac{\kappa_5}{\kappa_2} = S^H\sigma^3 \qquad \frac{\chi_6}{\chi_2} = \frac{\kappa_6}{\kappa_2} = \kappa^H\sigma^4$$

Measure the net-proton number fluctuations

- baryon number susceptibilities χ_i^B calculated on the lattice
- enhancement of susceptibilities near the critical point
- susceptibilities might be measurable as cumulants of baryon number distribution
- B-number not measurable, since no neutrons are measured
- Conflict!
  - susceptibilities are calculated in grand-canonical ensemble
  - cumulants are measured in real collisions which conserve B, have limited acceptance, and measure (almost) only protons
- many papers devoted to these subjects (!!!)

Data: enhanced net-proton number fluctuations at $\sqrt{s_{NN}} = 7.7$ GeV

- Not all baryons are measurable
- net-proton number as proxy for the net baryon number
- enhanced κ_4/κ_2 at $\sqrt{s_{NN}} = 7.7$ GeV
- not reproduced by theoretical calculations



[STAR collaboration: 2112:00240]

A toy model Monte Carlo simulation

- baryon number is conserved
- only protons and neutrons (and their antiparticles) in the simulations
- only a (fluctuating) part of incoming nucleons participate
- isospin of individual wounded nucleons is kept
- wounded nucleons have double-Gaussian rapidity distribution
  protons from this source fluctuate due to:
  - fluctuations of number of wounded nucleons
  - random number of protons out of wounded nucleons, track isospin
  - limited acceptance out of the whole rapidity distribution
- additionally produced $B\bar{B}$-pairs flat in rapidity
  (net) protons from this source fluctuate due to:
  - Poissonian fluctuations of $B\bar{B}$ pairs with mean proportional to N_{wound}
  - random number of protons and antiprotons ($p = 1/2$)
  - limited acceptance out of the whole rapidity distribution

$\Rightarrow$ **composition wounded/produced protons depends on energy, centrality, and rapidity window**

Rapidity distribution of wounded nucleons

$$\frac{dN_w}{dy}(y) = \frac{N_w}{2\sqrt{2\pi\sigma_y^2}} \left\{ \exp\left(-\frac{(y-y_m)^2}{2\sigma_y^2}\right) + \exp\left(-\frac{(y+y_m)^2}{2\sigma_y^2}\right) \right\}$$

Parameter settings:

- $\sigma_y = 0.8$
- obtain y_m from

$$N_{p-\bar{p}} = \frac{Z}{A} \int_{-y_b}^{y_b} \frac{dN_w}{dy}\, dy$$

where
$N_{p-\bar{p}}$ in $|y| < y_b = 0.25$
is taken from STAR:
PRC**79** (2009) 034909, PRC**96** (2017) 044904

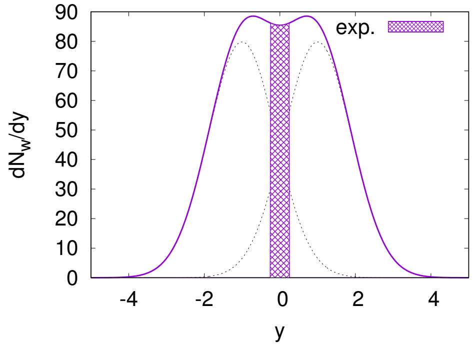


Illustration for: $y_m = 1$, $dy = 0.8$

Rapidity distribution of produced $N\bar{N}$ pairs

$$\frac{dN_{B\bar{B}}}{dy} = N_{B\bar{B}} \frac{C}{1 + \exp\left(\frac{|y| - y_m}{a}\right)}$$

Parameter settings:

- $C = \left(2a \ln\left(e^{y_m/a} + 1\right)\right)^{-1}$
- $a = \sigma_y / 10$
- obtain $N_{B\bar{B}}$ from

$$N_{\bar{p}} = \frac{1}{2} \int_{-y_b}^{y_b} \frac{dN_{B\bar{B}}}{dy}\, dy$$

where
$N_{\bar{p}}$ in $|y| < y_b = 0.25$
is taken from STAR:
PRC**79** (2009) 034909, PRC**96** (2017) 044904

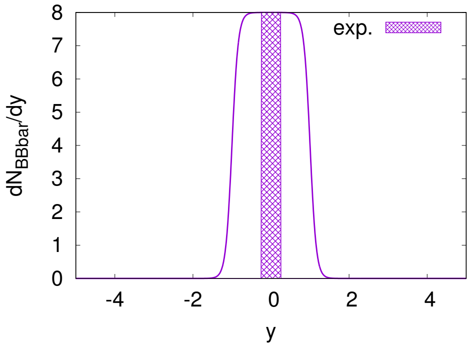


Illustration for: $y_m = 1$, $a = 0.08$

Other model faetures

Isospin determination

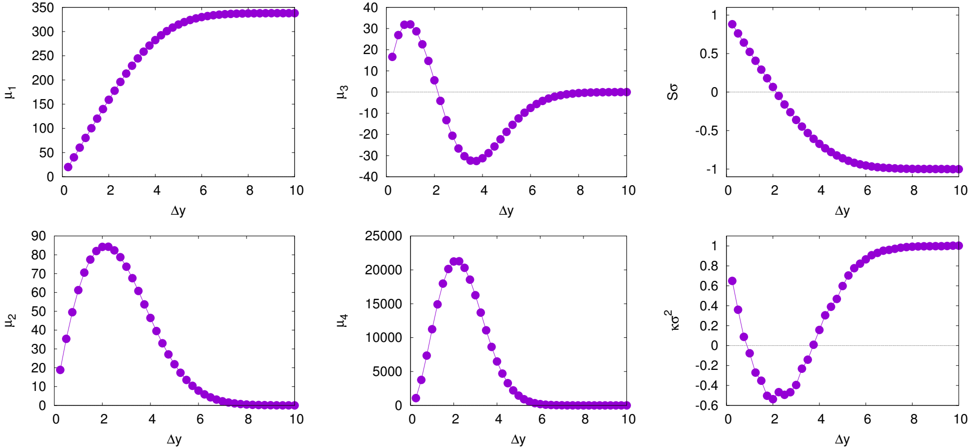
- Wounded nucleons remember their isospin. This feature can be turned off and on.
- Wounded proton number thus follows hypergeometric distribution.
- A produced nucleon becomes proton with probability 1/2.

Glauber Monte Carlo

- we use GLISSANDO 2
  [M. Rybczyński *et al.*, Comp. Phys. Commun. **185** (2014) 1759]
- centrality is determined based on deposited energy measure (analogically to experiment)

Exercise: Baryon number conservation

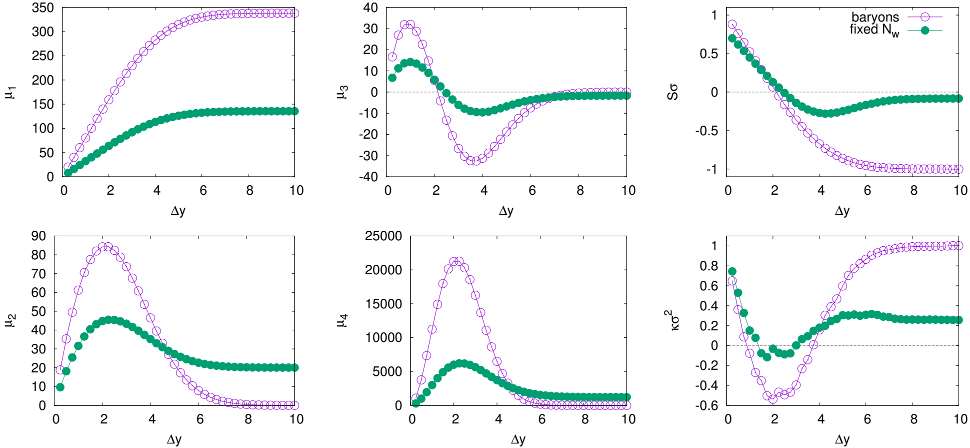
Moments of baryon number distribution around midrapidity given by Poissonian distribution.



$N_w = 338$, $N_{B\bar{B}} = 16.94$, $y_m = 1.019$, ($\sqrt{s_{NN}} = 19.6$ GeV), 5×10^7 events

Net-proton number: dependence on rapidity window width

Moments of net proton number distribution around midrapidity.

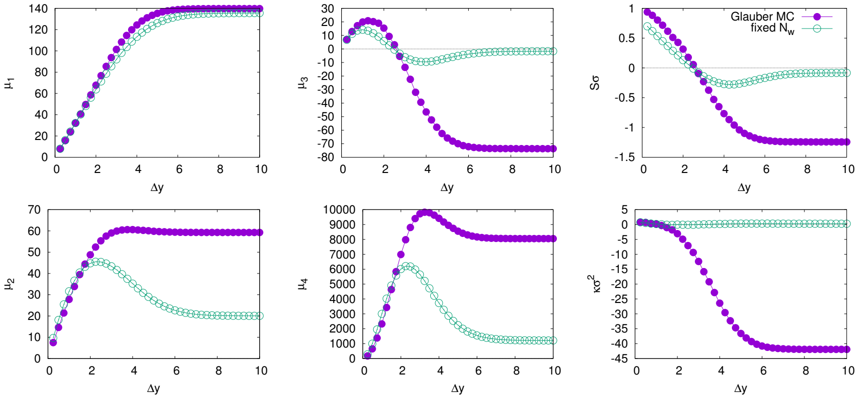


$N_w = 338$, $N_{B\bar{B}} = 16.94$, $y_m = 1.019$, ($\sqrt{s_{NN}} = 19.6$ GeV), 2×10^7 events

Dependence on Δy: fixed N_w vs. Glauber MC

Moments of $p - \bar{p}$ distribution around $y = 0$

$N_w = 338$
$N_{B\bar{B}} = 16.94$
$y_m = 1.019$
$(\sqrt{s_{NN}} = 19.6$ GeV$)$
2×10^7 events
Glauber MC:
1.2×10^6 events

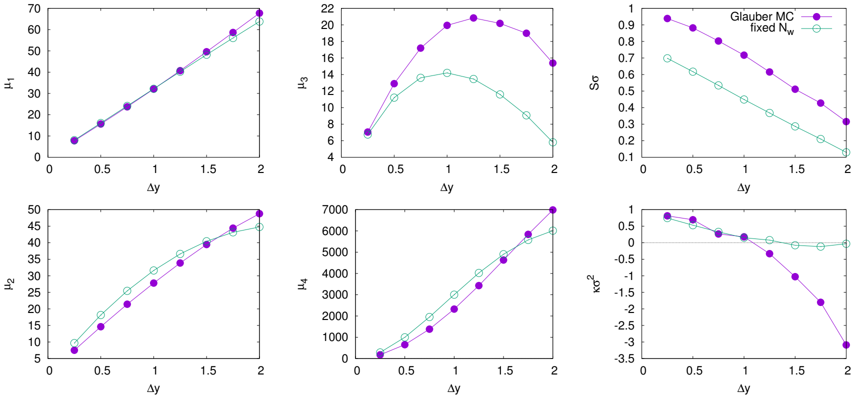


[see also: Braun-Munzinger, Rustamov, Stachel]

Dependence on Δy: fixed N_w vs. Glauber MC

Moments of $p - \bar{p}$ distribution around $y = 0$: zoom into detector coverage

$N_w = 338$
$N_{B\bar{B}} = 16.94$
$y_m = 1.019$
$(\sqrt{s_{NN}} = 19.6\text{ GeV})$
2×10^7 events
Glauber MC:
1.2×10^6 events

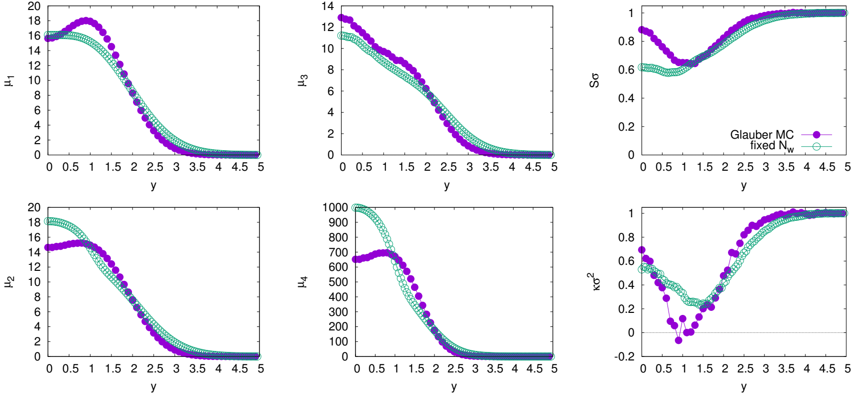


[see also: Braun-Munzinger, Rustamov, Stachel]

Net-proton number: dependence on rapidity

Moments of $p - \bar{p}$ distribution for $\Delta y = 0.5$

$N_w = 338$
$N_{B\bar{B}} = 16.94$
$y_m = 1.019$
$(\sqrt{s_{NN}} = 19.6 \text{ GeV})$
2×10^7 events
Glauber MC:
1.2×10^6 events



cf: [J. Brewer, S. Mukharjee, K. Rajagopal, Y. Yin, Phys. Rev. C 98 (2018) 061901]

Net-proton number: dependence on rapidity

Moments of $p - \bar{p}$ distribution for $\Delta y = 0.5$: zoom into detector coverage

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Glauber MC:
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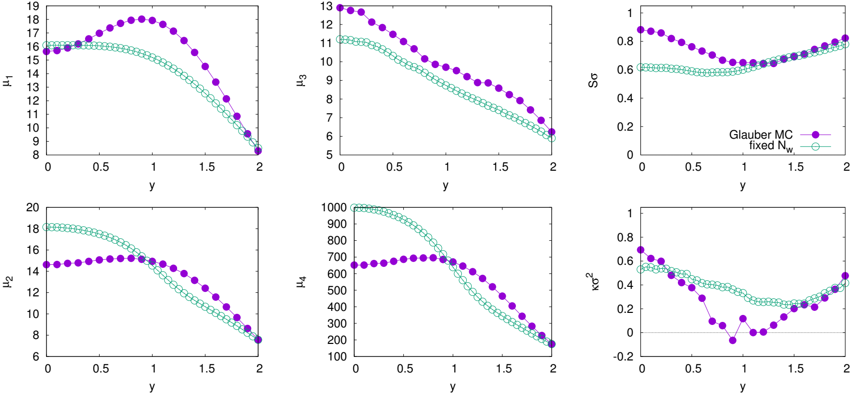
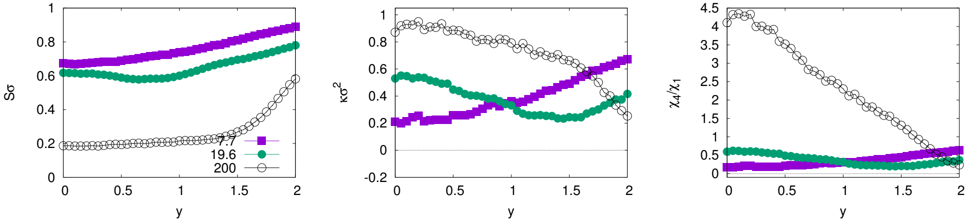


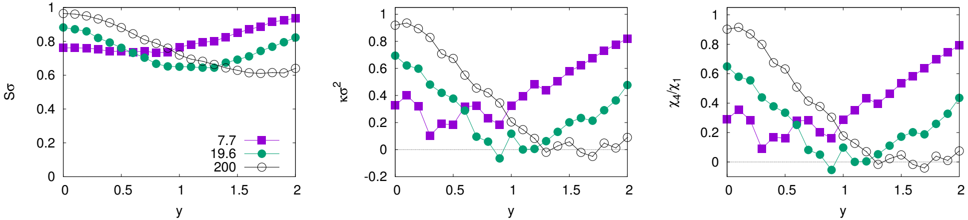
cf: [J. Brewer, S. Mukharjee, K. Rajagopal, Y. Yin, Phys. Rev. C 98 (2018) 061901]

Dependence on rapidity for different collision energies

Fixed $N_w = 338$, $N_{B\bar{B}} = 16.94$, $y_m = 1.019$, 2×10^7 events,

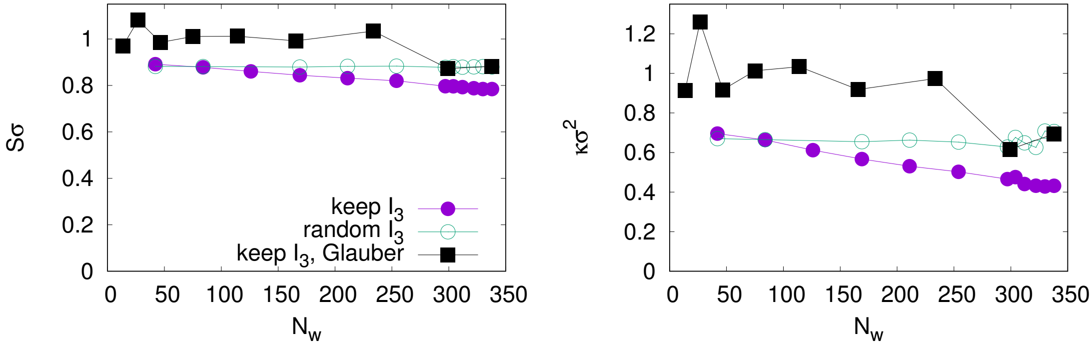


Glauber MC, 1.2×10^6 events



Net-proton number: dependence on centrality

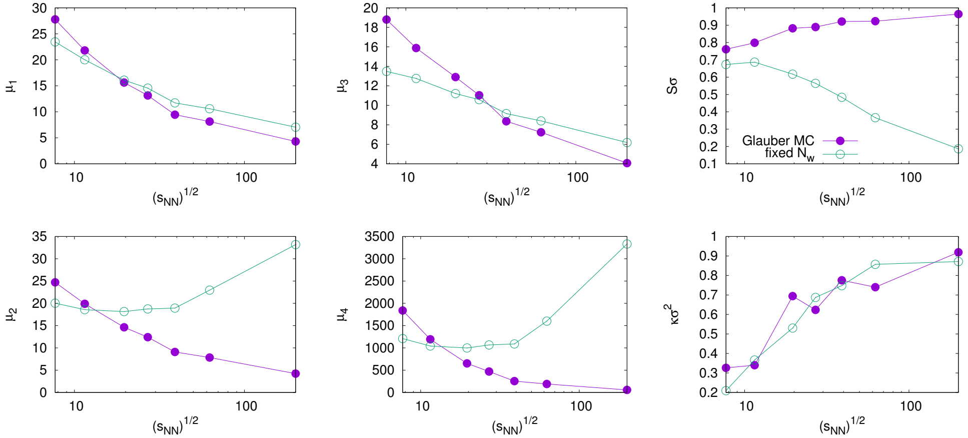
$\sqrt{s_{NN}} = 19.6$ GeV: $y_m = 1.019$, $N_{B\bar{B}}/N_w = 0.050$
Statistics: 2×10^7 for fixed N_w, $\sim 5 \times 10^5$ for Glauber MC



$S\sigma$ and $\kappa\sigma^2$ are lowered towards more central events of wounded protons nucleons remember their isospin.

Net-proton number: dependence on collision energy

rapidity bin $\Delta y = 0.5$ around $y = 0$
Statistics: 2×10^7 events for fixed N_w, 1.2×10^6 events for Glauber MC



The importance of produced $B\bar{B}$ pairs grows with increasing energy.

Net-proton number fluctuations from the statistical model

- Not calculable as derivatives of the partition function!
    - derivatives of $\log Z$ only contain fluctuations due to exchange with the heat bath
    - decays of resonances are random and may randomize proton number (even at fixed B)
- cumulants of proton and antiproton number via derivatives of the generating function

$$\left\langle (\Delta N)^l \right\rangle_c = \left. \frac{\mathrm{d}^l K(i\xi)}{\mathrm{d}(i\xi)^l} \right|_{\xi=0}$$

$$K(i\xi) = \ln \sum_{N=0}^{\infty} e^{i\xi N} P(N) = \sum_R \ln \left\{ \sum_{N_R=0}^{\infty} P_R(N_R) \left(e^{i\xi} p_R + (1-p_R) \right)^{N_R} \right\}$$

- $P_R(N_R)$: number probability of resonance R, furnished by statistical model
- Net-proton number cumulants obtained via

$$\left\langle (\Delta N_{p-\bar{p}})^l \right\rangle_c = \left\langle (\Delta N_p)^l \right\rangle_c + (-1)^l \left\langle (\Delta N_{\bar{p}})^l \right\rangle_c$$

Partial chemical equilibrium

- Statistical production can be used to describe hadron abundances and also their spectra
- (Simple) statistical model of interacting hadrons: interactions via inclusion of (free) resonance states [R. Dashen, S.K. Ma, H.J. Bernstein, Phys. Rev. 187 (1969) 345]

Chemical freeze-out

- Hadron abundances set by three (four) parameters: V, T_{ch}, μ_B, (γ_s)
- $T \sim 140 - 160$ MeV ($\sqrt{s_{NN}}$ dependent, above 7.7 GeV)

Kinetic freeze-out

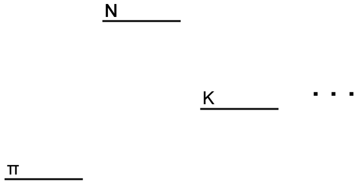
- Sets the p_T spectra
- need transverse expansion
- slope due to T_k and $\langle v_t \rangle$
- $T_k \sim 80 - 120$ MeV (also higher)

How to build a scenario with chemical and kinetic freeze-out?

- need to freeze the effective numbers of stable hadrons—projected numbers after decays of all resonances $N_h^{eff} = \sum_r p_{r\to h} \langle N_r \rangle$
- Assumption: at chemical freeze-out inelastic collisions stop and elastic continue

The chemical potentials

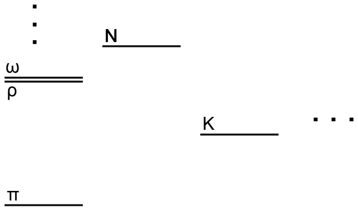
- ground state species do not change one into other $\Rightarrow$ chemical potential for each



[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B 378 (1992) 95]

The chemical potentials

- ground state species do not change one into other ⇒ chemical potential for each
- towers of resonances above every stable hadron species
- resonances always in equilibrium with ground state ⇒ it does not cost extra energy to produce or decay resonance into stable species
- resonance chemical potentials from those of stable hadrons, e.g. $\mu_\rho = 2\mu_\pi$, $\mu_\omega = 3\mu_\pi$



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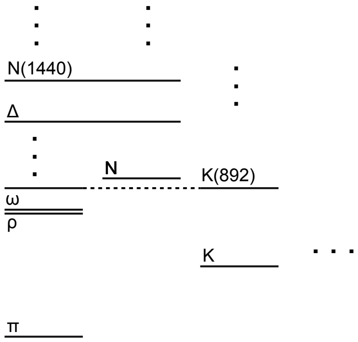
- resonances that decay into two different stable species, e.g. $\mu_\Delta = \mu_N + \mu_\pi$, $\mu_{K(892)} = \mu_\pi + \mu_K$



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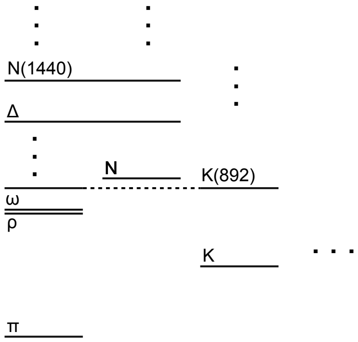
- resonances that decay into two different stable species, e.g. $\mu_\Delta = \mu_N + \mu_\pi$, $\mu_{K(892)} = \mu_\pi + \mu_K$
- Resonances with more decay channels, chain decays:

$$\mu_R = \sum_h p_{R\to h}\mu_h$$



[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B 378 (1992) 95]

Evolution of chemical potentials

Keep the (effective stable) particle numbers constant, as a function of temperature!

$$\langle N_h^{eff}\rangle = \sum_r p_{r\to h} V(T) n_r(T,\{\mu(T)\})\,, \qquad \frac{\mathrm{d}\langle N_h^{eff}\rangle}{\mathrm{d}T} = 0$$

$$-\frac{\frac{\mathrm{d}V}{\mathrm{d}T}}{V}\sum_r p_{r\to h} n_r(T) = \sum_r p_{r\to h}\frac{\mathrm{d}n_r(T)}{\mathrm{d}T}$$

Obtain the derivative of volume from entropy conservation: $0 = \mathrm{d}S/\mathrm{d}T = \mathrm{d}(sV)/\mathrm{d}T$

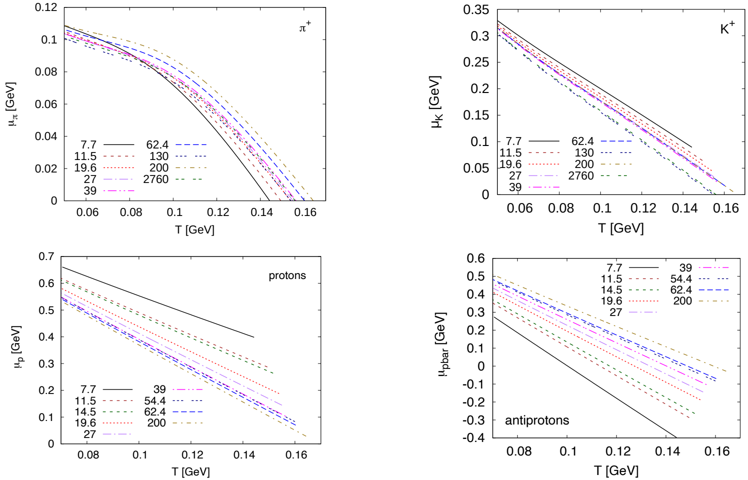
$$-\frac{\frac{\mathrm{d}V}{\mathrm{d}T}}{V} = \frac{\frac{\mathrm{d}s}{\mathrm{d}T}}{s}$$

Equations for the evolution of chemical potentials

$$\frac{\sum_r p_{r\to h}\frac{\mathrm{d}n_r(T,\{\mu(T)\})}{\mathrm{d}T}}{\mathrm{d}s/\mathrm{d}T} = \frac{1}{s}\sum_r p_{r\to h} n_r(T,\{\mu(T)\})$$

Evolution of chemical potentials: results

Start the evolution of chemical potentials at the chemical freeze-out
[STAR collab., Phys. Rev. C 96 (2017) 044904 and ALICE collab., Nucl. Phys. A 904-905 (2013) 531c]



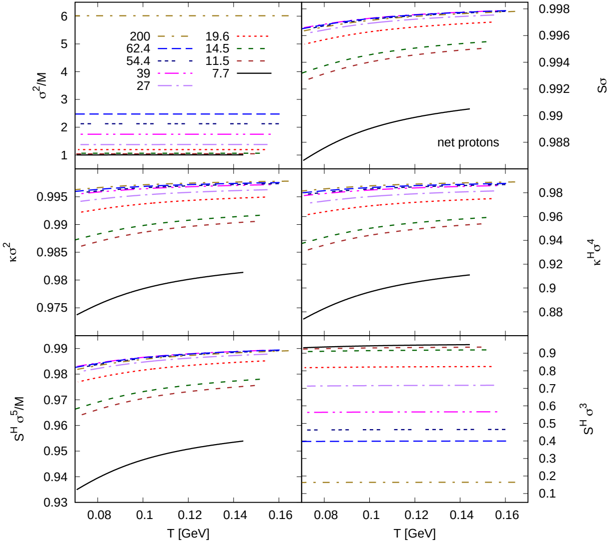
Net-proton number fluctuations from PCE

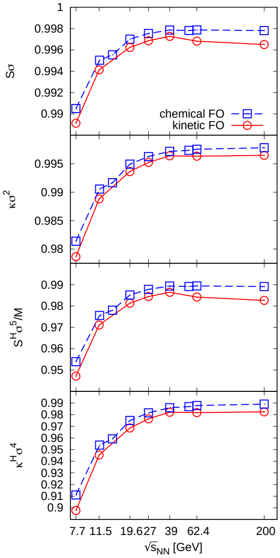
- Cumulants of the resonance number distributions

$$
\begin{aligned}
\langle N_R \rangle_c &= \frac{g_R V}{2\pi^2} m_R^2 T \sum_{j=1}^{\infty} \frac{(\mp 1)^{j-1}}{j} e^{j\mu_R/T} K_2\left(\frac{j m_R}{T}\right) , \\
\left\langle (\Delta N_R)^l \right\rangle_c &= \frac{g_R V}{2\pi^2} m_R^2 T \sum_{j=1}^{\infty} (\mp 1)^{j-1} j^{l-2} e^{j\mu_R/T} K_2\left(\frac{j m_R}{T}\right) .
\end{aligned}
$$

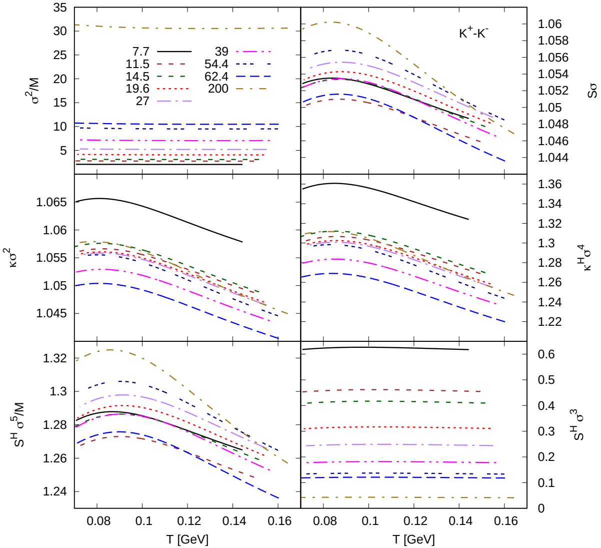
- first terms in the sums correspond to Boltzmann approximation (not BE or FD)
- In Boltzmann approximation, cumulants of all orders are the same!

Results for net-proton cumulants in PCE





Results for $K^+ - K^-$ cumulants in PCE



Conclusions

- Net baryon number fluctuations are sensitive to the statistical properties of the matter in the phase diagram.
- Only (net) proton number in limited detector acceptance is measurable—this involves other effects on which fluctuations depend.
- Exciting data on χ_4/χ_2 at $\sqrt{s_{NN}} = 7.7$ GeV.
- A "minimal" model for proton number fluctuations:
  - rapidity dependent composition through two components: wounded B and produced $B\bar{B}$
  - Glauber MC (GLISSANDO 2)

  Results from minimal model:
  - rapidity dependence of $\kappa\sigma^2$ with $\sqrt{s_{NN}}$-dependent minimum
  - baryon number conservation: decrease of $S\sigma$ and $\kappa\sigma^2$ with lower energies
- Results from Partial Chemical Equilibrium on net-proton number fluctuations
  [B. Tomášik, P. Hillmann, M. Bleicher, Phys.Rev.C 104 (2021) 044907]
  - volume-independent ratios of cumulants of net-proton number are almost temperature independent $\Rightarrow$ they reflect values at chemical freeze-out
  - experimental data on cumulants at low energies are not reproduced