Eigenvalue spectrum of 2+1 flavor QCD using highly improved staggered quarks in the continuum limit

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Reference: O. Kaczmarek, R.S.,S. Sharma, arXiv:2301.11610

New Trends in Thermal Phases of QCD, Prague, 14-17 April 2023

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- However $U_A(1)$ part of the chiral symmetry is anomalous hence it is not clear whether it is effectively restored along with its non-singlet part.
- There are some evidence that show $U_A(1)$ remains effectively broken at T_c in $2+1$ flavor QCD with physical quark mass m [A. Bazavov et al., 12, V. Dick et al., 15] even when $m \to 0$ [O. Kaczmarek, L. Mazur, and S. Sharma, 21] but also there are some contrary results [S. Aoki et. al., JLQCD coll, 15, 17, 21, B. Brandt et. al., 16, T. W. Chiu, 13].

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- The eigenvalue spectrum of Dirac operator contains valuable information about the fundamental properties of Quantum Chromodynamics (QCD).

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- It will be interesting to check the properties of the eigenvalue spectrum by carefully performing a continuum extrapolation, in the large volume limit.

Technical details

- In this work we use the gauge configurations for $2+1$ flavor QCD with physical quark masses, generated by the HotQCD collaboration using Highly Improved Staggered quark (HISQ)

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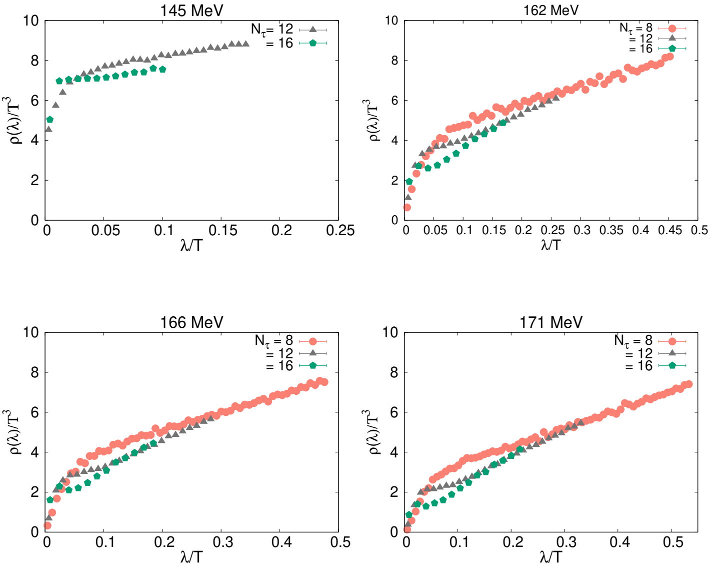
- We next measure $60 - 200$ eigenvalues of the massless HISQ Dirac matrix per configuration.

Our results: Eigenvalue density as a function of T



Characterizing Bulk modes

- The bulk eigenvalue density is characterized as

[S. Aoki, H. Fukaya, and Y. Taniguchi, 12]

$$\frac{\rho(\lambda)}{T^3} = \frac{\rho_0}{T^3} + \frac{\lambda}{T} \cdot \frac{c_1(T,m)}{T^2} + \frac{\lambda^2}{T^2} \cdot \frac{c_2(T,m)}{T} + \frac{\lambda^3}{T^3} c_3(T,m) \ .$$

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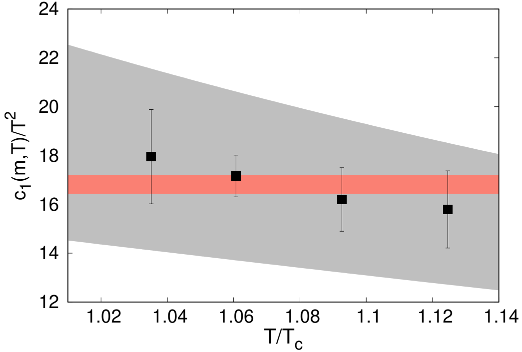
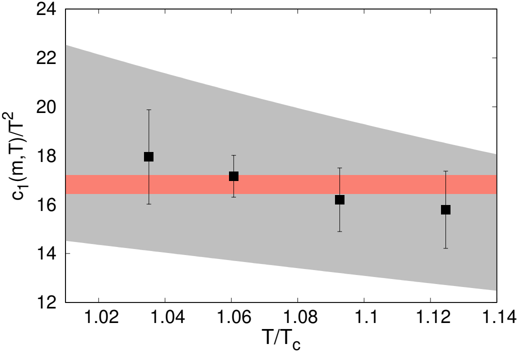
- The coefficients $c_{1,2,3}$ can in general be a function of the temperature T and the light-quark mass m.
- Earlier study assuming the restoration of singlet and non-singlet part for upto 6-point functions in scalar-pseudoscalar correlators gives [Aoki et.al. 2012] $c_{1,2} = \mathcal{O}(m^2)$ and $c_3 = \text{const} + \mathcal{O}(m^2)$.

Characterizing Bulk modes



- We perform a fit to the bulk part i.e. all eigenvalues $\lambda > \lambda_0$ with $\frac{\rho(\lambda)}{T^3} = \frac{\lambda}{T} \cdot \frac{c_1(T,m)}{T^2} + \frac{\rho_0}{T^3}$.



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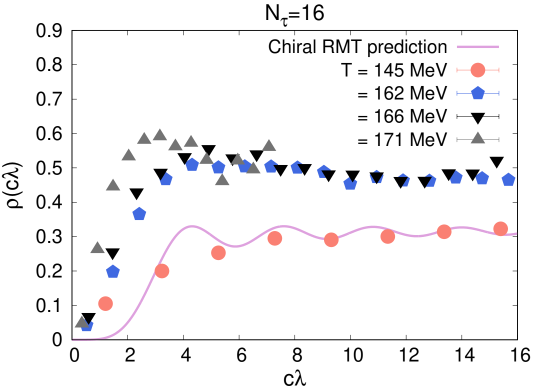
- Taking continuum ($\sim 1/N_\tau^2$) extrapolation of c_1 at different T, we get a m-independent part of the slope $c_1(m,T)/T^2 = 16.8(4) \rightarrow$ in addition to $\mathcal{O}(m^2)$ found earlier.

- Near zero modes distribution at zero temperature from chiral random matrix theory for $N_f = 2$ flavors and zero topological charge sector is distributed according to [G. Akemann, 2016]

$$\rho(c\lambda) = \frac{c\lambda}{2}\left[J_2^2(c\lambda) - J_3(c\lambda)J_1(c\lambda)\right] \quad , \; c = \langle 0|\bar{\psi}\psi|0\rangle V/T \; .$$



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- These quantities are defined as $\chi_\pi = \int d^4x \, \langle \pi^i(x) \pi^i(0) \rangle$ and $\chi_\delta = \int d^4x \, \langle \delta^i(x) \delta^i(0) \rangle$

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- In this work we measure $(\chi_\pi - \chi_\delta)/T^2$ at the four different temperatures above T_c, and perform a $\sim 1/N_\tau^2$ continuum extrapolation at each temperature.

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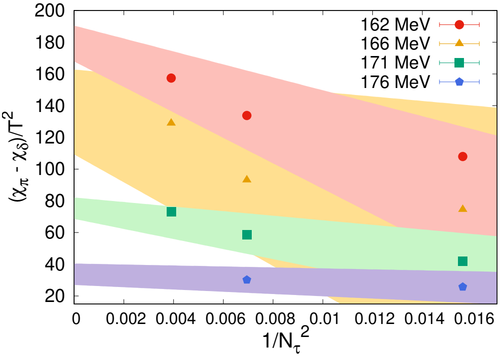

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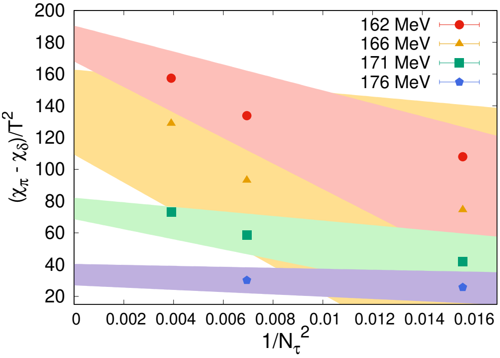

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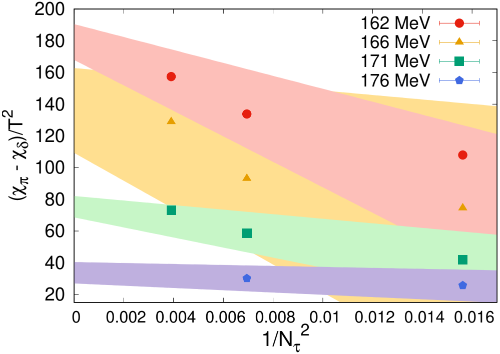

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- A linear extrapolation of the intercept gives a temperature around $1.14\ T_c$ when this observable goes to zero.
- $U_A(1)$ is effectively restored at temperature $\sim 1.14\,T_c$

Comparison with earlier works

- Studies with chiral fermions but on relatively small volumes and finite lattice spacings have reported effective restoration of $U_A(1)$ at T_c.

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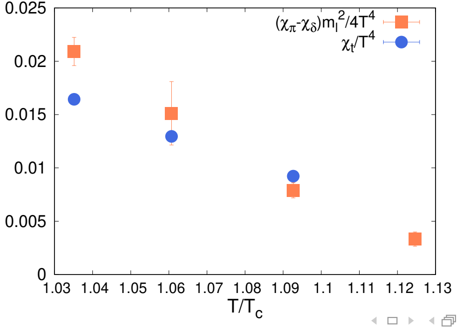
- Our study points towards an earlier effective restoration of $U_A(1)$ due to non-trivial interplay between interactions and disorder in QCD.

Chiral Ward identity

- In the chiral symmetry restored phase, $\chi_\sigma = \chi_\pi$ and $\chi_\delta = \chi_\eta$ hence one obtaines $\chi_\pi - \chi_\delta = 4\chi_{5,\text{disc}}$.



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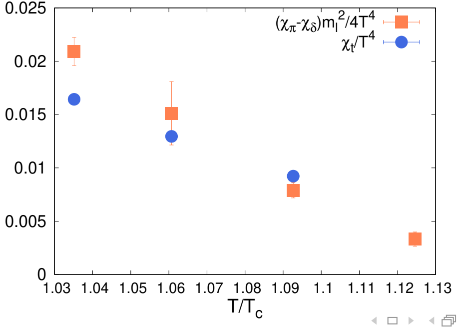
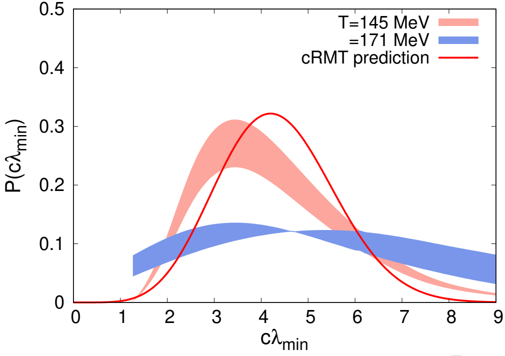
- Using chiral Ward identities it is known that $\chi_{5,\mathrm{disc}} = \chi_t/m^2$ where χ_t is the topological susceptibility of QCD. This allows relating the $U_A(1)$ breaking parameter to the topological susceptibility [L. Mazur, Ph.D thesis, 2021] through the relation, $1/4(\chi_\pi - \chi_\delta)m_l^2/T^4 = \chi_t/T^4$.



- For a chiral random matrix ensemble for $N_f = 2$ (at zero temperature) the lowest eigenvalue is distributed according to [Akemann, 2016],

$$P(x) = \frac{x}{2} e^{-\frac{x^2}{4}} \left[J_2^2(x) - J_3(x) J_1(x) \right] \; , \; x = c\lambda_{\text{min}} \; .$$



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- A fit to the unfolded level-spacing distribution $P(s)$ gives a level repulsion between the small s bulk modes that is quadratic similar to that of random matrices belonging to the Gaussian unitary ensemble.

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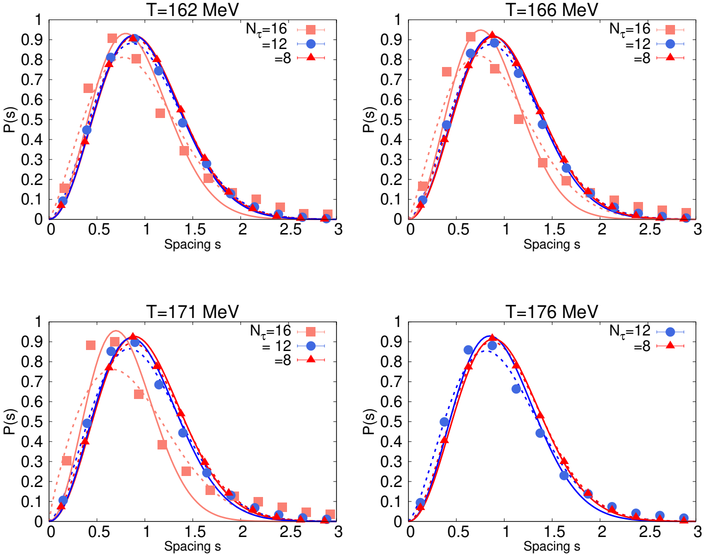

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- For small s, the level repulsion is still quadratic similar to GUE.
- In order to account for the long tail of the spacing distribution we fit it to $P(s) \sim s^2 \exp[(-\alpha s)]$ which falls off slowly than s^2 at large values of s parameterized by a fit parameter α.

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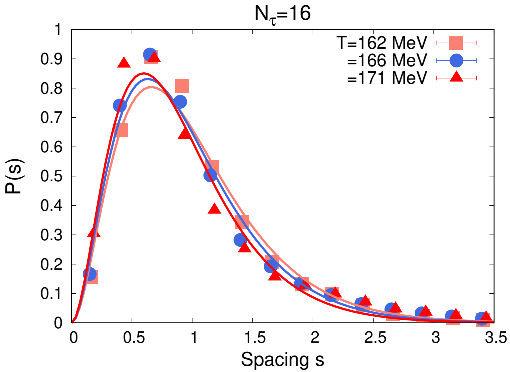

- The ansatz fits the lattice data quite well $\rightarrow$ reminiscent of mixing between the localized and delocalized modes at the band-edge of Anderson-like systems.

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- Bulk modes are delocalized following chiral RMT density distribution whereas the near-zero are localized following a different eigenvalue distribution.
- We find that at $T \sim 1.15\,T_c$ the bulk and near-zero modes completely separates.
- Incidentally this is the same temperature at which $U_A(1)$ is effectively restored.

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- We can visualize quarks as many-body interacting state moving in the disordered potential due to the instantons.
- At low $T \lesssim T_c$ instantons are strongly correlated resulting in cRMT like distribution of the small eigenvalues. For $T \gtrsim 1.15\ T_c$ correlations between instantons weaken $\rightarrow$ leads to separation of near-zero and bulk modes

Summary and Outlook

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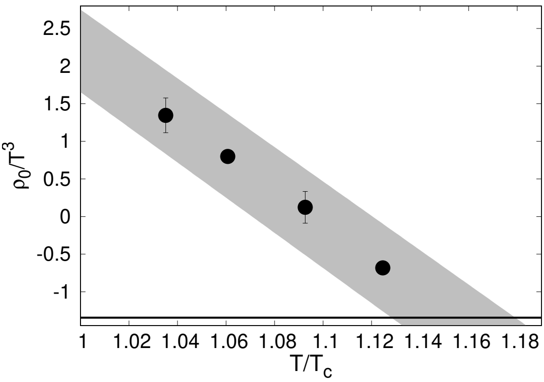
- Interplay of both these lead to separation of the near-zero peak out of the bulk modes which also happens to be around $T \sim 1.15\ T_c$.
- How different topological objects play a role in this phenomenon needs to be further investigated.

Thanks


2.5
2
1.5
1
0.5
0
-0.5
-1
ρ_0/T^3
1
1.02
1.04
1.06
1.08
1.1
1.12
1.14
1.16
1.18
T/T_c