

New Trends in Thermal Phases of QCD

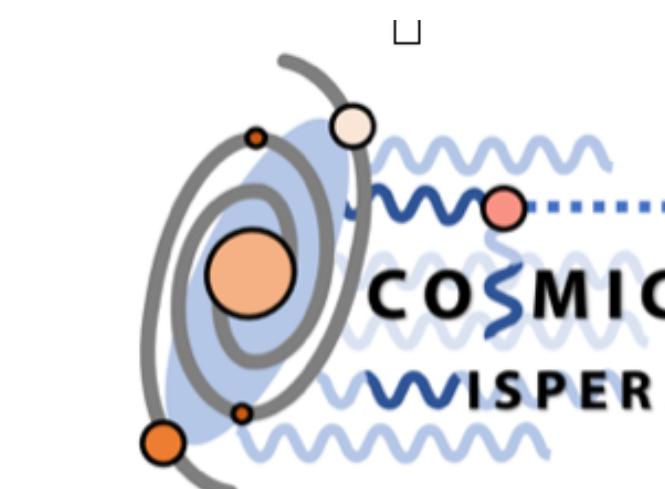
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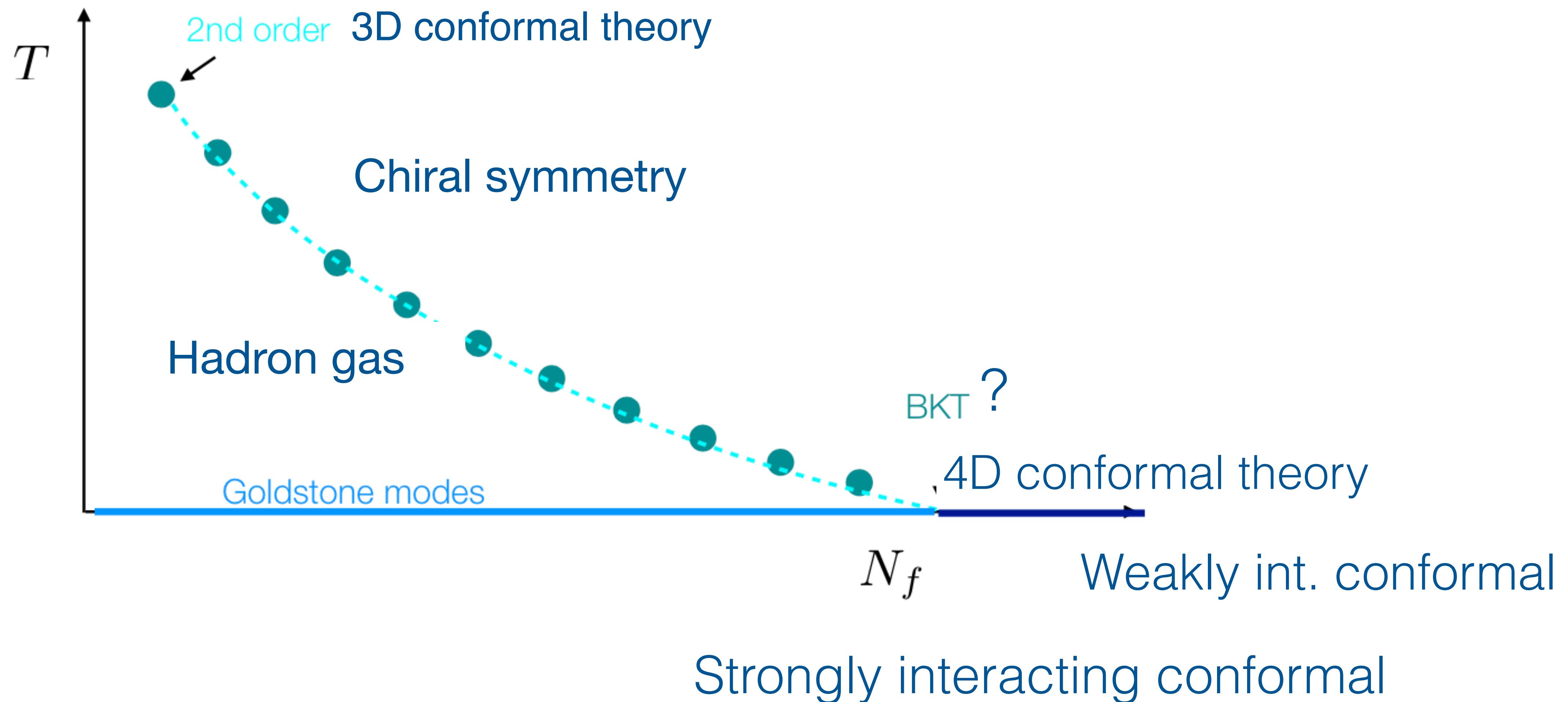
From the thermal QCD transition to the zero temperature QCD transition

Maria Paola Lombardo

INFN Firenze



AdS/CFT



.. prologue..

Euclidean finite temperature field theory

Imaginary time

and

Inverse
Temperature

pbc: bosons
abc: fermions

d-dimensional space

Continuous transition: diverging correlation length - dimensional reduction

Mode expansion and Decoupling

$$\phi(x, t) = \sum_{\omega_n=2n\pi T} e^{i\omega_n t} \phi_n(x) \quad \text{Bosons}$$

$$\psi(x, t) = \sum_{\omega_n=(2n+1)\pi T} e^{i\omega_n t} \psi_n(x) \quad \text{Fermions}$$

In the expression for the Action

$$S(\phi, \psi) = \int_0^{1/T} dt \int d^d x \mathcal{L}(\phi, \psi)$$

the integral over time can then be traded with a sum over modes, and we reach the conclusion that a $d+1$ statistical field theory at $T > 0$ is equivalent to a d -dimensional theory with an infinite number of fields.

When dimensional reduction is possible, only one boson (zero frequency) field survives

$$U(n)_L \times U(n)_R \cong SU(n) \times SU(n) \times U(1)_V \times U(1)_A$$



Universality class of the high T transition: theoretical (*lack of*)guidance

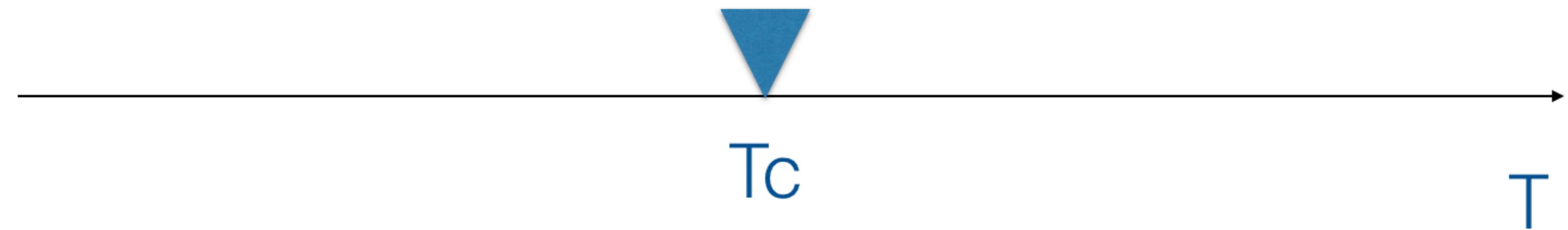
Parisen Toldin, Pelissetto, Vicari 2003

N_f	U(1) _A broken	U(1) _A restored
1	crossover or 1 st ord	$O(2) \rightarrow \mathbb{Z}_2$ or 1 st ord
2	$O(4) \rightarrow O(3)$ or 1 st ord Pisarski, Wilczek 1984	$U(2)_L \otimes U(2)_R \rightarrow U(2)_V$ or 1 st ord
≥ 3	1 st ord ?	1 st ord

$N_f=2$

$m=0$

$O(4)$ 3D IR fixed point



$N_f=2$

T

The magnetic equation of State:

$$h = M^\delta f(t/M^{1/\beta}).$$

$M \equiv \bar{\psi}\psi$, $h \equiv m_q$, $t \equiv T - T_c$, m_q is the quark mass and T_c is the critical temperature

Three strategies to identify the scaling behaviour:

- direct comparison with the Equation of State
- the study of the dependence of the pseudo-critical temperatures on the breaking field, also known as scaling of pseudo-critical temperatures
- definition of RG invariant quantities, which do not depend on the breaking field at the critical point.

Byproduct: critical temperature in the chiral limit

Significant source of scaling violations:

additive linear mass corrections to $\bar{\psi}\psi$

Analysis tool : scaling of the singular part of the Free Energy

Assumption:

$$\text{Free Energy} = \text{Singular} + \text{Regular}$$



Analysis tool : scaling of the singular part of the Free Energy

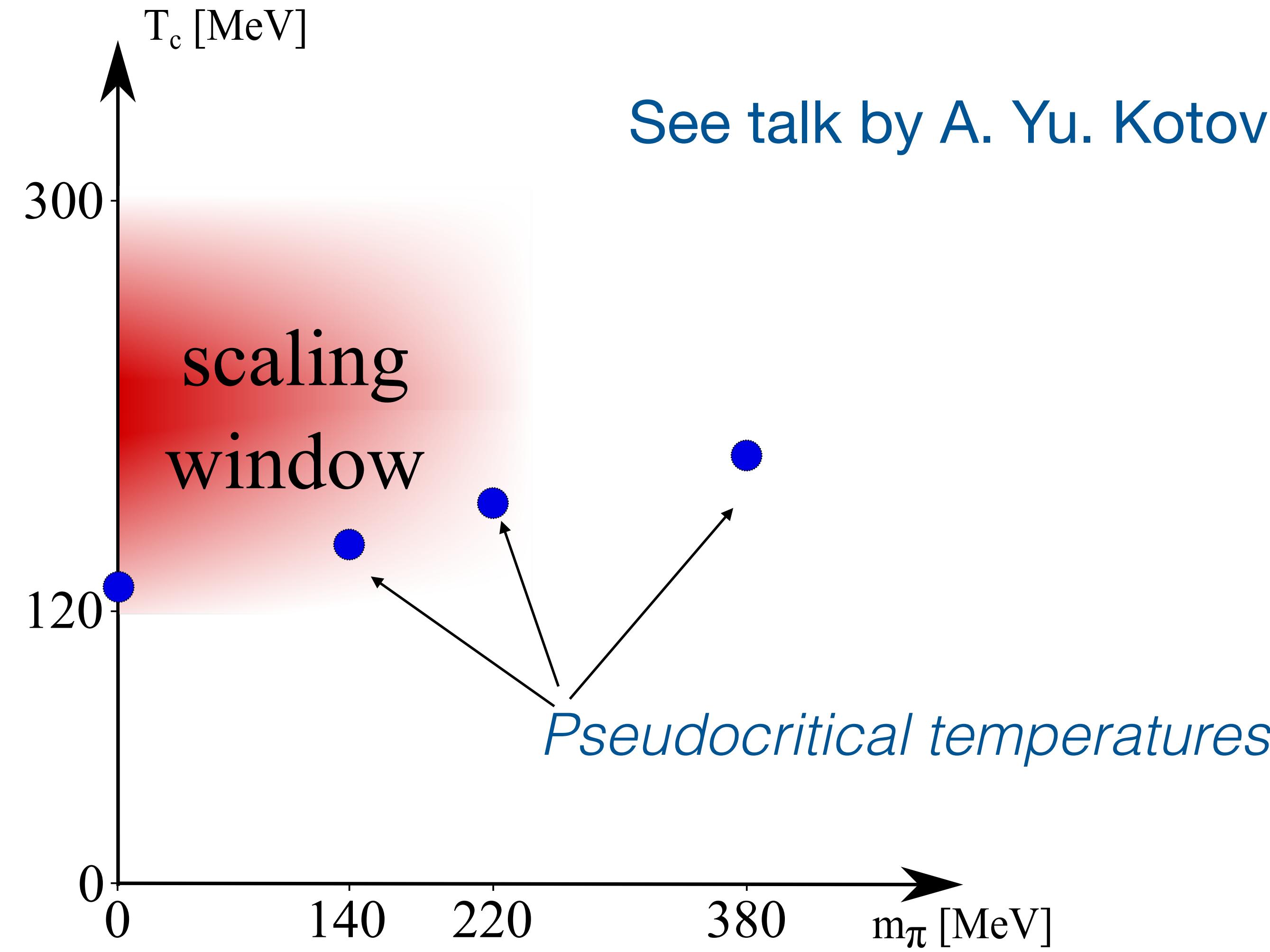
Assumption: Free Energy = Singular + Regular

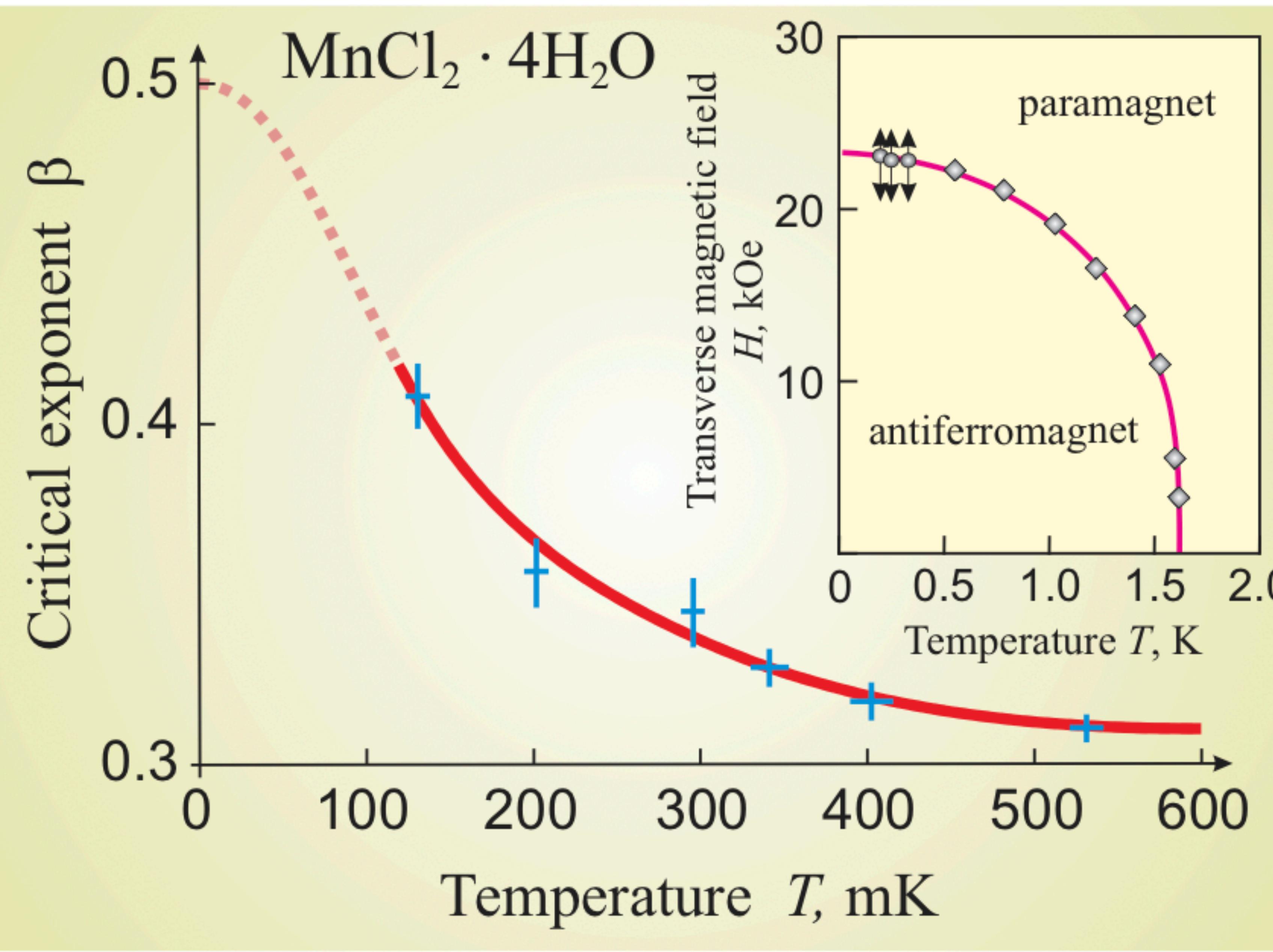


$$|t| > G$$

Mean field:
Interaction dominated,
 $\xi^2 G$ Small

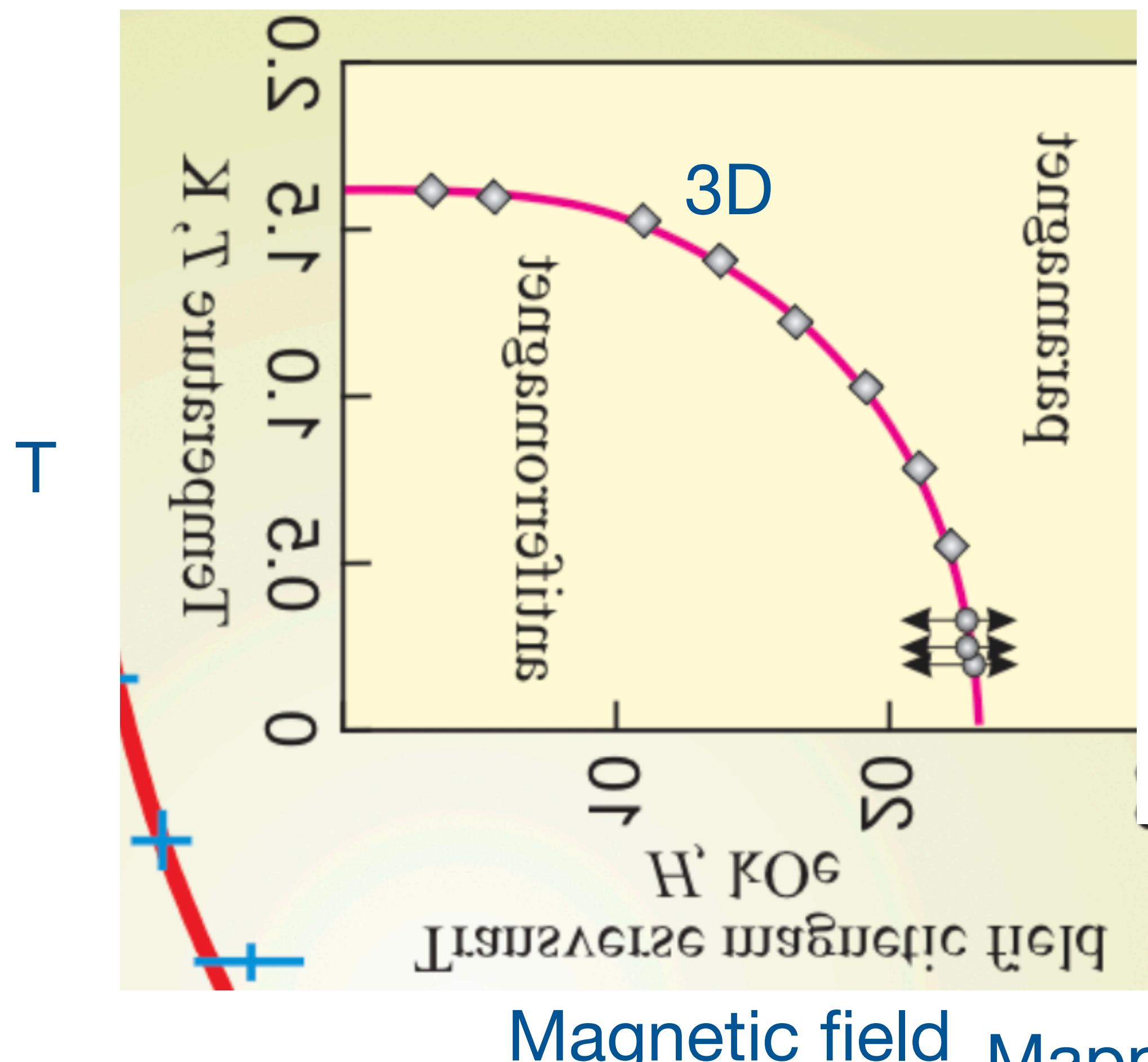
A sketch of the O(4) 3D scaling window for high T QCD



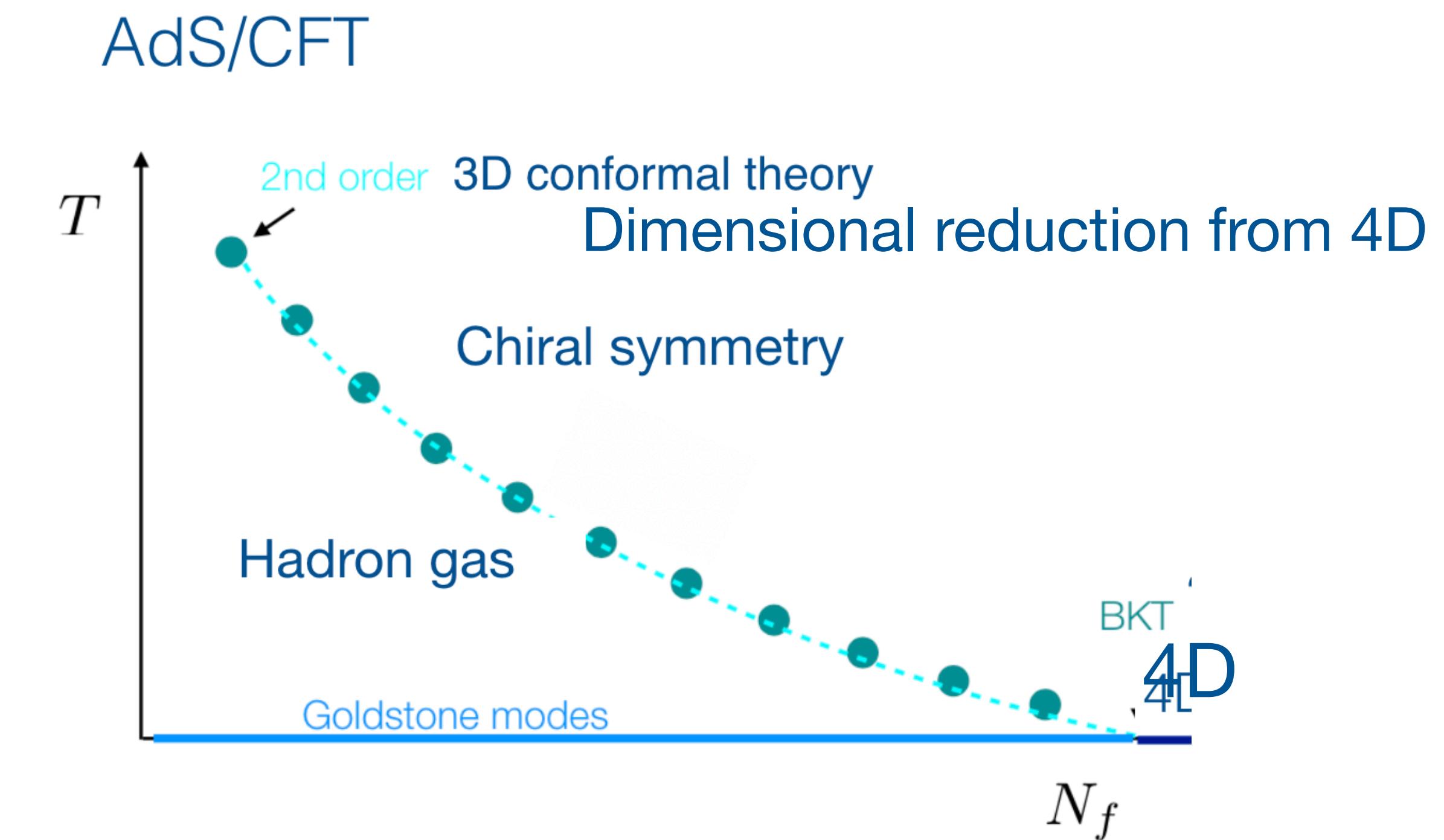


Vasin, Ryzhov, Vinokur
2015, Nature Sc.Rep.

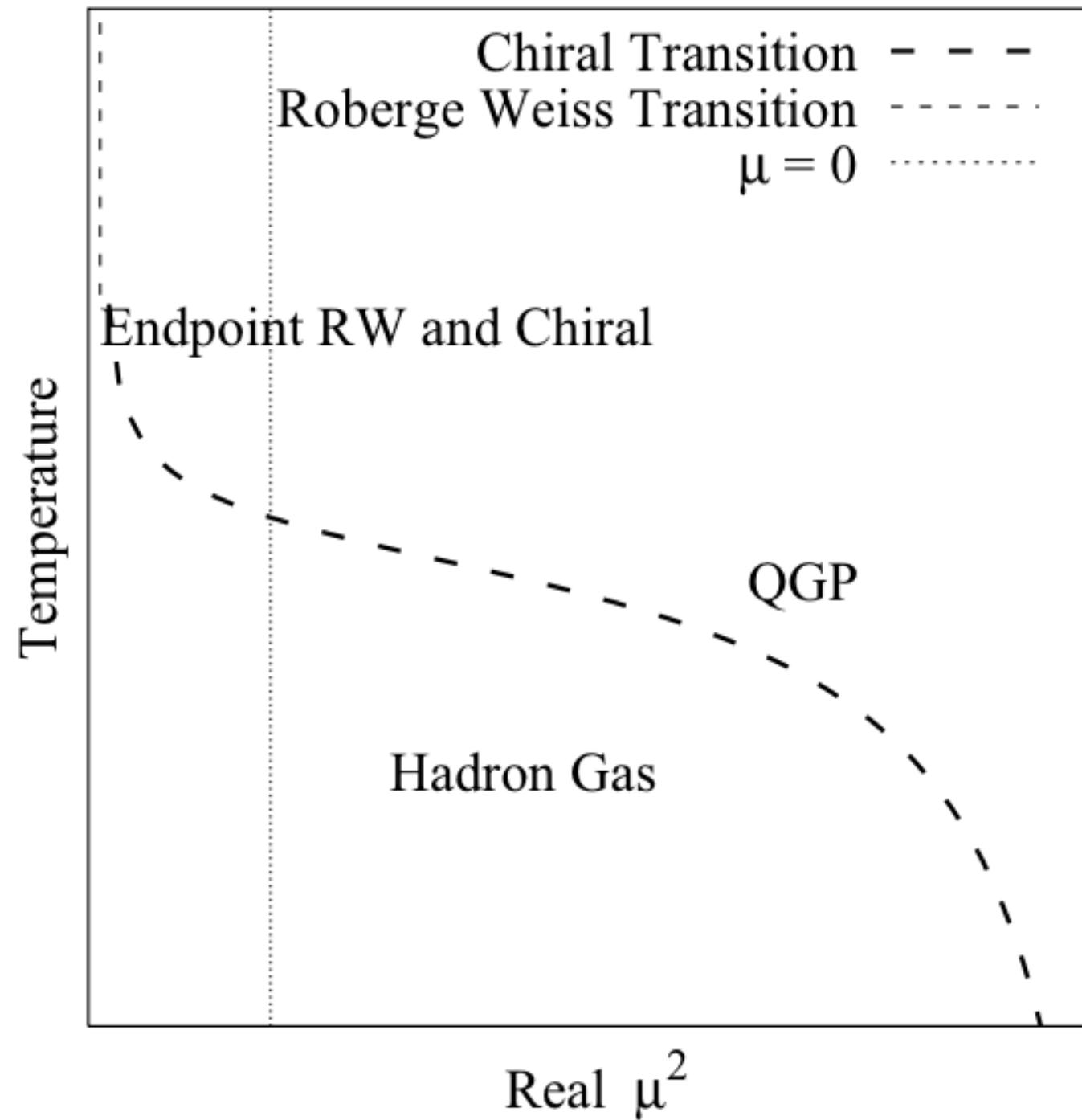
Continuous evolution of critical exponents along the critical line



Mapping to 3D+1
4th dimension:
imaginary time



Strongly coupled QGP and singularities



T_{RW} approx. 207 MeV

$$T = T_{RW} \quad n(\mu_I) = A\mu_I(\mu_I^c{}^2 - \mu_I^2)^\alpha$$

Di Renzo, D'Elia, MpL 2007

The conformal window



The QCD beta function, the infrared fixed point and

The zero temperature phase transition

$$\mu \frac{\partial}{\partial \mu} \alpha(\mu) = \beta(\alpha) \equiv -b \alpha^2(\mu) - c \alpha^3(\mu) - d \alpha^4(\mu) - \dots ,$$

$$b = \frac{1}{6\pi} (11N - 2N_f)$$

$$c = \frac{1}{24\pi^2} \left(34N^2 - 10NN_f - 3\frac{N^2 - 1}{N}N_f \right) .$$

Asymptotically free: $b > 0$ ($N_f < \frac{11}{2}N$)

For $N = 3$ c changes sign when $N_f > 8.05$

Infrared fixed point:

$$\alpha_* = -\frac{b}{c} .$$

Running coupling

$$b \log \left(\frac{q}{\mu} \right) = \frac{1}{\alpha} - \frac{1}{\alpha(\mu)} - \frac{1}{\alpha_*} \log \left(\frac{\alpha (\alpha(\mu) - \alpha_*)}{\alpha(\mu) (\alpha - \alpha_*)} \right) ,$$

Close to $N_f = 16$ the infrared fixed point is small

defining

$$\Lambda = \mu \exp \left[\frac{-1}{b\alpha_*} \log \left(\frac{\alpha_* - \alpha(\mu)}{\alpha(\mu)} \right) - \frac{1}{b\alpha(\mu)} \right] ,$$

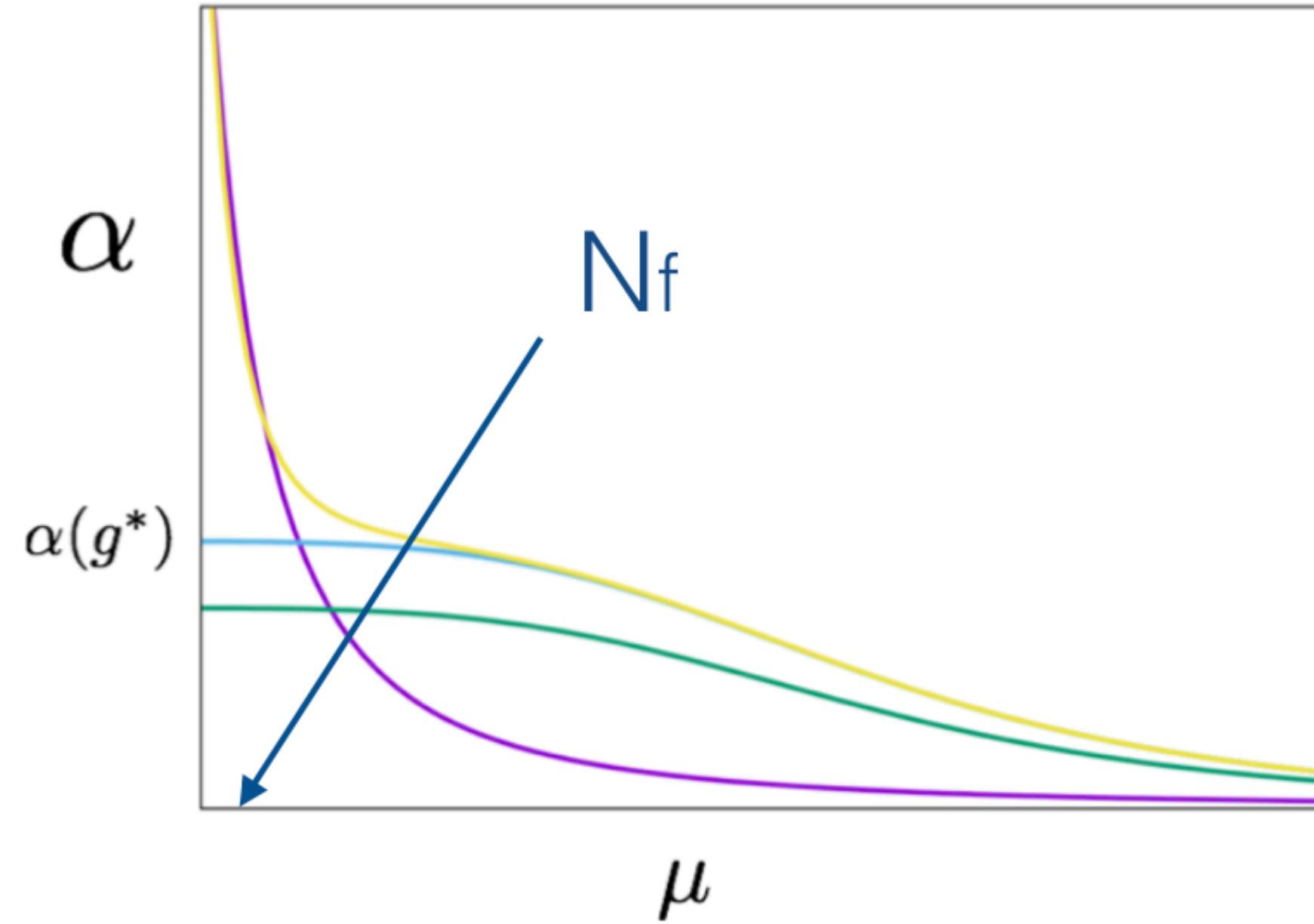
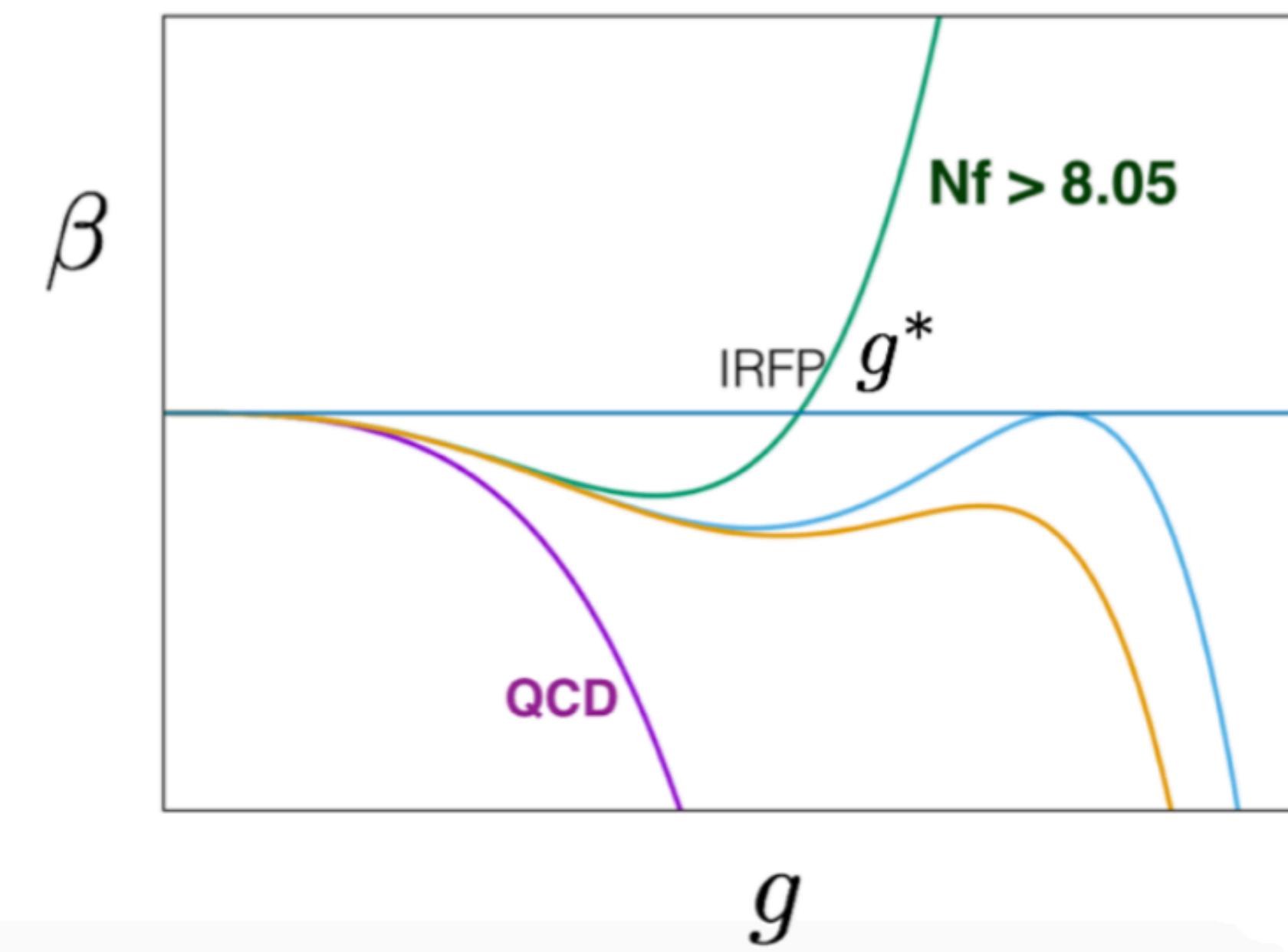
Running coupling:

$$\alpha \approx \frac{1}{b \log \left(\frac{q}{\Lambda} \right)} ,$$

$$\alpha \approx \frac{\alpha_*}{1 + \frac{1}{e} \left(\frac{q}{\Lambda} \right)^{b\alpha_*}} .$$

UV

Close to IR fixed point



$$\alpha_c \equiv \frac{\pi}{3 C_2(R)} = \frac{2\pi N}{3(N^2 - 1)}$$

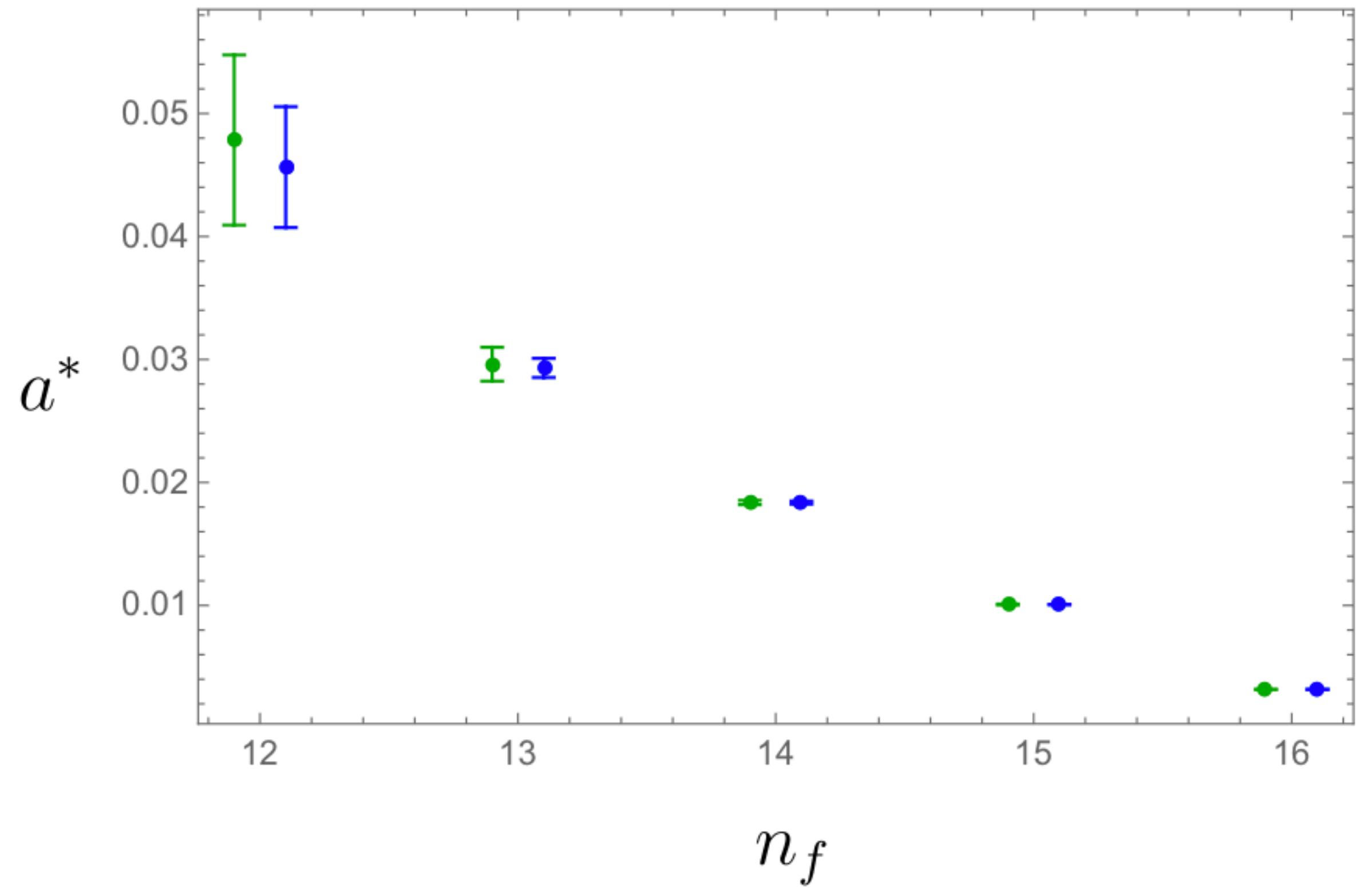
When

$$N_f > N_f^c = N \left(\frac{100N^2 - 66}{25N^2 - 15} \right)$$

$$\alpha(g^*) < \alpha_c$$

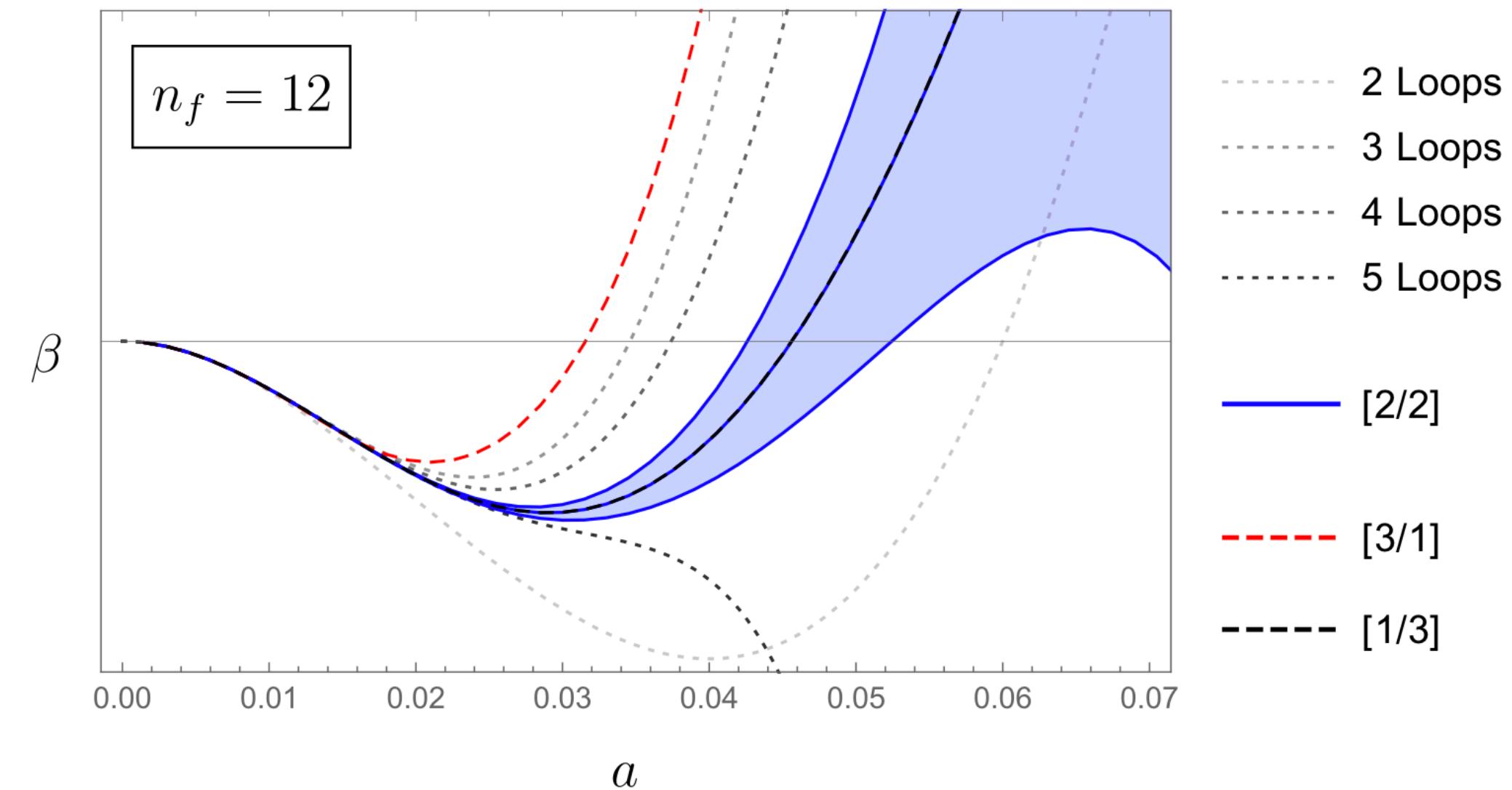
chiral symmetry is unbroken

IR fixed point

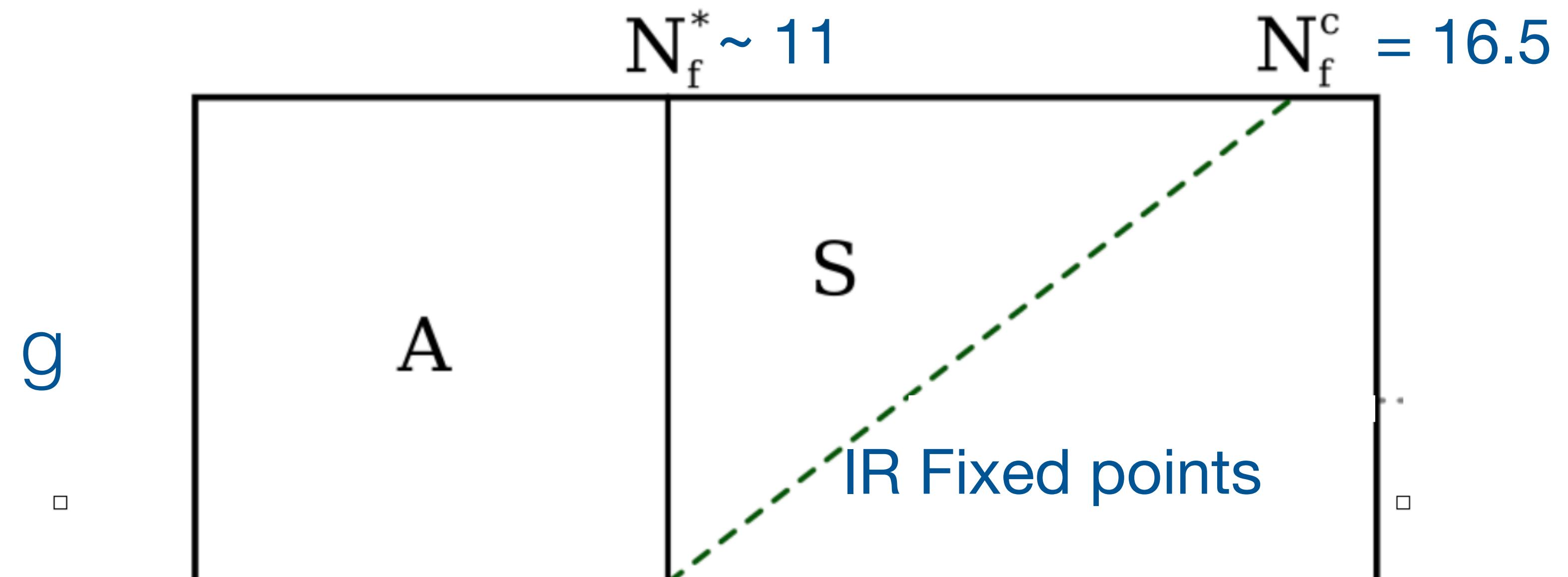


Di Pietro, Serone 2020

similar results Shrock, Dietrich 2014-2022



Phases of $N_c=3$ QCD at zero temperature



The chiral condensate in the conformal, mass deformed theory

$$\langle \bar{q}q \rangle \sim m^{\eta_{\bar{q}q}}$$

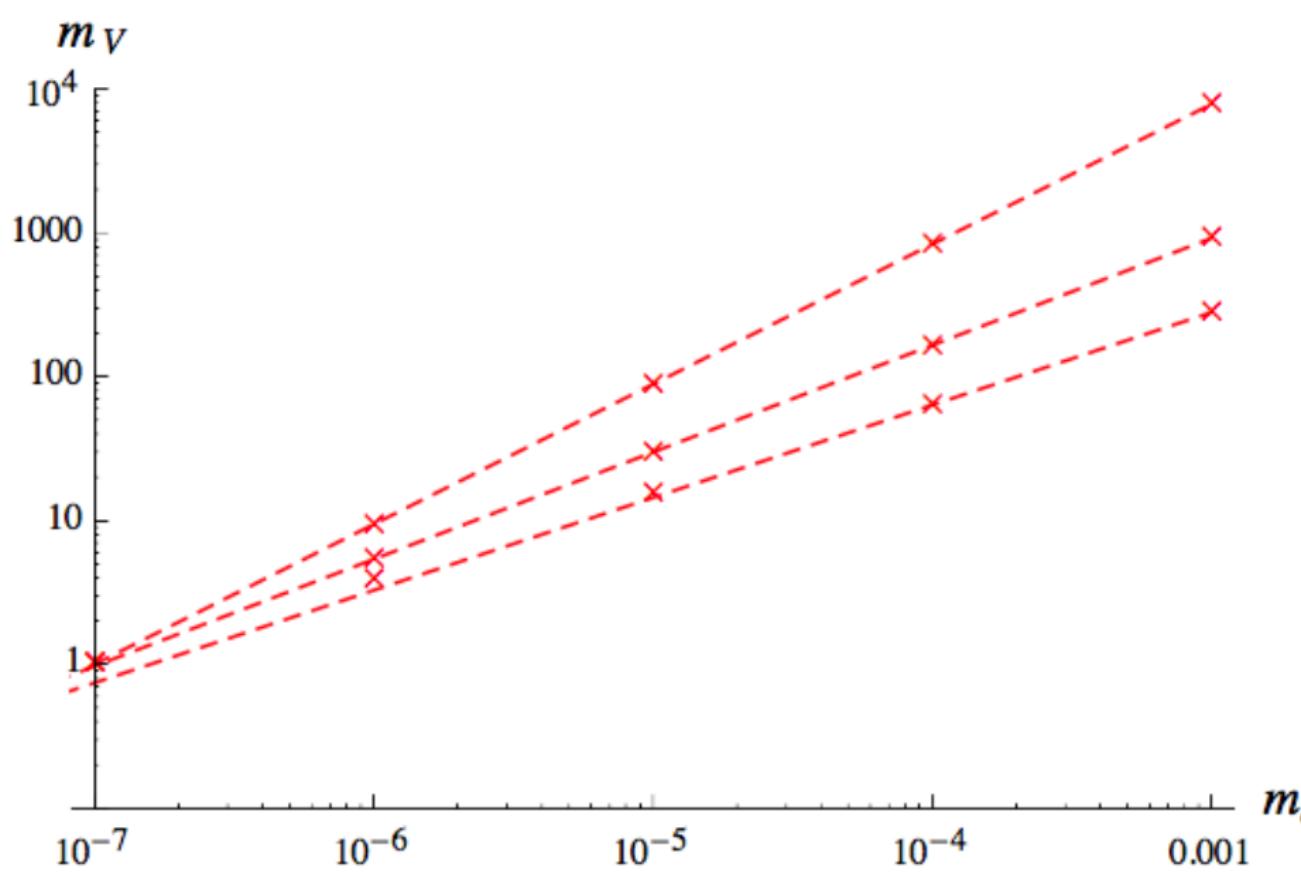
$$\eta_{\bar{q}q} = (3 - \gamma_*) / (1 + \gamma_*)$$

Del Debbio, Zwicky 2010



Conformal scaling (valid also for the masses)

$$M_H = c_H m^{1/y_h}$$



Nfc
Power law Scaling with
anomalous dimension

Status $N_f = 12$

Slide from M.Serone at “Newton 1665”

Ref.	$ \gamma^* $	Method
This work	0.320(85)	PB coupling
[44]	0.345(47)	PB conformal
[45]	0.23(6)	Gradient flow
[46]	0.47(10)	Top. susceptibility
[47]	0.33(6)	
[48]	0.32(3)	Dirac eigenmodes
[49]	0.235(46)	
[49]	0.235(15)	Finite-size scaling
[50]	0.45(5)	
[50]	0.403(13)	

Table 2: Comparison between the results of our Padé-Borel (PB) resummation for the mass-anomalous dimension for QCD with $n_f = 12$ –both using the coupling expansion and the Banks-Zaks conformal expansion, and averaging over all available Padé approximants in each case— and lattice results.

[44] - Carosso, Hasenfratz, Neil 1806.01385

[45] - Aoki et al. 1601.04687

[46] - Cheng, Hasenfratz, Petropoulos, Schaich, 1301.1355

[47] - Lombardo, Miura, Nunes da Silva, Pallante, 1410.0298

[48] - Cheng et al, 1401.0195

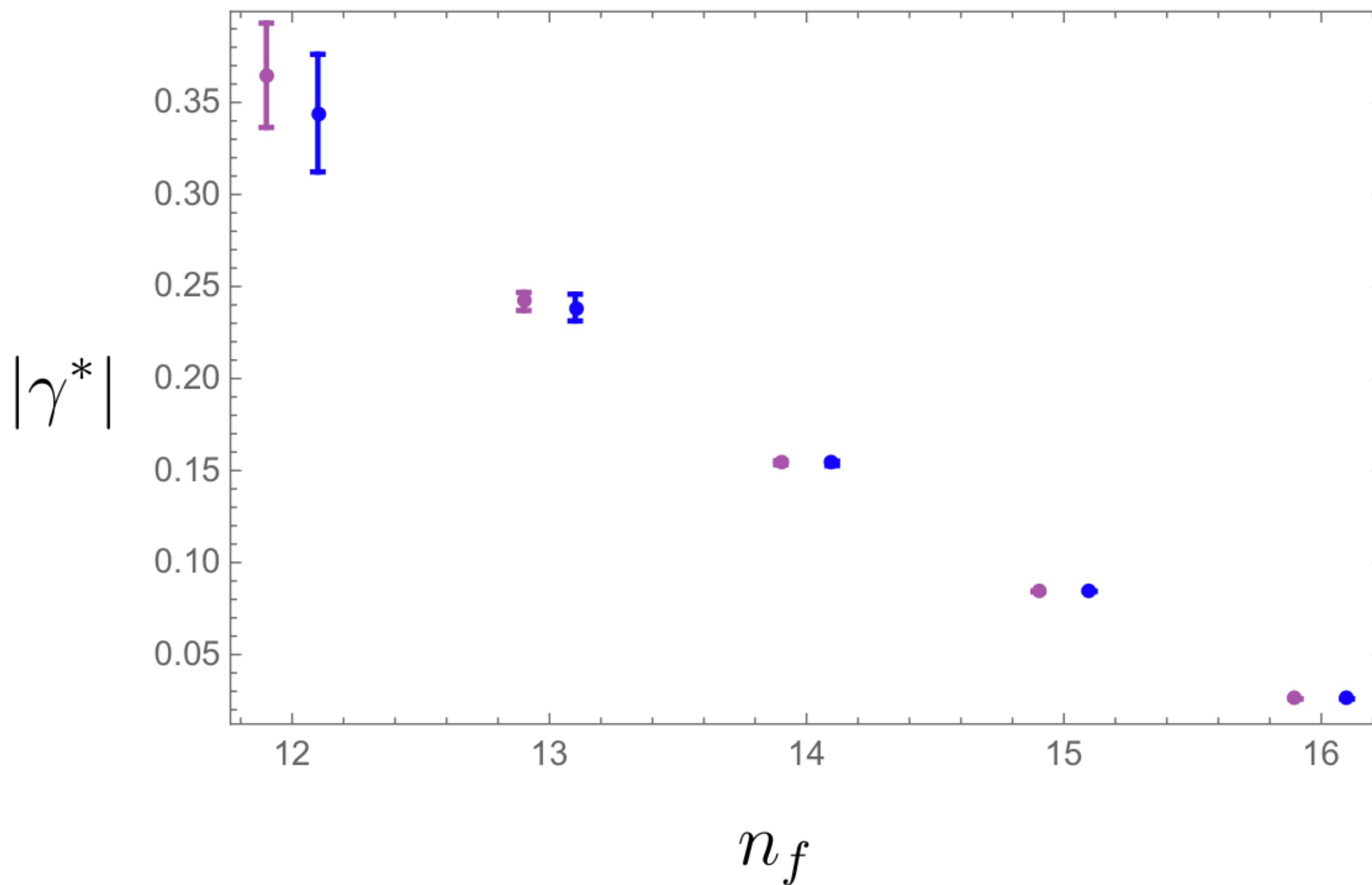
[49] - Aoki et al, 1207.3060

[50] - Appelquist et al, 1106.2148

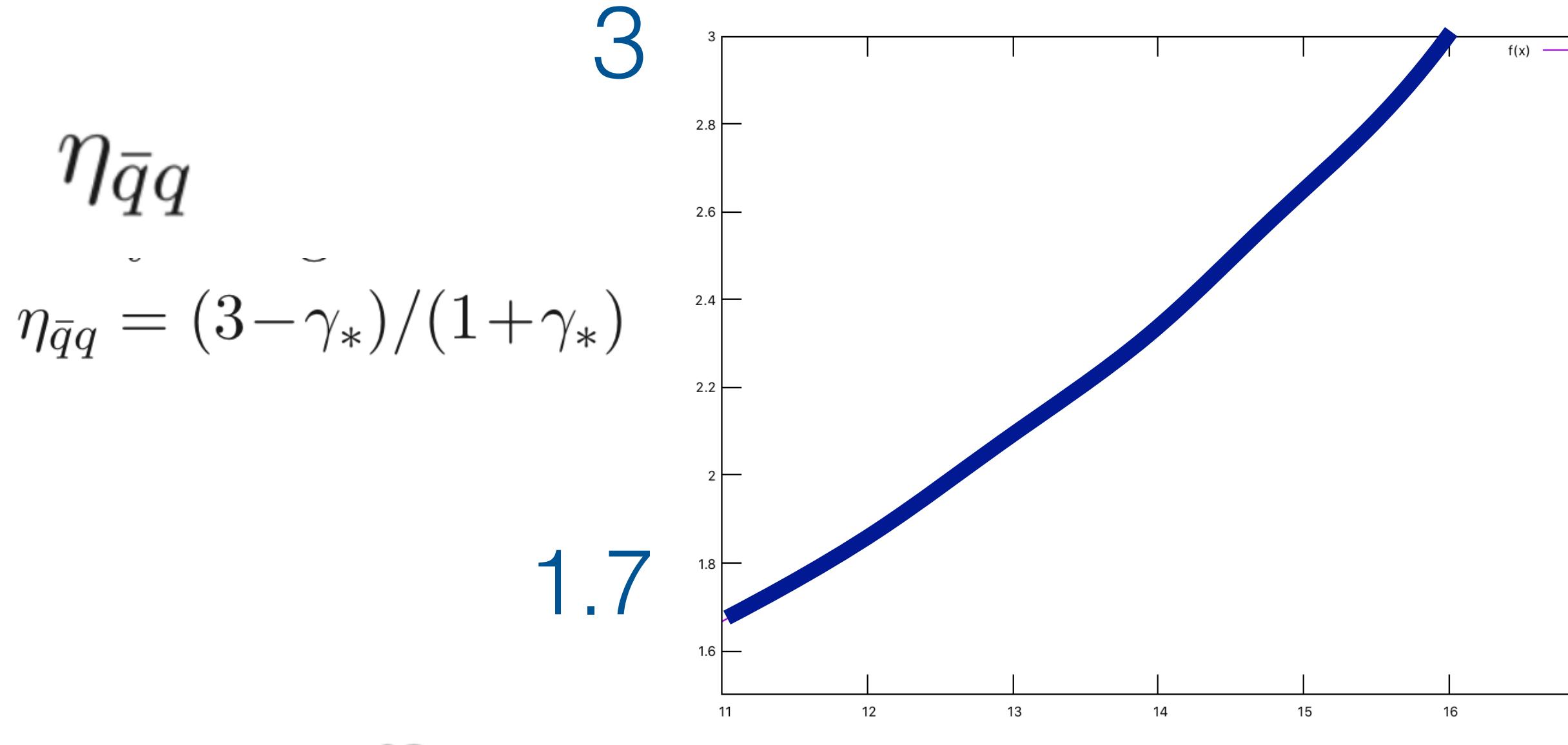
[from 2003.01742]

One group claims non-conformality

Anomalous dimension



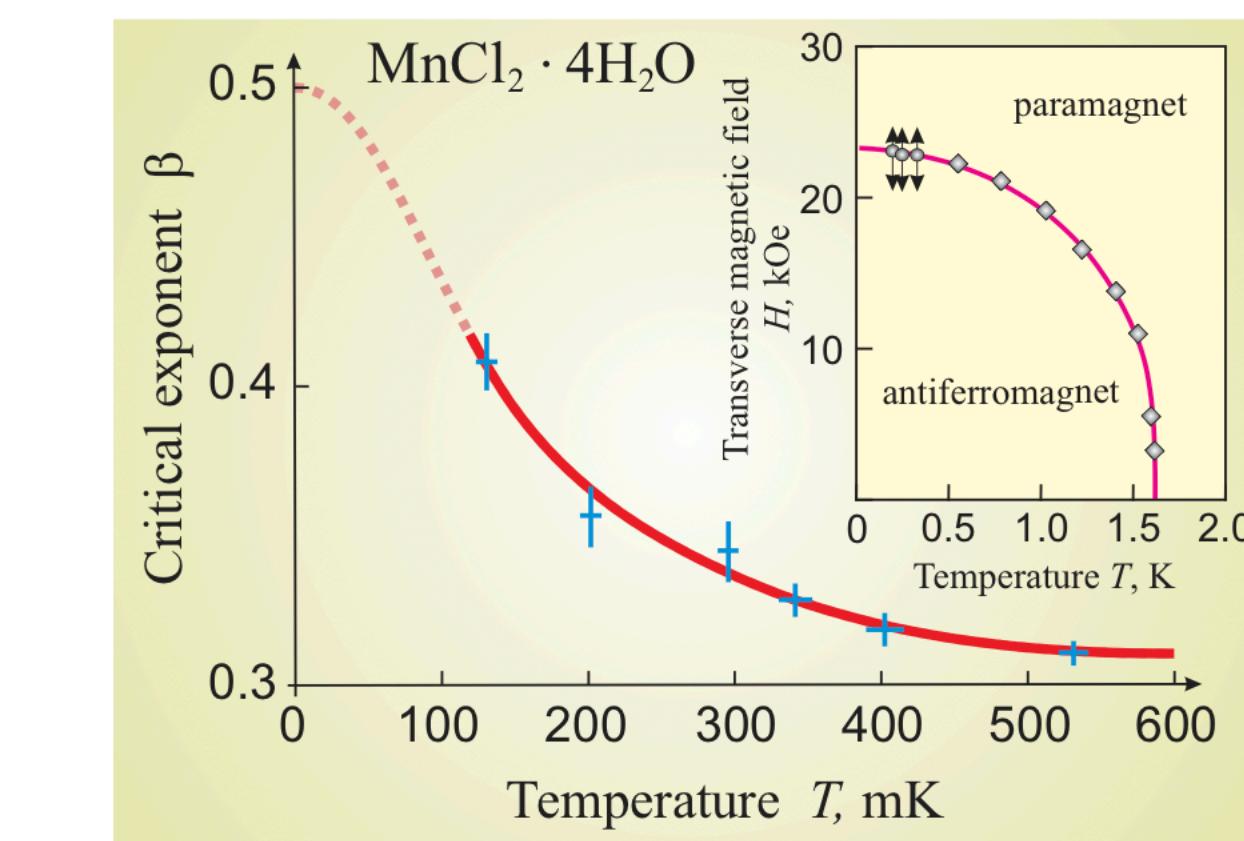
The chiral condensate in the conformal mass deformed theory



Expected $\eta_{\bar{q}q} = 1$ at the transition

$$\langle \bar{q}q \rangle \sim m^{\eta_{\bar{q}q}}$$

$$1 \leq \eta_{\bar{q}q} \leq 3$$

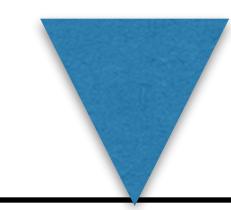


Critical line in the T , N_f plane:

From the high temperature critical region to the conformal window?

$m=0$

3D IR fixed point

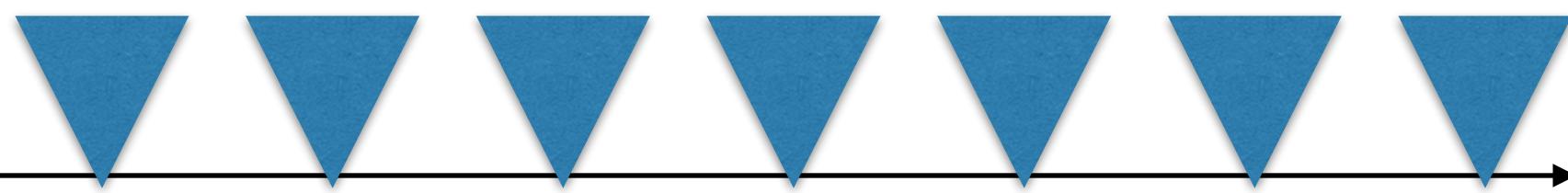


T_c

T

$N_f=2$

4D IR fixed points



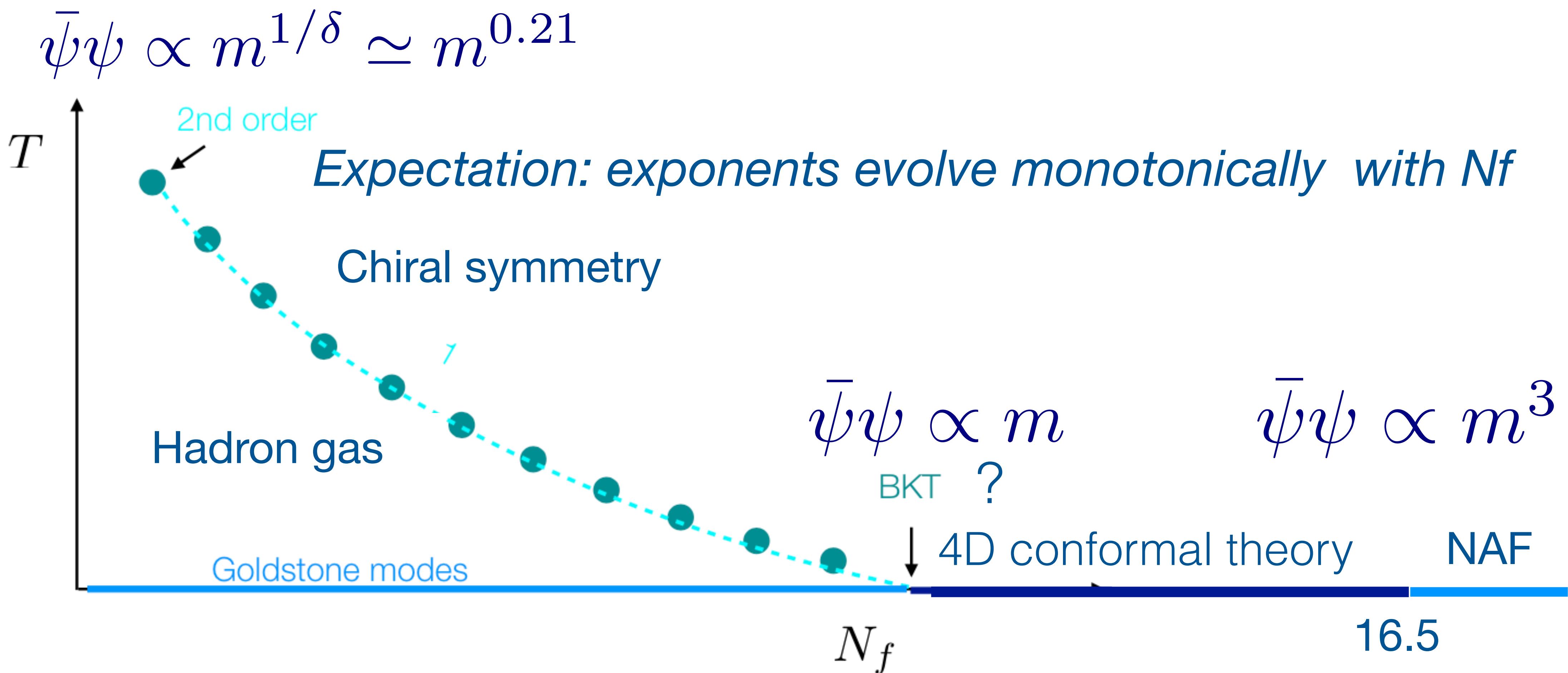
N_{fc}

N_f

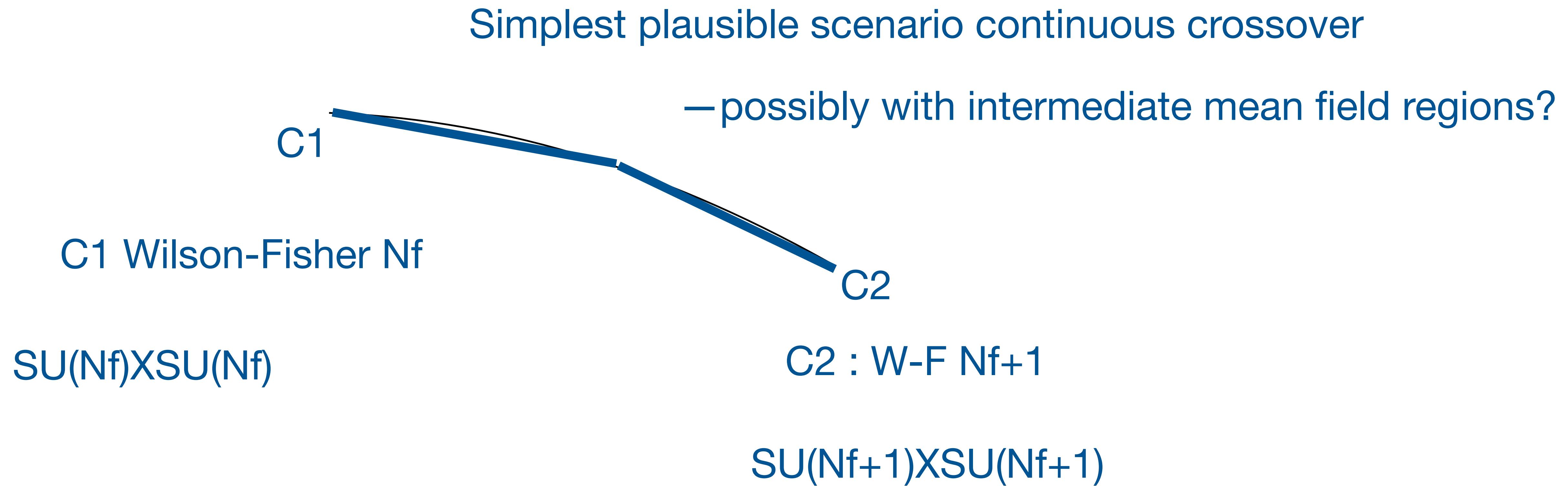
$T=0$

Scaling of the condensate

: no match btwQCD at $T_c \leftrightarrow$ conformal window

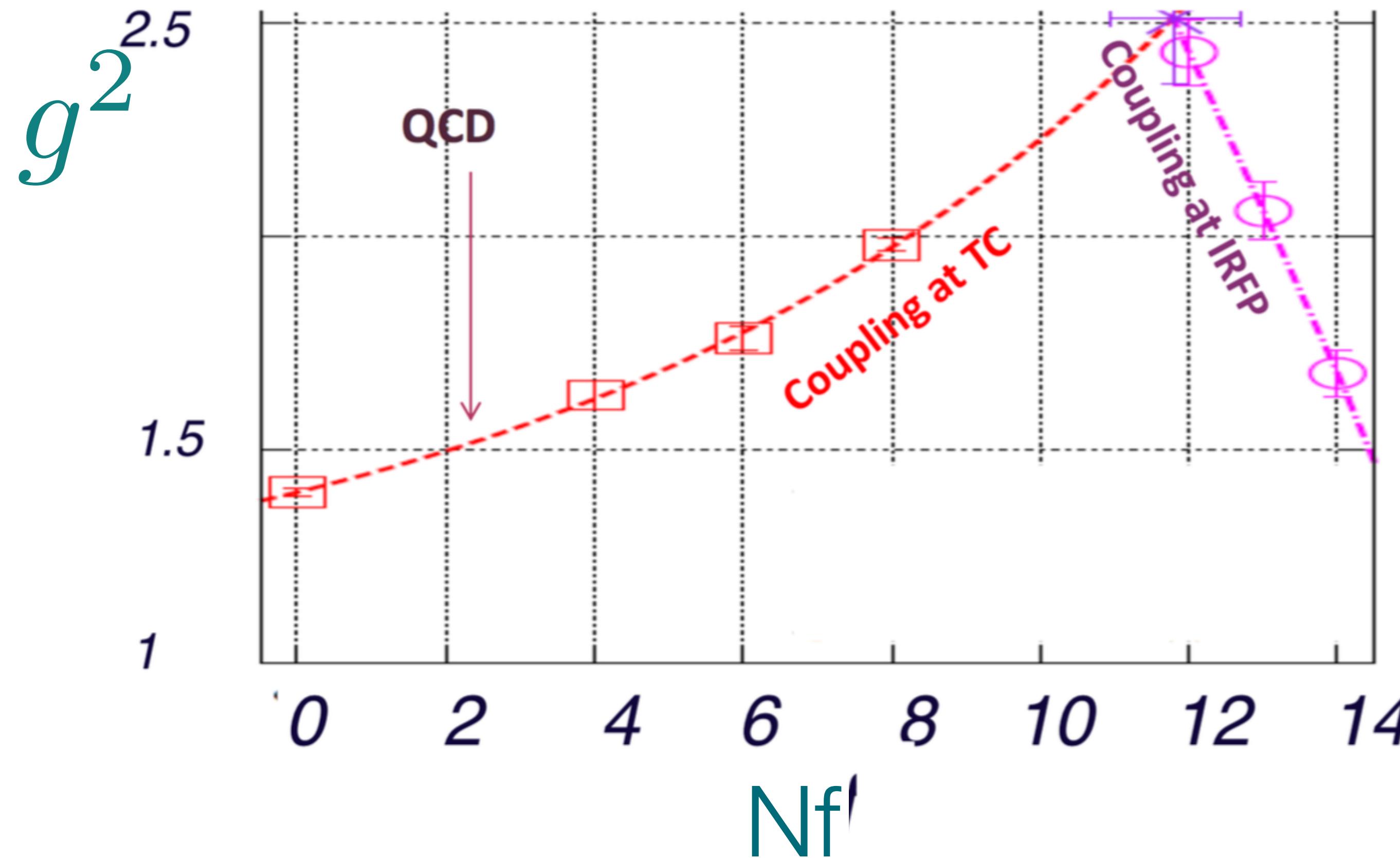


Different (integer) N_f and N_f+1 with own's universality classes interpolated by quark mass



Coupling at the transition and conformal dynamics

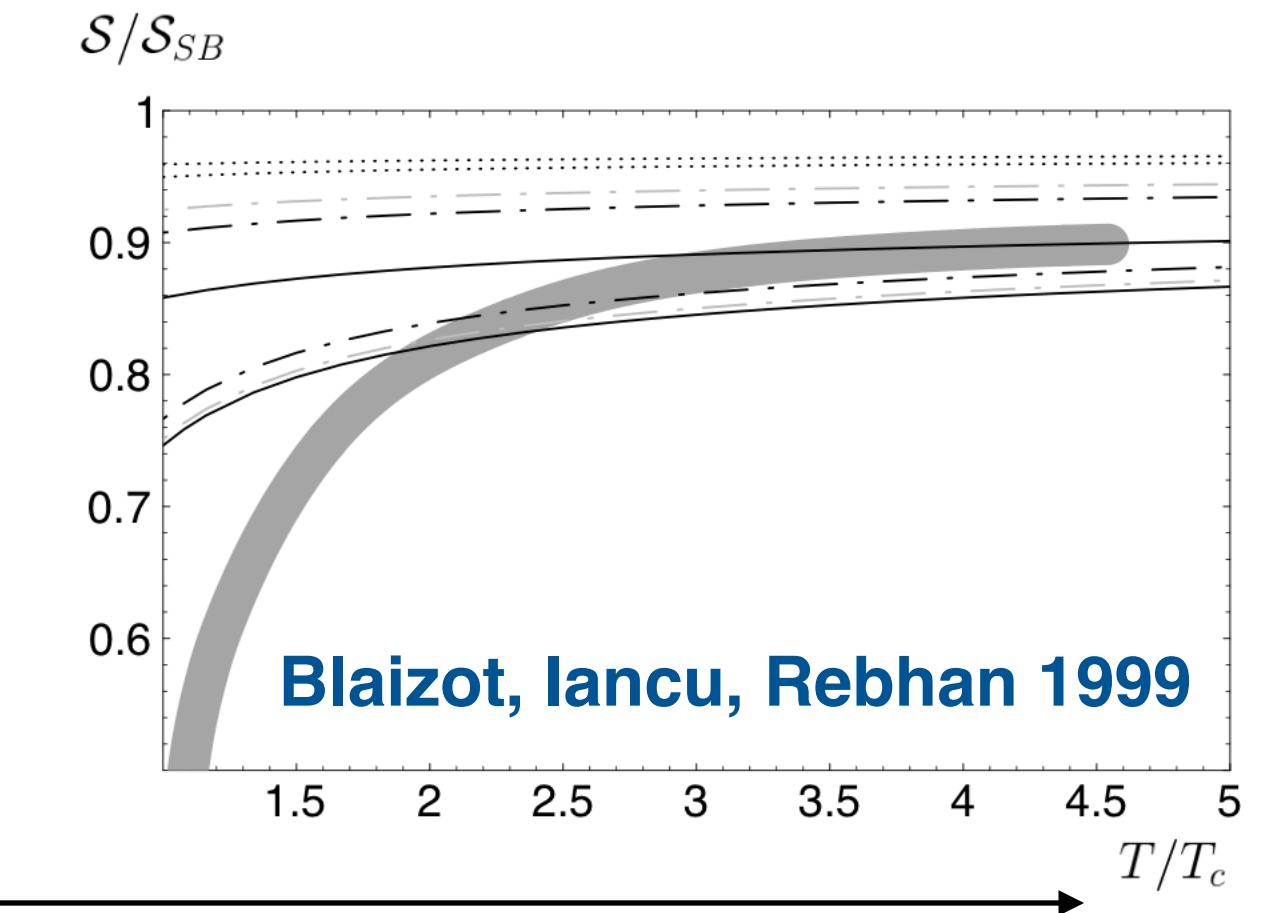
“Analogy” between thermal transition for $N_f=2$ and $N_f=14$?



The coupling increases with N_f in the broken phase and decreases with N_f in the conformal window

Miura, MpL 2013

Thermal behaviour clearly different



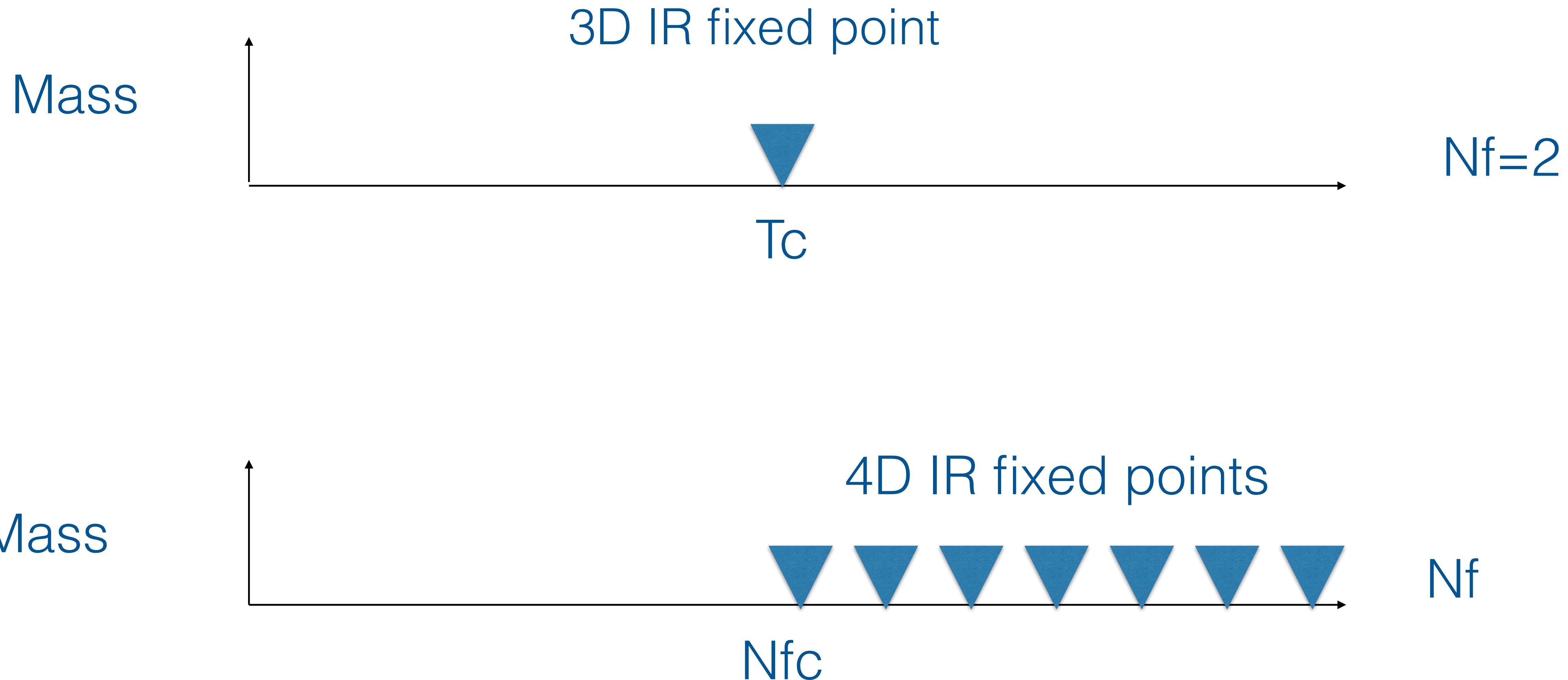
$m=0$

$N_f=2$

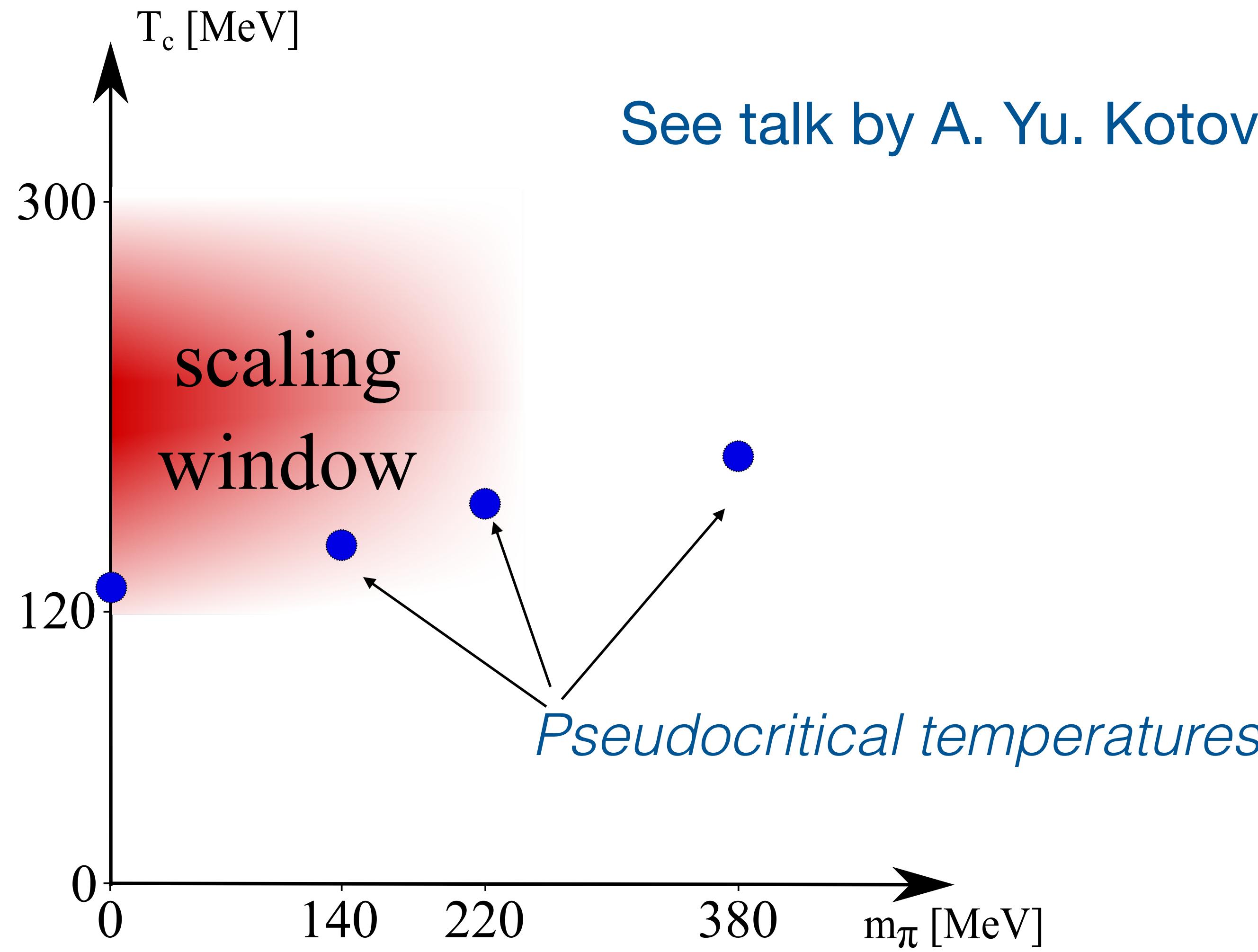


Add a mass term : conformality broken

compare theories
within the scaling window



Kotov, Trunin, MpL



The chiral condensate in the conformal, mass deformed theory

$$\langle \bar{q}q \rangle \sim m^{\eta_{\bar{q}q}}$$

$$\eta_{\bar{q}q} = (3 - \gamma_*) / (1 + \gamma_*)$$

Del Debbio, Zwicky 2010

The chiral condensate in QCD close to Tc

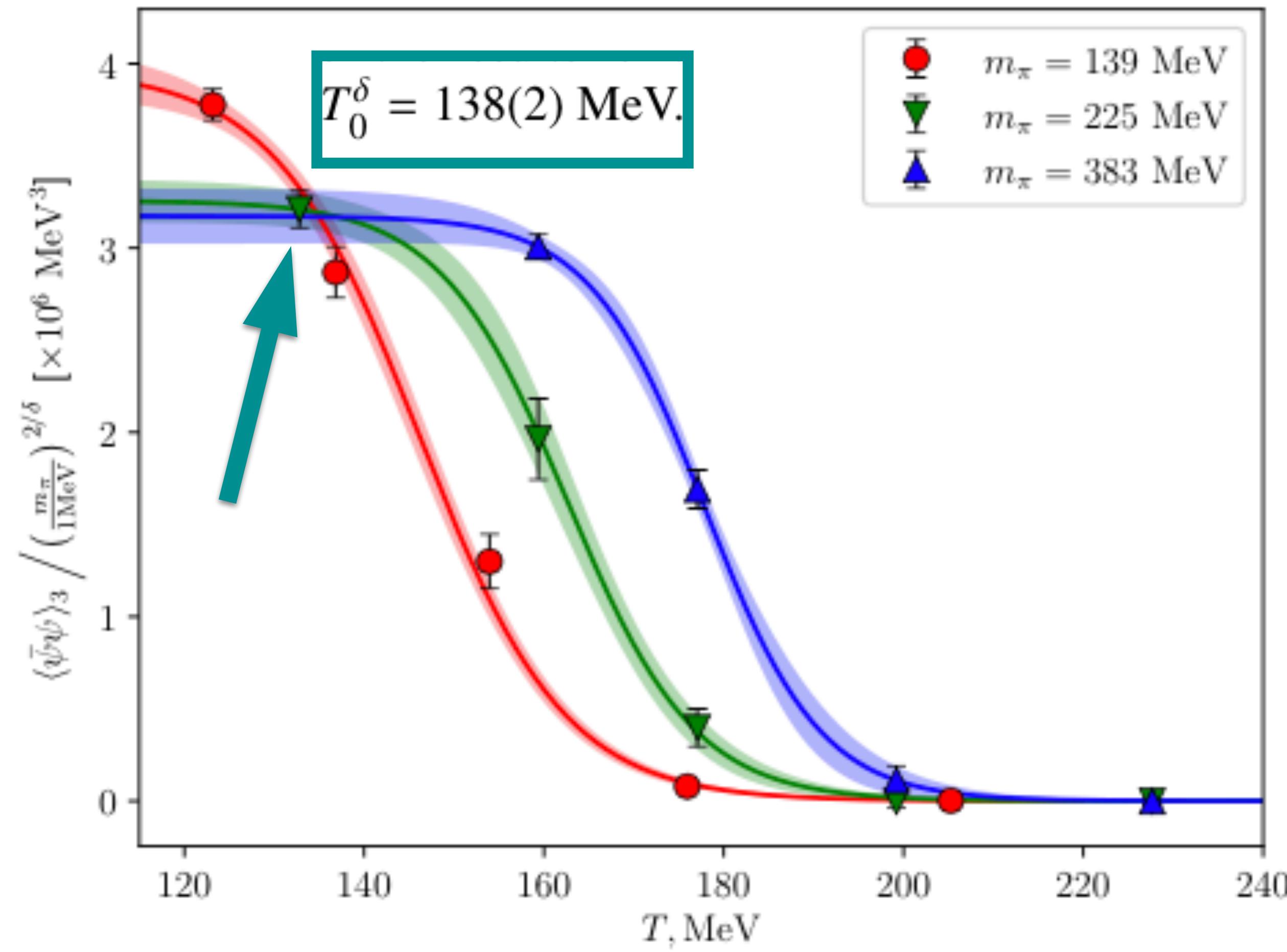
$$\langle q\bar{q} \rangle \approx m^{\frac{1}{\delta}} - A(T - T_c)m^{\frac{1}{\delta} - \frac{1}{\beta\delta}}$$

Perhaps for $T > T_c$: $\langle \bar{\psi}\psi \rangle \propto m^{1/\delta_{eff}}$

$$m_1 < m < m_2$$

Baseline

Scaling at the critical point: searching for $\langle \bar{\psi} \psi \rangle_3 (T = T_0) = A m_\pi^{2/\delta}$



$$\delta = \delta(O(4)3D) = 4.8$$

$$R_\pi = \chi_T^{-1} / \chi_L^{-1} \quad (\text{reminiscent of the Kouvel-Fisher parameter})$$

- For a continuous symmetry $h = M^\delta f(t/M^{1/\beta})$ becomes:

$$h_a = M_a M^{\delta-1} f(t/M^{1/\beta})$$

$$t = T - T_c$$

- The response to an external field h_a is given by the inverse susceptibility $(\chi^{-1})_{ab} = \partial h_a / \partial M_b$

$$(\chi^{-1})_{ab} = \delta_{ab} M^{\delta-1} f(x) + (\delta - 1) \frac{M_a M_b}{M^2} M^{\delta-1} f(x) - \frac{x}{\beta} \frac{M_a M_b}{M^2} M^{\delta-1} f'(x)$$

$$x = t/M^{1/\beta}$$

- We can separate longitudinal and transverse susceptibilities $\chi_L = \partial M / \partial h$ and $\chi_T = M/h$

$$\chi_T^{-1} = M^{\delta-1} f(x), \quad \chi_L^{-1} = M^{\delta-1} \left(\delta f(x) - \frac{x}{\beta} f'(x) \right)$$

- Define $R_\pi = \chi_T^{-1} / \chi_L^{-1}$

$$\frac{1}{R_\pi(t, m)} = \delta - \frac{x}{\beta} \frac{f'(x)}{f(x)}, \quad R_\pi(0, m) = \frac{1}{\delta}$$

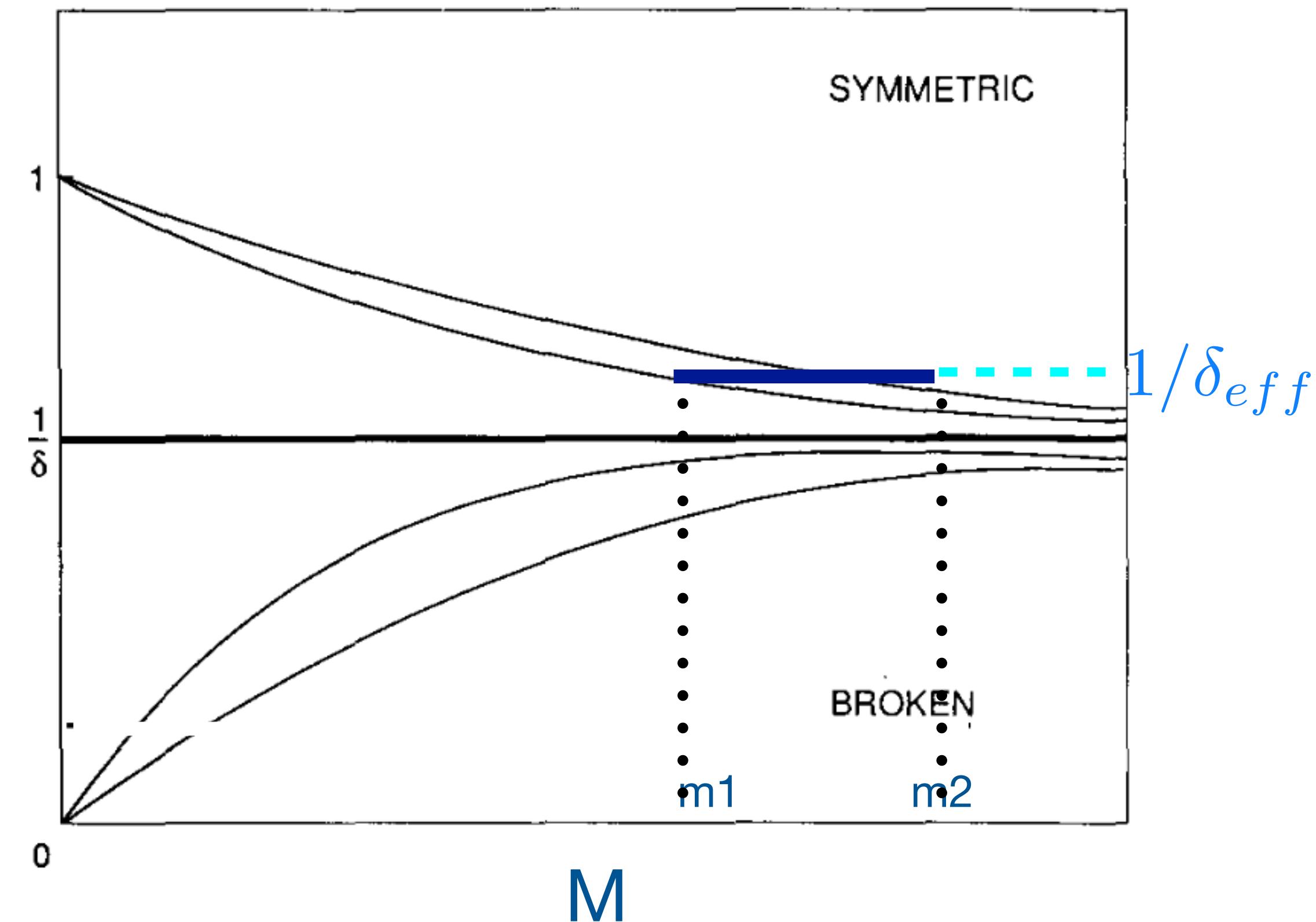
Kogut, Kocic, MpL, 1992
 Karsch, Laermann, 1992
 HotQCD, 2020 - 2021

$R_\pi = \chi_T^{-1}/\chi_L^{-1}$ and an effective δ exponent

Chiral symmetry

R_π

Goldstone

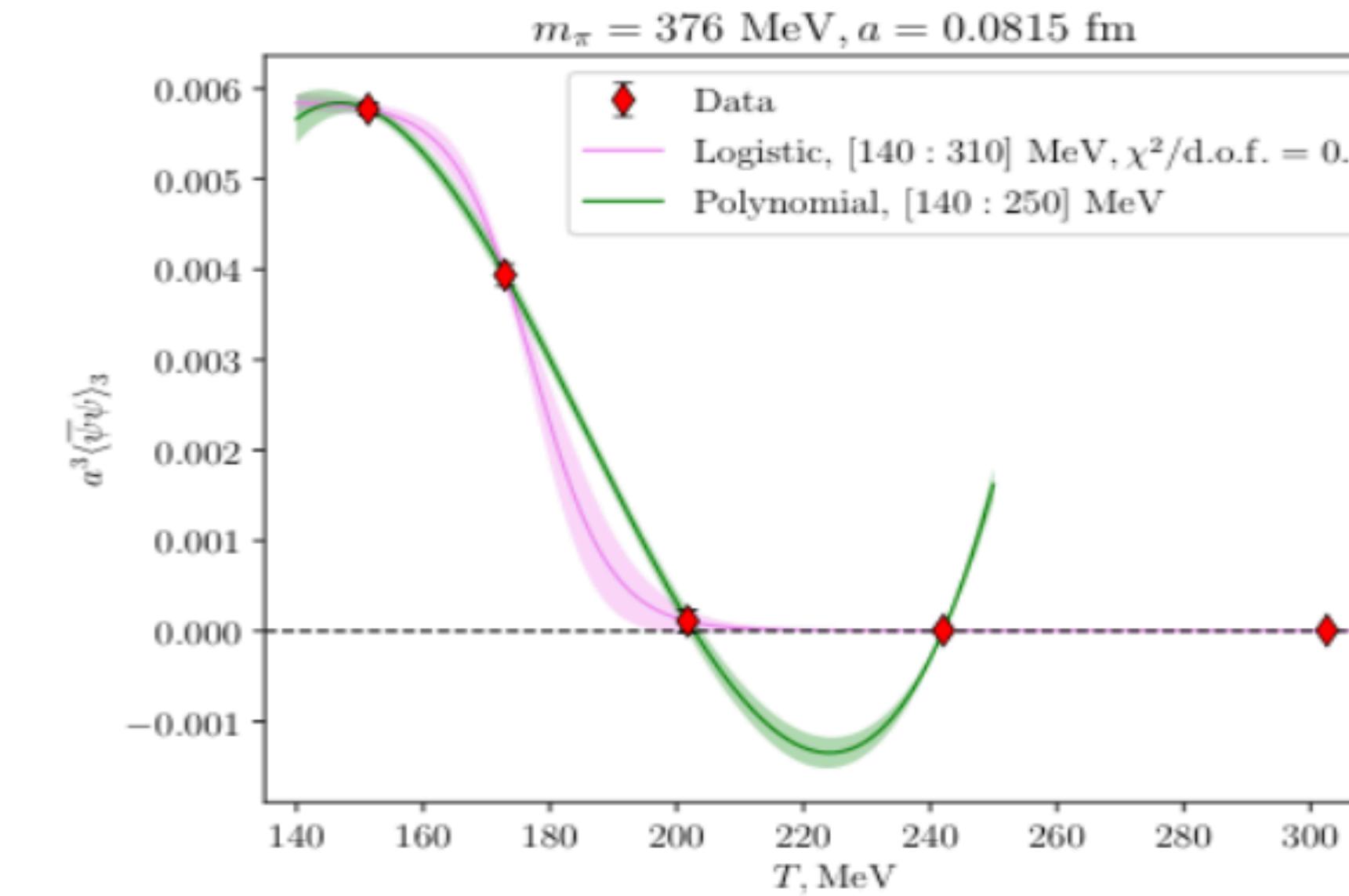
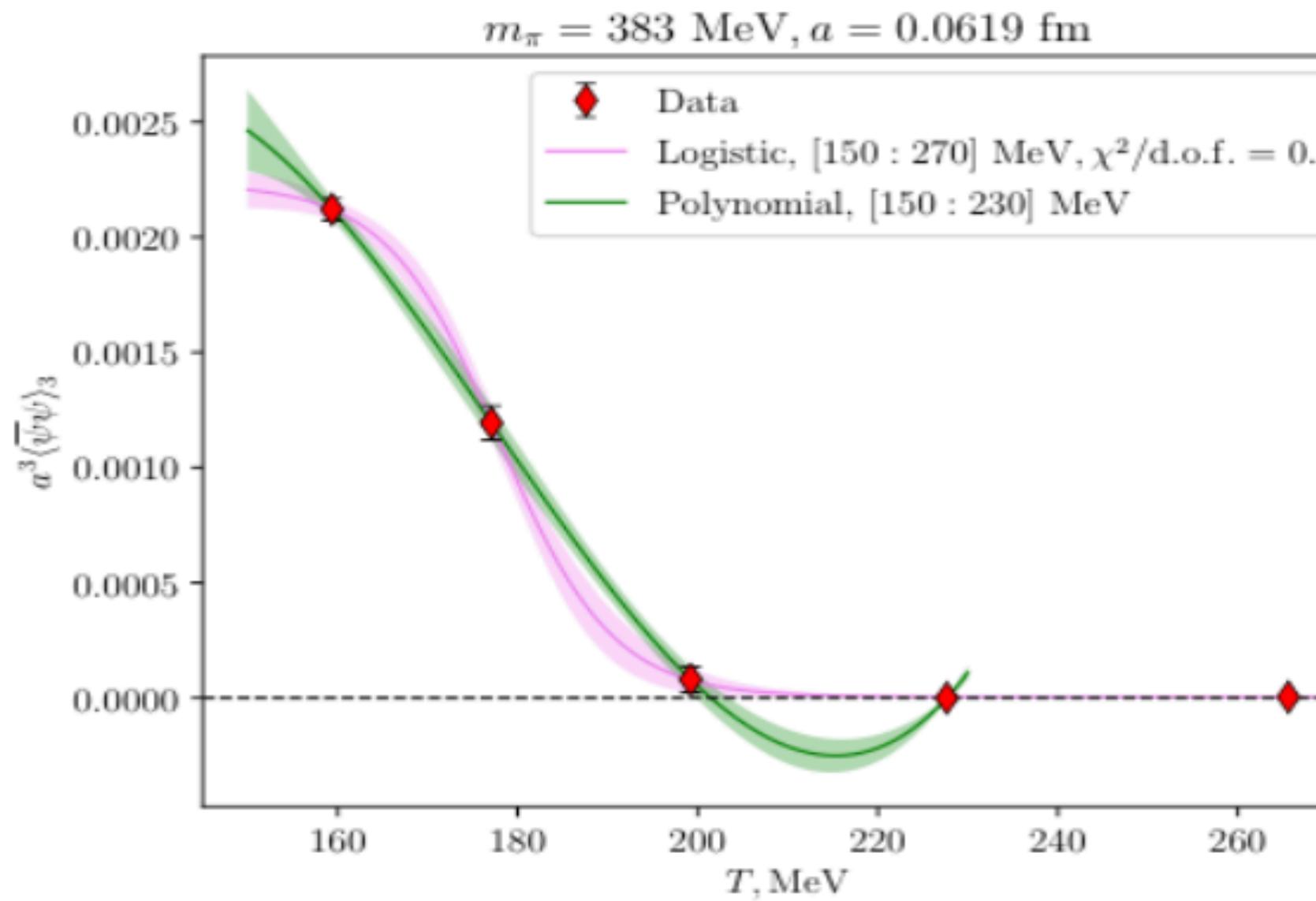
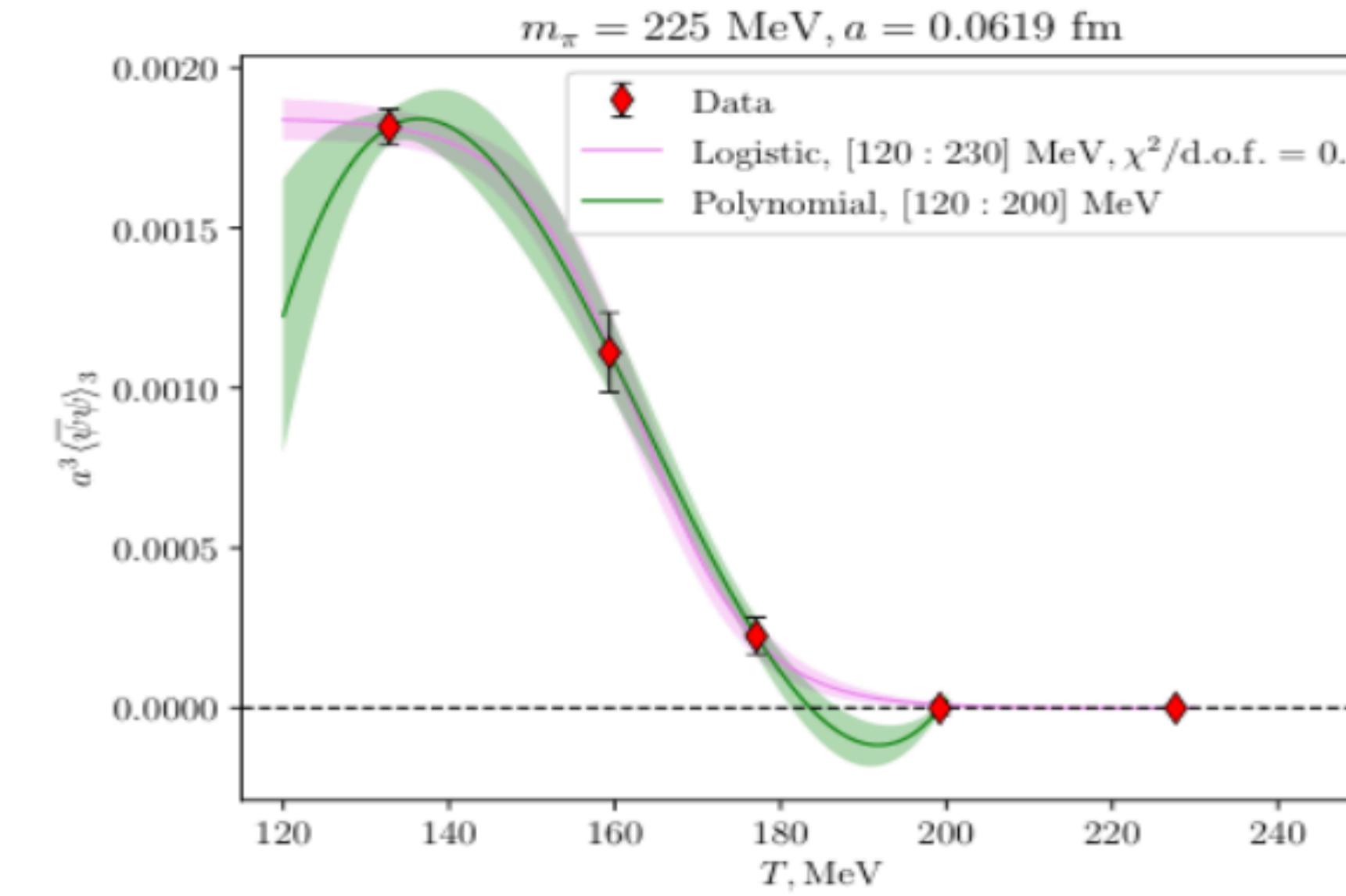
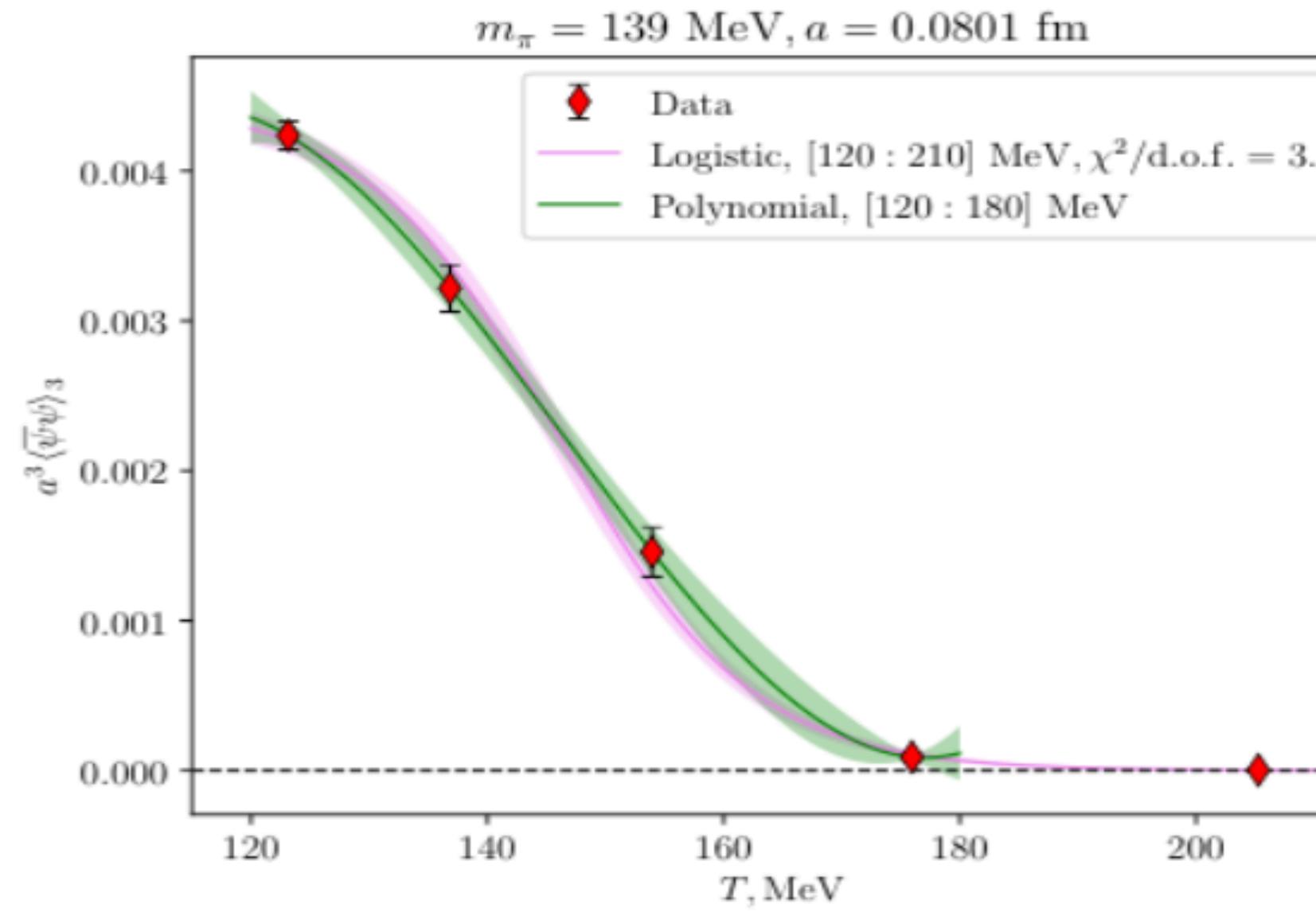


Within the scaling window of the transition one may identify two mass scales m_1 and m_2 with a quasi-conformal scaling of the order parameter

$$M \propto m^{1/\delta_{eff}}$$

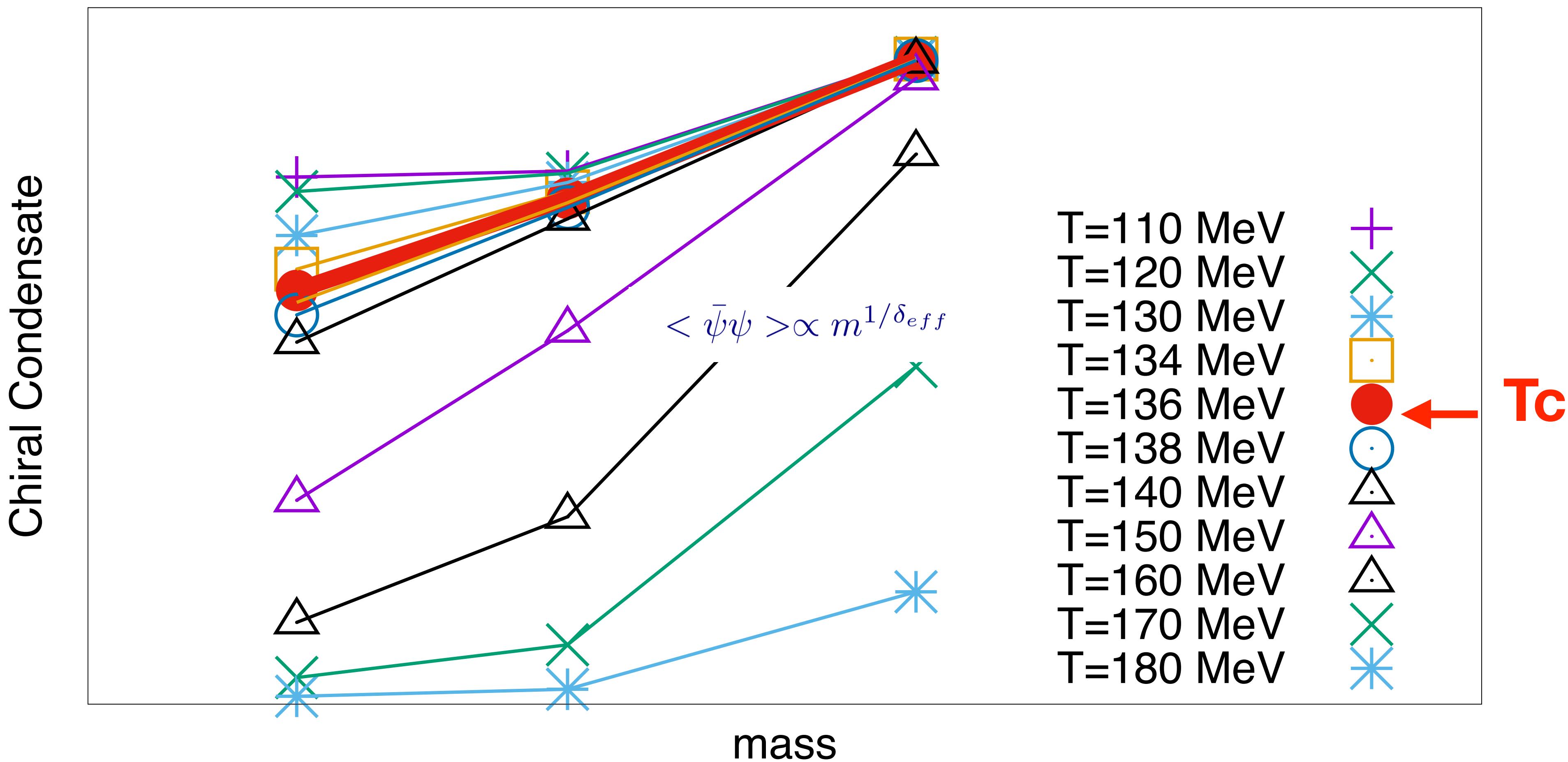
$$m_1 < m < m_2$$

Fits to the chiral condensate



Log-Log plot: straight line at T_c , with slope $1/\delta$

(fits w logistic curves produce results for different temperatures)



Possible to identify an effective exponent in some mass window above T_c

$$\delta_{eff} : 1 < \delta_{eff} < \delta$$

Summary

The critical line in the T, N_f plane

The critical exponent δ is about 4.8 for $N_f=2$, approaches 1 at the onset of conformality and decreases smoothly till $\delta = 1/3$ before loosing AF.

The coupling at the scale of the critical temperature increases with N_f and matches the IR fixed point at the onset of conformality

sQGP and conformality

For any N_f in the broken phase there is an N_f whose IRFP matches the coupling at T_c .

The critical exponents changes monotonously from $N_f=2$ till $N_f=16$, with δ in the broken phase always larger than the corresponding exponent in the conformal phase.

The scaling behaviour may be compatible with near-conformality in T -dependent mass intervals

$$m_{IR}(T) < m < m_{UV}(T)$$

but it is not immediate to relate this with the conformal window