# AAMP XX meeting

## including the special session on spectral theory

Prague, August 28 - 31, 2023

# → Book of Abstracts ←

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**On the self-adjointness of two-dimensional relativistic shell interactions**

*Badreddine Benhellal*

*University of Oldenburg, Germany*

badreddine.benhellal@uni-oldenburg.de

**Abstract:** In this talk, we consider the self-adjointness of the two-dimensional Dirac operator coupled with a combination of electrostatic and Lorentz scalar shell interaction of constant strength $\varepsilon$ and $\mu$, supported on a closed Lipschitz curve. We shall present several new explicit ranges of $\varepsilon$ and $\mu$ for which there is a unique self-adjoint realization with domain included into $H^{\frac{1}{2}}$. A more precise analysis is carried out for curvilinear polygons, for which the result depends on the minimal corner opening. Compared to the preceding works in the domain, we employ two new technical ingredients: the explicit use of the Cauchy transform on non-smooth curves and the explicit characterization of the Fredholmness for singular integral operators.

**Ground states of the NLSE on hybrids**

*Filippo Boni*

*Scuola Superiore Meridionale, Italy*

filippo.boni@unina.it

**Abstract:** In this talk we present some recent results about nonlinear models on hybrids, i.e. singular structures obtained gluing together manifolds of different dimension at junction points. In particular, we consider a Nonlinear Schrödinger Equation on a simple hybrid structure, obtained gluing together a halfline and a plane at a point. The linear part of the equation is given by a self-adjoint operator acting as the Laplacian far from the junction point and satisfying proper boundary conditions at that point, while the nonlinearity is of power type. We investigate the existence of ground states at fixed mass of the associated NLS energy in the $L^2-$subcritical case. The results have been obtained in collaboration with R. Adami, R. Carlone and L. Tentarelli.

**Virtual levels and virtual states of linear operators in Banach spaces. Applications to Schrödinger operators.**

*Nabile Boussaid*

*Laboratoire de Mathématiques de Besançon, France*

nabile.boussaid@univ-fcomte.fr

**Abstract:** In this talk I will present a joint work with Andrew Comech (Texas A&M).

Our primary interest is on the limiting absorption principle. Such a tool is useful in spectral analysis. It is also used to obtain dispersive estimates and then to analyze long time evolutions of associated nonlinear problems.

In dimension 1 and 2, in contrast with higher dimensions, the free linear Schrödinger operator has no limiting absorption principle. This makes all this classical perturbative approach much more involved.

The absence of limiting absorption principle (LAP) in the vicinity of some point is equivalent to the presence of a virtual level at this point. But it is also known that virtual levels are unstable by perturbations leading to bifurcation of eigenstates.

Our work is an attempt to understand the different characterizations of virtual levels and to provide limiting absorption principle for small perturbations of the free Schrödinger operators in dimension 1 and 2.

**Compactly supported perturbations of a 2D Magnetic Dirac operator**

*Vincent Bruneau*

*Institut de Mathématique de Bordeaux, France*

vincent.bruneau@math.u-bordeaux.fr

**Abstract:** We consider the 2D Dirac operator with a constant magnetic field, perturbed by a compactly supported electric potential. For potentials of definite sign, the spectrum is given by discrete eigenvalues which accumulate to the so-called Dirac-Landau Levels (the eigenvalues of the unperturbed Magnetic Dirac operator, which are of infinite multiplicity). As for the Magnetic Schrödinger operator, we prove that near each Dirac-Landau Level, the distribution of these eigenvalues involves the logarithmic capacity of the support of the potential. We will discuss the difficulties that arise in dealing with perturbations by an obstacle (e.g. on the exterior of a compact set, with Infinite Mass boundary condition).

**Time dependent point interactions in 2D**

*Raffaele Carlone*

*Università degli studi di Napoli, Italy*

raffaele.carlone@unina.it

**Abstract:** This talk will analyze a two-dimensional Schrdinger equation with a time-dependent point interaction, representing a model for the dynamics of a quantum particle subject to a point interaction whose strength varies in time. It will illustrate results about the global well-posedness of the associated Cauchy problem under general assumptions on the potential and the initial datum. Then, for a monochromatic periodic potential (which also satisfies a suitable no-resonance condition), the asymptotic behavior of the survival probability of a bound state of the time-independent problem will be investigated.

In the second part of the talk, it will be discussed the dynamics of a polaron (a quantum particle coupled to bosonic fields) in the quasi-classical regime: in such a regime, the effective dynamics for the quantum particles are approximated by the one generated by a time-dependent point interaction, i.e., a singular time-dependent perturbation of the Laplacian supported in a point.

Joint work with M. Correggi, L. Tentarelli, M. Falconi, M. Olivieri, R. Figari, W. Borrelli

**Integrability, spacetime dynamics and field theories**

*Francisco Correa*

*Universidad Austral Valdivia, Chile*

francisco.correa@uach.cl

**Abstract:** In this talk we will review some recent developments connecting integrable models, gravitation, soliton theory and field theories. The relationship between the AKNS hierarchy and anti-de Sitter three dimensional gravity will be discussed as well as integrable features of relativistic particles and complex solitons having real conserved quantities.

**Absence of eigenvalues of biharmonic operators with non-self-adjoint potentials via the method of multipliers**

*Lucrezia Cossetti*

*University of the Basque Country, Bilbao, Spain*

lucrezia.cossetti@ehu.eus

**Abstract:** Originally arisen to understand characterising properties connected with dispersive phenomena, in the last decades the method of multipliers has been recognized as a useful tool in Spectral Theory, in particular in connection with proof of absence of point spectrum for self-adjoint and non-self-adjoint operators. In this talk we will see the developments of the method reviewing some recent results concerning self-adjoint and non-self-adjoint Schrödinger operators in different settings and relativistic Pauli and Dirac operators. Moreover we will show how this technique can be fruitful developed to obtain similar results for higher order operators. In particular special emphasis will be given to the case of perturbed bilaplacian. The talk is based on joint works and ongoing projects with L. Fanelli and D. Krejčiřík.

**Curvature-induced bound states in soft quantum waveguides and dot arrays**

*Pavel Exner*

*Doppler Institute for Mathematical Physics and Applied Mathematics, Prague, Czech Republic*

exner@ujf.cas.cz

**Abstract:** We consider Schrödinger operators with a regular attractive potential extended in one direction, a potential 'ditch' or an array of potential wells, and investigate the discrete spectrum which arises if the potential support is not straight. A few related problems will be also mentioned.

**Supersymmetric Quantum Mechanics and a Variational Principle for Orthonormal Polynomials on the Whole Real Line**

*Joshua Feinberg*

*University of Haifa, Technion, Israel*

joshua@physics.technion.ac.il

**Abstract:** A peculiar property of polynomials which are orthogonal with respect to some weight function along the whole real line is that they do not obey a Sturm-Liouville type differential equation, with the unique exception of Hermite polynomials. (This is in contrast with systems of polynomials orthogonal on a compact or semi-compact segment.) By combining methods of supersymmetric quantum mechanics and random matrix theory, we show that such polynomials obey a system of Hartree-Fock equations, familiar from the theory of non-interacting Fermi gases. We demonstrate these equations for the specific example of the continuous Hahn polynomials. These Hartree-Fock equations reduce to the Schrödinger equation for the harmonic oscillator in the case of Hermite polynomials.

**Powers of discrete Laplacians and Hardy-type inequalities**

*Borbala Gerhat*

*Czech Technical University, Prague, Czech Republic*

borbala.gerhat@fjfi.cvut.cz

**Abstract:** We study the existence of non-trivial lower bounds for positive powers of the discrete Dirichlet Laplacian on the half line. Unlike in the continuous setting, where both $-\Delta$ and $(-\Delta)^2$ admit a Hardy-type inequality, their discrete analogues exhibit a different behaviour. While the discrete Laplacian is subcritical, its square is critical. The threshold where the criticality of $(-\Delta)^\alpha$ first appears turns out to be $\alpha = 3/2$. We provide corresponding (non-optimal) Hardy-type inequalities in the subcritical regime. Moreover, for the critical exponent $\alpha = 2$, we employ a remainder factorisation strategy to derive a discrete Rellich inequality on a subspace (with a weight improving upon the classical Rellich weight).

Based on joint work with D. Krejčiřík and F. Štampach.

**Investigation of two-particle energy levels by considering Dunkl formalism**

*Hassan Hassanabadi*

*Shahrood University of Technology, Shahrood, Iran*

h.hasanabadi@shahroodut.ac.ir

**Abstract:** We use the Dunkl derivative to obtain the coordinate representation of the momentum operator and Hamiltonian. Then we study the one-dimensional box problem for a particle with mass m confined to the one-dimensional infinite wall and we apply the results for a two-body system as a Boson and Fermion and we show that the results are thought provoking. In collaboration with J. Kříž

**Spectral transitions for two-dimensional Dirac operators with $\delta$-shell potentials**

*Markus Holzmann*

*Graz University of Technology, Austria*

holzmann@math.tugraz.at

**Abstract:** This talk is devoted to the discussion of the self-adjointness and spectral properties of a family of Dirac operators with singular $\delta$-shell potentials supported on curves in $\mathbb{R}^2$. For a three-parameter group of coefficients of the singular interaction the self-adjoint realizations are described. It turns out that for a critical combination of the coupling constants the nature of the spectral properties changes abruptly: If the interaction support is a closed and compact curve, then there is an additional point in the essential spectrum in the critical case. If the interaction support is a straight line, then an interval that is contained in the continuous spectrum collapses in the critical case to an eigenvalue of infinite multiplicity. Moreover, in the critical case, there is a lack of Sobolev regularity in the domain of the self-adjoint realization.

**Conditionally exactly solvable Dirac potential, including $x^{1/3}$ pseudoscalar interaction**

*Artur Ishkhanyan*

*Institute for Physical Research, Ashtharak, Armenia*

aishkhanyan@gmail.com

**Abstract:** We study the solution of the 1D Dirac equation for a pseudoscalar potential, which includes an interaction term proportional to $x^{1/3}$ and a $\sim 1/x$ term with a fixed strength. This is a conditionally exactly solvable singular potential, symmetric with respect to the origin. A feature of the potential is that the effective potential for the Schrödinger equation, to which the Dirac equation can be reduced, splits into two known potentials: the first Stillinger potential and the second Exton potential for the positive and negative semiaxes, respectively. We present the solution and analyze the energy spectrum and wave functions of the bound states. Our results reveal notable differences between the Schrödinger and Dirac behaviors.

**Quantum systems at the brinks: threshold states for discrete Schrdinger operators**

*Michal Jex*

*Czech Technical University, Prague, Czech Republic*

michal.jex@fjfi.cvut.cz

**Abstract:** (joint work with František Štampach)

One of the crucial properties of a quantum system is the existence of bound states. The existence of eigenvalues outside the essential spectrum is well understood. Their eigenfunctions exhibit exponential decay, and their existence is linked to the energy gap. However, the situation at the threshold is much more subtle. There are two challenging problems for the states at the threshold-their existence and asymptotic behaviour. In this talk we address the existence.

We present necessary and sufficient conditions for discrete Schrdinger operators to have a zero-energy bound state. Our sharp criteria show that the existence and non-existence of zero-energy ground states depends strongly on the dimension and the asymptotic behaviour of the potential. There is a spectral phase transition with dimension four being critical. Furthermore, we show how the situation changes for the upper edge of the essential spectrum.

# Supersymmetric Quantum Mechanics as an Algebraic Method in Physics

*Georg Junker*

*ESO, Germany*

gjunker@eso.org

**Abstract:** Supersymmetric quantum mechanics (SUSY QM) was introduced in the 1970s as a simple (0+1)- dimensional model for supersymmetric quantum field theories. However, soon it became an area of intense research by its own, with many applications in quantum mechanics, statistical physics, optics, solid-state physics, etc. Even in financial mathematics it has received some applications. The purpose of this lecture is to revisit the algebraic properties of SUSY QM and its potential applications in quantum physics, with some focus on more recent results related to relativistic quantum mechanics.

**Band gaps in periodic media with small resonators**

*Andrii Khrabustovskyi*

*University of Hradec Králové, Czech Republic*

andrii.khrabustovskyi@uhk.cz

**Abstract:** We investigate spectral properties of the Neumann Laplacian on a periodic unbounded domain, which is obtained by inserting into $\mathbb{R}^n$ $m$ families of $\varepsilon$-periodically distributed small resonators. We demonstrate that the spectrum of this operator has at least $m$ gaps provided the period $\varepsilon$ is sufficiently small and the resonators are appropriately scaled. When $\varepsilon$ tends to zero, the first $m$ gaps converge to some intervals whose location and lengths can be controlled by a suitable choice the resonators, while other gaps (if any) go to infinity. An application to the theory of photonic crystals is discussed. This is a joint work with E. Khruslov (ILTPE, Kharkiv, Ukraine).

**On the spectrum of a perturbed periodic differential systems of arbitrary order with weight**

*Volodymyr Khrabustovskyi*

*Ukrainian State University of Railway Transport, Kharkiv, Ukraine*

v_khrabustovskyi@ukr.net, khrabustovsky@kart.edu.ua

**Abstract:** In the space $L^2_W(\mathbb{R}^1)$ we consider the operator $L$ corresponding to the system

$$\ell[y] = \lambda W(t)y,$$

where $\ell$ is a symmetric differential expression of an arbitrary order $r$ having periodic matrix coefficients and a periodic matrix weight $W(t) > 0$. Also in $L^2_{W+\Delta}(\mathbb{R}^1)$ we consider the perturbed operator $L_p$, which corresponds to the perturbed system

$$(\ell + m)[y] = \lambda(W(t) + \Delta(t))y,$$

where $m$ is symmetric matrix differential expression of order $\leq r$, $\Delta(t)$ is a matrix perturbation of the weight such that $W(t) + \Delta(t) > 0$.

First we prove that, under suitable assumptions on $m$ and $\Delta$, one has the equality $\sigma_{\text{ess}}(L_p) = \sigma(L)$. Then we derive the conditions implying the prescribed edge of the gap in $\sigma_{\text{ess}}(L_p)$ either being or not being an accumulation point of eigenvalues of $L_p$.

Also, we obtain the conditions providing the absence of the eigenvalues of $L_p$ within the regions of $\sigma(L)$ with maximal multiplicity (including the endpoints of these regions). For perturbed scalar periodic Sturm-Liouville operators related results can be found in [1].

The obtained results are new even for perturbed systems with constant coefficients, and some of them are new for scalar Hill equation.

## References

[1] J. Behrndt, P. Schmitz, G. Teschl, G. Trunk, Perturbations of periodic Sturm-Liouville operators, Adv. Math. 422 (2023), 109022.

**Quantum soft waveguides with resonances induced by broken symmetry**

*Sylwia Kondej*

*University of Zielona Góra, Poland*

s.kondej@if.uz.zgora.pl

**Abstract:** We consider two-dimensional, non-relativistic quantum system with asymptotically straight soft waveguide. We show that the local deformation of the symmetric waveguide can lead to the emerging of the embedded eigenvalues living in the continuous spectrum. The main problem of this paper is devoted to the analysis of weak perturbation of the symmetric system. We show that the original embedded eigenvalues turn to the second sheet of the resolvent analytic continuation and constitute resonances. We describe the asymptotics of the real and imaginary components of the complex resonant pole depending on deformation. Finally, we generalize the problem to three dimensional system equipped with a soft layer.

**Self-adjointness problem for Sturm-Liouville and 1D Schrödinger operators with distributional coefficients**

*Volodymyr Mikhailets*

*Institute of Mathematics of the CAS, Czech Republic*

mikhailets@math.cas.cz

**Abstract:** We introduce and investigate symmetric Sturm-Liouville operators on the line under minimal assumptions on the regularity of the coefficients. Two Povzner-Wiengoltz-type theorems will be presented. Conditions on measure-valued potential which provide that 1D Schrödinger operator is self-adjoint are found.

**Spectral stability and instability of solitary waves of the Dirac equation with concentrated nonlinearity**

*Diego Noja*

*University of Milano-Bicocca, Italy*

diego.noja@unimib.it

**Abstract:** In the talk is presented the detailed study of the spectrum of linearization operator at solitary waves for the nonlinear Dirac equation with Soler-type nonlinearity concentrated at one point. We then consider two different perturbations of the nonlinearity which break the $\mathbf{SU}(1,1)$ symmetry: the first preserving and the second breaking the parity symmetry, showing the respective effect on the linear stability of eigenvalues. These are the results of a collaboration with N. Boussaid, C. Cacciapuoti, R. Carlone, A. Comech and A. Posilicano.

**Matrix Dirac-like facttorizations associated to Schrödinger factorizations**

*Javier Negro*

*Universidad de Valladolid, Spain*

jnegro@fta.uva.es

**Abstract:** We extended the factorization method of scalar shape-invariant Schrödinger Hamiltonians to a class of Dirac-like matrix Hamiltonians. The intertwining operators of the Schrödinger equations have been implemented in the Dirac-like shape invariant equations. We have considered also anti-intertwining operators changing the sign of energy. The Dirac-like Hamiltonians can be obtained from reduction of higher dimensional spin systems.Some examples have been worked out.

**Massive and massless two-dimensional Dirac particles in electric quantum dots**

*Luis Miguel Nieto*

*Universidad de Valladolid, Spain*

luismiguel.nieto.calzada@uva.es

**Abstract:** In this talk we analyze the confinement properties of charged particles of a Dirac material in the plane when it is subjected to an electrostatic potential well (an electric quantum dot). The study focuses on the effect of mass and angular momenta on these confining properties. To have a global picture of the confinement, both the bound and the resonance states are considered. To show that they are physically significant, the resonances are examined by means of (a) the Wigner time delay of the scattering states, as well as (b) through the complex eigenvalues of the outgoing states. By adjusting the intensity of the well potential, electron captures and atomic collapses are observed for critical values. In these processes, the bound states of the discrete spectrum become resonances of the continuous spectrum or vice versa. For massive charges, the phenomenon of atomic collapse keeps the number of bound levels in the quantum dot below a maximum value. In the massless case, the bound states have zero energy and occur only for some discrete values of the potential depth, as is known. We also show that although the intensity of the resonances for massive particles is not significantly influenced by angular momenta, on the contrary, for massless particles they are quite sensitive to angular momenta, as it is the case for graphene.

In addition, we also investigate how the energy spectrum of an electric quantum dot is affected by the fact that the charged particle has a different effective mass inside and outside of it. Depending on the different values we give to the mass, we have found different properties. Specifically, when the mass is positive but lighter inside the quantum dot than outside it, the spectrum increases and splits into two types of states separated by a gap. Conversely, if the mass inside the quantum dot is heavier than the mass outside, the spectrum has fewer states and requires stronger fields. Finally, the case of negative mass inside the quantum dot and positive mass outside it, or vice versa, gives rise to a new spectral curve of edge states.

Work done in collaboration with S. Kuru, J. Negro, S. Salamanca, and L. Sourrouille.

# Estimates for Periodic and Antiperiodic Eigenvalues of the Schrödinger Operator with a $PT$-symmetric Optical Potential

*Cemile Nur*

*Yalova University, Turkey*

cnur@yalova.edu.tr

**Abstract:** We provide estimates for the eigenvalues of nonself-adjoint Sturm-Liouville operators with periodic and antiperiodic boundary conditions for a shift of the special potential $4\cos^2 x + 4iV\sin 2x$ that is a PT-symmetric optical potential, especially when $|c| = |\sqrt{1-4V^2}| < 3$ or equally $0 \leq V < \sqrt{10}/2$. We obtain some useful equations for calculating the periodic and antiperiodic eigenvalues of the Schrödinger operator. We even approximate complex eigenvalues by the roots of some polynomials derived from some iteration formulas. We discuss our results by explaining the connection between the periodic, antiperiodic, Dirichlet and Neumann eigenvalues of the Schrödinger operator. Moreover, we give a numerical example with error analysis.

**Spectrum of the Dirac operator in thin structures**

*Thomas Ourmières-Bonafos*

*Aix-Marseille University, France*

thomas.ourmieres-bonafos@univ-amu.fr

**Abstract:** We study the spectrum of the Dirac operator with infinite mass boundary conditions posed in a tubular neighborhood of a hypersurface of the Euclidean space (in any space dimension). We look for an asymptotic expansion of the eigenvalues of this operator when this tubular neighborhood becomes thin. We prove that there exists an effective operator (posed on the hypersurface) driving the asymptotics of the eigenvalues. This operator is of geometric nature and involves the curvatures of the hypersurface. I will start by discussing the case of waveguides in dimension two and then I will address a similar problem in any space dimension. Joint works with W. Borrelli, P. Briet, N. Kerraoui, D. Krejčiřík and V. Lotoreichik.

**The double Dyson index effect in non-Hermitian tridiagonal matrices**

*Mauricio Pato*

*Universidade de São Paulo, Brasil*

mauriciopato70@gmail.com

**Abstract:** The Dyson index $\beta$ plays an essential role in random matrix theory as it labels the so-called "three fold way" that refers to the symmetries satisfied by the ensembles under unitary transformations. As it is known, its 1,2 and 4 values denote the Orthogonal, the Unitary and the Symplectic classes whose matrix elements are real, complex and quaternion numbers, respectively. It functions therefore as a measure of the number of independent non-diagonal variables. On the other hand, in the case of the $\beta$-ensembles, which are the tridiagonal form of the theory, it can assume any real positive value loosing this way that function. Our purpose, however is to show that, when the Hermitian condition of the real matrices generated with a given value of $\beta$ is removed, and, as a consequence, the number of non-diagonal independent variables doubles, non-Hermitian matrices exist that asymptotically behave as if they had been generated with a value $2\beta$. So, it is as if the $\beta$ index were, in this way, again operative. It is shown that this effect happens for the three tridiagonal ensembles, namely, the $\beta$-Hermite, the $\beta$-Laguerre and the $\beta$-Jacobi ensembles.

**On the origin of Minnaert resonances**

*Andrea Posilicano*

*University of Insubria, Como, Italy*

andrea.posilicano@uninsubria.it

**Abstract:** We consider the appearance of what are called "Minnaert resonances" in the scattering of sound waves in a medium with a small inhomogeneity enjoying a high contrast of both its mass density and bulk modulus. This phenomenon is explained in terms of the behavior, as the size of the inhomogeneity decreases to zero, of the norm resolvent limit of a related frequency-dependent Schrödinger operator, the limit being not trivial if and only if the frequency coincides with that of Minnaert.

**Norm resolvent convergence of discretisations of the Laplacian on domains**

*Olaf Post*

*University of Trier, Germany*

olaf.post@uni-trier.de

**Abstract:** In this talk, I will discuss some new results of norm resolvent convergence of discretisations of Laplacians on Euclidean domains. The main idea is to use certain triangulations and suitable splines for the discretisation. We completely avoid using Fourier methods.

**Factorization Method For Symmetry Determination: Relativistic Generalization**

*Sergio Salamanca*

*Universidad de Valladolid, Spain*

sergio.salamanca@uva.es

**Abstract:** The purpose of this talk is to showcase a section of our research based on the applications of the factorization method as a way to determine the symmetries of quantum and classical superintegrable systems. In particular, we are going to consider the application of the factorization method to relativistic systems with constant curvature. The results obtained help us analyze the Evans and Winternitz systems as a generalization of central potential systems. We summarize by comparing the results obtained with the ones given by other methods as well as giving a natural interpretation of the classical symmetries obtained in terms of action-angle variables.

**S-Matrix Poles and SUSY transformations of the Rosen-Morse II Potential**

*Carlos San Millan Carpintero*

*Universidad de Valladolid, Spain*

carlos.san-millan@uva.es

**Abstract:** Within the list of solvable Hamiltonians in Non-Relativistic Quantum Mechanics, one of them is the Rosen-Morse II potential. This potential is not symmetric, so it does not commute with the parity operator. Additionally, it is described by hypergeometric functions and Jacobi polynomials. In this talk, we will discuss the results obtained by studying the poles of the S-Matrix, obtaining a special kind of poles. Then different SUSY (Supersymmetry) transformations are applied, which will allow us to provide solutions to new potentials based on the obtained results in the S-Matrix analysis.

**A shape optimization problem in relativistic quantum mechanics**

*Tomás Sanz-Perela*

*Autonomous University of Madrid, Spain*

tomas.sanz@uam.es

**Abstract:** Dirac operators defined on domains of the Euclidean space are used in relativistic quantum mechanics to describe particles that are confined in a region. A remarkable example in dimension 3 is the MIT bag operator, used to model confinement of quarks in hadrons, and a fundamental topic in mathematical physics concerns the analysis of the spectral gap and its associated shape optimization problem. This consists on minimizing the first squared eigenvalue among all domains with prescribed volume, and it is conjectured that the ball is the optimal domain.

In this talk I will describe a recent work —in collaboration with N. Arrizabalaga, A. Mas, and L. Vega — in which we propose an approach towards this open problem. We have studied a family of Dirac operators $\{\mathcal{H}_\tau\}_{\tau\in\mathbb{R}}$ defined on $\Omega \subset \mathbb{R}^3$ and with boundary conditions that depend on a real parameter $\tau$. This family contains the MIT bag operator (when $\tau = 0$), while some well-known operators arise in the limits as $\tau \to \pm\infty$. We parametrize the spectrum of the family $\{\mathcal{H}_\tau\}_{\tau\in\mathbb{R}}$ through a collection of increasing smooth curves, and we study the limits $\tau \to \pm\infty$. To do it, a main tool consists of rewriting the problem in terms of boundary integral operators on $\partial\Omega$.

Thanks to the analysis at the limits $\tau \to \pm\infty$, we manage to establish (for large values of the parameter) the optimality of the ball for the shape optimization problem involving $\mathcal{H}_\tau$. This is expected to hold for all the parametrization and thus solve the open problem for $\tau = 0$.

**Quantum walk state transfer on a hypercube**

*Stanislav Skoupý*

*FNSPE CTU, Prague*

Stanislav.Skoupy@fjfi.cvut.cz

**Abstract:** We present state transfer on a hypercube by means of a quantum walk where the sender and the receiver vertices are marked by a weighted loops. First, we analyse search for a single marked vertex, which can be used for state transfer between arbitrary vertices by switching the weighted loop from the sender to the receiver after one run-time. Next, state transfer between antipodal vertices is considered. We show that one can tune the weight of the loop to achieve state transfer with high fidelity in shorter run-time in comparison to the state transfer with a switch. Finally, we investigate state transfer between vertices of arbitrary distance. It is shown that when the distance between the sender and the receiver is at least 2, the results derived for the antipodes are well applicable. If the sender and the receiver are direct neighbours the evolution follows a slightly different course. Nevertheless, state transfer with high fidelity is achieved in the same run-time.

**Tunneling in soft waveguides: closing a book**

*David Spitzkopf*

*Nuclear Physics Institute CAS, Řež, Czech Republic*

spitzkopf98@gmail.com

**Abstract:** In this talk, we present recent new results in the field of quantum waveguides - Schrödinger operators in curved regions - with 'soft' walls. We investigate the properties of their spectrum in two dimensions in the setting of the generalized bookcover shape, that is, Schrödinger operator with the potential in the form of a ditch consisting of a finite curved part and straight asymptotes which are parallel or almost parallel pointing in the same direction. We show how the eigenvalues accumulate when the angle between the asymptotes tends to zero. In case of parallel asymptotes the existence of a discrete spectrum depends on the ditch profile. We prove that it is absent in the weak-coupling case, on the other hand, it exists provided the transverse potential is strong enough. We also present a numerical example in which the critical strength can be assessed.

**Approximation of Dirac operators with general $\delta$-shell potentials supported on curves and surfaces**

*Christian Stelzer*

*Graz University of Technology, Austria*

christian.stelzer@tugraz.at

**Abstract:** In this talk we study the approximation of Dirac operators with general $\delta$-shell potentials supported on the boundary of a possibly unbounded domain in 2D or 3D. These singular potentials are used as idealized replacements for potentials which are strongly localized in a neighbourhood of the support of the $\delta$-shell potential. In order to justify these replacements we show that Dirac operators with $\delta$-shell potentials can indeed be approximated by Dirac operators with squeezed potentials, where we observe a well-known non-linear renormalization in the limit. In contrast to existing literature where convergence in the strong sense was proven for similar problems, we derive sufficient conditions for convergence in the norm resolvent sense. Moreover, for the special cases of electrostatic, Lorentz-scalar and magnetic $\delta$-shell potentials we provide explicit conditions in terms of the interaction strengths.

This talk is based on joint work with Jussi Behrndt and Markus Holzmann.

**Schrödinger operators with generalized oblique transmission conditions in $\mathbb{R}^2$**

*Georg Stenzel*

*Graz University of Technology, Austria*

gstenzel@math.tugraz.at

**Abstract:** The considered transmission conditions are similar to those associated with Schrödinger operators with $\delta'$-potentials, where in addition to the normal derivative a purely imaginary contribution of the tangential derivative on a sufficiently smooth closed curve in $\mathbb{R}^2$ is prescribed. Using boundary triple techniques, self-adjointness as well as spectral properties are studied. In particular, it is shown that for certain combinations of the parameters, the discrete spectrum of this class of Schrödinger operators is countable and unbounded from below.

This is joint work with Jussi Behrndt and Markus Holzmann.

**New classes of quadratically integrable systems with vector potentials: non-subgroup type cases**

*Libor Šnobl*

*Czech Technical University in Prague, Czech Republic*

libor.snobl@fjfi.cvut.cz

**Abstract:** We study quadratic integrability of systems with velocity dependent potentials in three-dimensional Euclidean space. Unlike in the case with only scalar potential, quadratic integrability with velocity dependent potentials does not imply separability in the configuration space. The leading order terms in the pairs of commuting integrals can either generalize or have no relation to the forms leading to separation in the absence of a vector potential. We call such pairs of integrals generalized, to distinguish them from the standard ones, which would correspond to separation. In this presentation we shall focus on three cases of generalized non-subgroup type integrals, namely elliptic cylindrical, prolate / oblate spheroidal and circular parabolic integrals, together with one case not related to any coordinate system. We find two new integrable systems, non-separable in the configuration space, both with generalized elliptic cylindrical integrals. In the other cases, all systems found were already known and possess standard pairs of integrals. In the limit of vanishing vector potential, both new systems reduce to free motion and therefore separate in every orthogonal coordinate system.

**Asymptotics of Robin eigenvalues for non-isotropic peaks**

*Marco Vogel*

*University of Oldenburg, Germany*

marco.vogel@uni-oldenburg.de

**Abstract:** Consider an open set $\Omega \subset \mathbb{R}^N$ and the operator $Q^\alpha_\Omega$ acting as the Laplacian $u \mapsto -\Delta u$ on $\Omega$ with the Robin boundary condition $\partial_\nu u = \alpha u$ on $\partial\Omega$, where $\partial_\nu$ is the outward normal derivative and $\alpha > 0$ is the so called Robin parameter. We are interested in the strong coupling asymptotics of the Robin eigenvalues $E_j(Q^\alpha_\Omega)$. First we discuss the cases where $\Omega$ has a smooth/Lipschitz boundary or has an isotropic peak. After that we move on to non-isotropic peaks. More precisely: Let $\Omega \subset \mathbb{R}^3$ be an open set such that

$$\Omega \cap \{x_3 < \delta\} = \left\{(x_1, x_2, x_3) \in \mathbb{R}^2 \times (0, \delta) : \left(\frac{x_1}{x_3^p}, \frac{x_2}{x_3^q}\right) \in (-1, 1)^2\right\} \subset \mathbb{R}^3,$$

$$\Omega \setminus \{x_3 \le \delta\} \text{ is a bounded Lipschitz domain,}$$

for some $\delta > 0$ and $1 < p < q < 2$. If a set satisfies the first condition one says that it has a non-isotropic peak at 0. We will take a glimpse on how to prove that for large $\alpha$ the $j$th eigenvalue behaves as $E_j(Q^\alpha_\Omega) \approx \mathcal{A}_j \alpha^{\frac{2}{2-q}}$, where the constants $\mathcal{A}_j < 0$ are eigenvalues of a one dimensional Schrödinger operator which depends on $p$ and $q$.

# WRONG-SIGN ANHARMONIC OSCILLATOR INTERPRETED AS A UNITARY AND STABLE QUANTUM SYSTEM

*Miloslav Znojil*

*NPI CAS, Řež*

znojil@ujf.cas.cz

**Abstract:** The 1D potential $V(x) = \omega^2 x^2 + g x^4$ with $x \in \mathbb{R}$ and $g > 0$ is popular, first of all, due to its role played in perturbation theory. It is less well known that the same potential also plays the equally important role of an illustrative example in the so called non-Hermitian (or, more precisely, quasi-Hermitian) reformulation of quantum mechanics. In the latter case one may pre-select a "wrong-sign" coupling $g < 0$ and keep the "coordinate" complex (say, with variable $s \in \mathbb{R}$ in $x = x(s) = s - \mathrm{i}\varepsilon$ using a constant shift $\varepsilon > 0$). In the talk we will review the corresponding older results and outline their recent extension in which (1) the parameters are allowed to vary with time, and in which (2) the necessary reconstruction of unitarity is achieved via an innovated, "hybrid" reformulation of the quantum mechanics of textbooks.

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