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→ Book of Abstracts ←

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Two parallel quantum wires in 2D, with non-constant δ-potential, as a model of DNA interaction.

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Abstract: We discuss a system of two parallel quantum wires, modelled by δ interaction of a constant coupling strength across all wire, except for a local region on each wire, where it is varied. Through the construction of a test-function, we prove the existence of discrete spectrum and investigate a dependence of function's energy levels on shift between perturbed regions of potential. The system can be used as a simplest model of interaction between long molecules (for example DNA).

Bifurcation of thresholds in essential spectrum produced by a small non-Hermitian hole

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Abstract: We consider a self-adjoint multi-dimensional elliptic operator in an infinite tube such that all coefficients of this operator depend only on the variable on the cross-section of the tube. The only dependence on the longitudial variable is via the second derivative in the operator. The essential spectrum of such operator contains certain thresholds, both internal ones and the bottom of the spectrum. Then in the tube we cut out a small hole and impose on its boundary a Robin condition with a complex-valued coefficient. The main result of the work describe how the mentioned thresholds bifurcate into the eigenvalues and resonances under the mentioned singular perturbation.

Hidden symmetries in superintegrable systems and enveloping algebras

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Abstract: The notion of hidden symmetry algebra arising in the context of exactly solvable systems is reconsidered from the perspective of subspaces of commuting polynomials in the enveloping algebras of Lie algebras that generate quadratic (and eventually higher-order) algebras. As a consequence of the construction, these algebras are independent on the choice of specific realizations by vector fields/differential operators of the underlying Lie algebra, which allows us to propose an alternative approach to analyze polynomial algebras as certain subspaces of enveloping algebras that commute with a given Hamiltonian. Based on the relation of superintegrability with exact solvability, a method that connects the underlying Lie algebra with algebraic integrals of motion is considered, leading to alternative choices of realizations that provide new explicit models with the same symmetry algebra.

Complex supersymmetry in graphene

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Abstract: Supersymmetric Quantum Mechanics (SUSY QM) is a technique that maps solutions of an initial Schrdinger equation to solutions of a final Schrdinger equation. Moreover, charge carriers in graphene at low energy are described by the Dirac-Weyl equation. The interaction of charge carriers with a magnetic field perpendicular to the graphene layer can be added with the minimal coupling rule. In this work, we apply the SUSY QM method using complex factorization energy ϵ to generate solutions of the Dirac-Weyl equation under specific external magnetic fields. Since a first iteration of the technique leads to complex magnetic fields, it is necessary to go up to a third iteration to produce a Hermitian Dirac-Weyl Hamiltonian with a real magnetic field. The above technique is known as Schrödinger SUSY QM. There is also a matrix version of SUSY QM that applies to Dirac-like systems. We show that matrix SUSY QM contains the Schrödinger SUSY QM, when using complex parameters.

In collaboration with Alonso Contreras-Astorga and David J. Fernández C.

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Zigzag graphene nanoribbons in inhomogeneous magnetic fields via SUSY QM

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Abstract: Departing from the free graphene nanoribbon system with zigzag edges, we constructed families of Dirac Hamiltonians with their eigenspinors and spectra for nanoribbons under inhomogeneous magnetic fields using supersymmetric quantum mechanics. Moreover, we studied the range of the parameters introduced by the supersymmetric algorithm where the solutions are regular and satisfy the zigzag boundary conditions.

Work in collaboration with Luis Hernández.

The authors acknowledge the support of CONACyT with the grant FORDECYT-PRONACES/61533/2020.

Time evolution of coherent electron states in tilted anisotropic Dirac materials

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Abstract: In this talk, we will discuss the effect of the Dirac cone tilt of anisotropic two-dimensional materials on the time evolution of coherent electron states in the presence of electric and magnetic fields. By performing a canonical transformation that maps the anisotropic Dirac-Weyl Hamiltonian with tilted Dirac cones to an effective and isotropic Dirac Hamiltonian under these fields, we obtain the well-known Landau levels spectra and wave functions, with which the Wigner matrix representation of Landau and coherent states is computed. The results show that the Wigner function for both Landau and coherent electron states depends on the so-called valley index. Also, the time evolution shows that the interplay of the Dirac cone tilt and the electric field also affects the uncertainties of both position and momentum according to the valley index. These findings may help to understand the generation of coherent electron states under the interaction with electromagnetic fields in tilted anisotropic Dirac materials.

Co-authors: Yonatan Betancur-Ocampo, Thomas Stegmann

Three-body closed chain of interactive (an)harmonic oscillators and the algebra $sl(4)$

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Abstract: In this talk we discuss 2- and 3-body oscillators in d-dimensions with quadratic and sextic pairwise potentials which depend on relative distances between particles and possess zero total angular momentum . In general, a three-body harmonic oscillator is a 6-parametric system depending on three arbitrary masses and three spring constants; it is an exactly-solvable problem with spectra linear in three quantum numbers and with hidden algebra $sl(4)$. We pay special attention to the atomic, where one mass is infinite, and molecular, where two masses are infinite, limits for which the system is non-integrable. The first and second order integrals of the 3-body oscillator for unequal masses are described. It is shown that for certain relations involving masses and spring constants the system becomes maximally (minimally) superintegrable in the case of two (one) relations. The classical case is briefly discussed as well.

The presentation is based on J. Phys. A: Math. Theor. 53 (2020) 055302

Quantum Information Processing and Related

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Abstract: We introduce some recent researches related to quantum information processing, including quantum coherence and quantum entanglement, quantum nonlocality, quantum information masking, quantum uncertainty relations, as well as tensor network compressed sensing and machine learning.

Self-Adjointness of the Dirac Operator on Sectors with Singular Interactions

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Abstract: We consider Dirac operators with electrostatic and Lorentz-scalar delta potentials supported on leads, dividing the plane into sectors. Separation of variables leads to an analysis of the deficiency indices in the cases where there are two or three leads. The essential spectrum for distinguished self-adjoint extensions is also determined.

REAL POVMS ON THE PLANE: INTEGRAL QUANTIZATION, NAIMARK THEOREM AND LINEAR POLARISATION OF THE LIGHT

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Abstract: Two aspects of the elementary example of real POVMs on the Euclidean plane are presented, namely their status as quantum observables and their role as quantizers in the integral quantization procedure. The compatibility of POVMs in the ensuing quantum formalism is discussed, and a Naimark dilation is found for the quantum operators. A physical situation is discussed, where is described the linear polarization of the light with the use of Stokes parameters. In particular, the case of sequential measurements in a real bidimensional Hilbert space is addressed. Finally, a necessary condition for the compatibility of two dichotomic POVMs is found.

Work in collaboration with Roberto Beneduci (Universit della Calabria), Emmanuel Frion, and Amedeo Perri

Generalized Fermion Sampling

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Abstract: We introduce generalized fermion sampling, by extending the computational problem FermionSampling of accurately estimating multichannel coincidence rates, resulting from injecting fermions at different times, either deterministically or randomly, into multiple inputs of an interferometer. Specifically, we construct an algorithm to solve FermionSampling by partitioning arrival times into nite-interval time bins, then use group functions to evaluate coincidence rates, and finally assess algorithmic complexity in terms of the number of complex-number arithmetic operations. Our major conclusion is that generalized FermionSampling is computationally hard when the times of arrival are uniformly random, challenging the paradigm that fermion sampling would be computationally efficient. Our proof hinges on the requirement of computing a hard matrix immanant, which always appears in cases where at most $\lceil \frac{n}{2} \rceil$ particles arrive at the same time. For n fermions, the number of arithmetic operations required to compute this hard matrix immanant scales as the product of a few terms, with one term involving a number of arithmetic operations that scales like the Catalan number C_n when n is even, or approximately so if n is odd.

This is joint work with:

Dylan Spivak (Department of Mathematical Sciences, Lakehead University)

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Freezable quantum states or bound states in the continuum for time dependent potentials

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Abstract: In this work, we study bound states in the continuum (BICs). We first explore time dependent potentials generated via supersymmetric quantum mechanics. These potentials have a quantum state with the property that after a certain time t0, when the potential does not longer change, the evolving state becomes a BIC and its probability distribution freezes. We further consider the case of a system supporting two BICs. The framework to add more of these states is presented.

Work in collaboration with Alonso Contreras Astorga and Alfredo Raya

Exact Christoffel-Darboux Expansions: A New, Multidimensional, Algebraic, Eigenenergy Bounding Method

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Abstract: Although the Christoffel-Darboux representation (CDR) plays an important role within the theory of orthogonal polynomials, and many important bosonic and fermionic, multidimensional, Hermitian and Non-Hermitian, systems can be transformed into a moment equation representation (MER), the union of the two into an effective, algebraic, eigenenergy bounding method has been overlooked. This particular fusion of the two representations (CDR and MER), defines the Orthonormal Polynomial Projection Quantization - Bounding Method (OPPQ-BM), as developed here. We use it to analyze several one dimensional and two dimensional systems, including the quadratic Zeeman effect for strong-superstrong magnetic fields. For this problem, we match or surpass the excellent, but intricate, results of Kravchenko et al (1996 Phys. Rev. A 54 287) for a broad range of magnetic fields, without the need for any truncations or approximations. The methods developed here apply to any linear, partial differential equation eigen-parameter problem, hermitian or non hermitian. We discuss various one and two dimensional systems. Preliminary results have been published (2021 Physica Scr. 96 075201)

Conservation laws in unitary, non-Hermitian, and Lindblad dynamics [1,2].

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Abstract: Recent experimental progress on open quantum systems has delineated three regimes for their dynamics. The coherent, unitary dynamics of a closed quantum system; the incoherent, trace-preserving Lindblad dynamics of a quantum system coupled to a bath; and the coherent dynamics of a non-Hermitian Hamiltonian. While the first two have been extensively studied in the literature, conservation laws in the presence of a non-Hermitian Hamiltonian or non-unitary quantum walk have not been extensively explored. Here, I will present a general analysis of conserved observables in such systems, and an analytical, recursive procedure to obtain a complete set of such observables. I will show discuss the consequences of such conserved quantities for electrical circuits with gain and loss, and in a PT-symmetric qudit, simulated by single-photon interferometry. Our results spell out nonlocal conservation laws in non-unitary dynamics and provide key elements that will underpin the self-consistent analyses of non-Hermitian quantum many-body systems that are forthcoming.

[1] Z. Bijan et al., Phys. Rev. Research 2, 022039(R) (2020).

[2] F. Ruzicka et al., arXiv:2104.11265.

Geometric approximations of point interactions

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Abstract: In this talk we demonstrate how to approximate $1d$ Schrödinger operators with δ-interactions by the Neumann Laplacian on a narrow waveguide-like domain. Namely, we consider the domain consisting of a straight narrow strip and a small protuberance with "room-and-passage" geometry. We show that in the limit when perpendicular size of the strip tends to zero, and the protuberance is appropriated scaled, the Neumann Laplacian on this domain converges in (a kind of) norm resolvent sense to the above singular Schrödinger operator. The estimates on the rate of this convergence are derived. We also proof the Hausdorff convergence of spectra. The talk is based on the preprint [1]. If time permits, we will also discuss geometric approximations of $1d$ Schrödinger operators with δ'-interactions [1].

1. A. Khrabustovskyi, O. Post, A geometric approximation of δ-interactions by Neumann Laplacians, arXiv:2104.10463 [math.SP].

2. G. Cardone, A. Khrabustovskyi, δ'-interaction as a limit of a thin Neumann waveguide with transversal window, J. Math. Anal. Appl. 473 (2) (2019), 1320-1342

Soliton-like behaviour in non-integrable systems

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Abstract: We present a general scheme for constructing robust excitations (soliton-like) in non-integrable multicomponent systems [1,2]. By robust, we mean localised excitations that propagate with almost constant velocity and which interact cleanly with little to no radiation. We achieve this via a reduction of these complex systems to more familiar effective chiral field-theories using perturbation techniques and the Fredholm alternative [2]. As a specific platform, we consider the generalised multicomponent Nonlinear Schrodinger Equations (MNLS) with arbitrary interaction coefficients [1]. This non-integrable system reduces to uncoupled Korteweg-de Vries (KdV) equations, one for each sound speed of the system. This reduction then enables us to exploit the multi-soliton solutions of the KdV equation which in turn leads to the construction of soliton-like profiles for the original non-integrable system. We demonstrate that this powerful technique leads to the coherent evolution of excitations with minimal radiative loss in arbitrary non-integrable systems. These constructed coherent objects for non-integrable systems bear remarkably close resemblance to true solitons of integrable models. Although we use the ubiquitous MNLS system as a platform, our findings are a major step forward towards constructing excitations in generic continuum non-integrable systems.

[1] R. Nimiwal, U. Satpathi, V. Vasan, M. Kulkarni, arXiv:2101.01651 (2021)

[2] S. Swarup, V. Vasan, M. Kulkarni, J. Phys. A: Math. Theor. 53, 135206 (2020)

On the Wess-Zumino Model - A Supersymmetric Field Theory

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Abstract: In this talk I would consider some important aspects of the Wess-Zumino model (WZM) - an important example of a supersymmetric field theory having global or rigid supersymmetry. I would consider its constrained dynamics and the Hamiltonian and Path Integral quantizations. I would discuss the role of this theory in the construction of supergravity as well as in the construction of RNS superstring theory.

Maxwell-Chern-Simons-Higgs Theory

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Abstract: In this talk, I would consider the constrained dynamics of the Maxwell-Chern-Simons-Higgs theory and study its Hamiltonian quantization under appropriate gauge-fixing conditions.

Choreographic motions on algebraic Lemniscate

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Abstract: For one 3-body and two 5-body planar choreographic motions on the same algebraic lemniscate by Bernoulli we found explicitly a maximal possible set of (particular) Liouville integrals, 7 and 15, respectively, (including the total angular momentum), which Poisson commute with the corresponding Hamiltonian along the trajectory. Thus, these choreographies are particularly maximally superintegrable. It is conjectured that the total number of (particular) Liouville integrals is maximal possible for any odd number of bodies $(2n + 1)$ moving choreographically (without collisions) along the lemniscate, thus, the corresponding trajectory is particularly, maximally superintegrable. Some of these Liouville integrals are presented explicitly. The limit of infinite n is studied: it is predicted that one-dimensional liquid with nearest-neighbor interactions occurs, it moves along algebraic lemniscate and it is characterized by infinitely many constants of motion.

The inverted harmonic oscillator revisited

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Abstract: We discuss the quantum dynamics of a harmonic oscillator as well as its inverted counterpart in Schrodinger picture. The inverted harmonic oscillator is formally obtained from the harmonic oscillator by the replacement $\omega \to i\omega$, if this replacement is applied to find the energy eigenvalues, we would get imaginary values for the Hermitian Hamiltonian. This explicitly demonstrates the subtle points involved with the inversion of the harmonic oscillator by redefinitions of variables. To do this, we introduce a scaling operator to connect the inverted harmonic to an anti PT symmetric harmonic oscillator. Finally, we investigate the coherent states problem and derive their properties.

Rational extension of the many particle systems

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Abstract: The search for the exactly solvable/ quasi-exactly solvable (QES) potentials has been boosted greatly due to the recent discovery of exceptional orthogonal polynomials (EOPs). Unlike the usual orthogonal polynomials, these EOPs start with degree $m \geq 1$ and still form a complete orthonormal set with respect to a positive definite inner product defined over a compact interval. This remarkable work leads to the discovery of several new systems whose solutions are written in terms EOPs. Such systems are known as rational extension. Many of the exactly solvable one particle systems have been extended rationally. In this talk we would like to discuss rational extension of some of the many particle systems.

In the first model, we construct the rational extension of the truncated Calogero-Sutherland model [1]. We analytically solve this rationally extended model to find the exact solutions. The spectrum remains unchanged but the eigenfunctions are completely different and are written in terms of EOPs. In another model [2] we consider rational extension of a QES N-particle Calogero model with harmonic confining interaction. Such QES many particle system, whose radial part of the effective potential yields a supersymmetric partner of the radial harmonic oscillator, is constructed by including new long-range interactions to the rational Calogero model. An infinite number of bound state energy levels are obtained for this system under certain conditions. We also calculate the corresponding bound state wave functions in terms of EOPs.

[1] R K Yadav, A Khare, N Kumari, B P Mandal, Rationally extended many-body truncated Calogero-Sutherland model, Annals of Physics, 400, 189 (2019)

[2] B Basu-Mallick, B P Mandal, P Roy, Quasi exactly solvable extension of Calogero model associated with exceptional orthogonal polynomials, Annals of Physics, 380, 206 (2017).

Goldstone bosons and the Englert-Brout-Higgs mechanism in non-Hermitian theories

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Abstract: In recent work, Alexandre, Ellis, Millington and Seynaeve have extended the Goldstone theorem to non- Hermitian Hamiltonians that possess a discrete antilinear symmetry such as PT and possess a continuous global symmetry. They restricted their discussion to those realizations of antilinear symmetry in which all the energy eigenvalues of the Hamiltonian are real. Here, we extend the discussion to the two other realizations possible with antilinear symmetry, namely energies in complex conjugate pairs or Jordan-block Hamiltonians that are not diagonalizable at all. In particular, we show that under certain circumstances it is possible for the Goldstone boson mode itself to be one of the zero-norm states that are characteristic of Jordan-block Hamiltonians. While we discuss the same model as Alexandre, Ellis, Millington and Seynaeve, our treatment is quite different, though their main conclusion that one can have Goldstone bosons in the non-Hermitian case remains intact. We extend our analysis to a continuous local symmetry and find that the gauge boson acquires a nonzero mass by the Englert-Brout-Higgs mechanism in all realizations of the antilinear symmetry, except the one where the Goldstone boson itself has zero norm, in which case, and despite the fact that the continuous local symmetry has been spontaneously broken, the gauge boson remains massless.

Hidden algebras for shape invariant nonseparable and nondiagonalizable models

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Abstract: A shape invariant nonseparable and nondiagonalizable two-dimensional model with quadratic complex interaction, first studied by Cannata, Ioffe, and Nishnianidze. I will discuss recent work in which the system is re-examined with the purpose of exhibiting its hidden algebraic structure. Four ladder operators are used as building blocks for constructing gl(2) generators, acting within the set of associated functions belonging to the Jordan block corresponding to a given energy eigenvalue. This analysis is extended by constructing a sp(4) nd osp(1/4) superalgebra. Another shape invariant nonseparable and nondiagonalizable three-dimensional model was introduced by Bardavelidze, Cannata, Ioffe, and Nishnianidze. I will also discuss hidden symmetry algebra and the description of the associated states that form Jordan blocks. These works are in collaboration with Christiane Quesne and based on arxiv 2010.1573, arxiv 2010.1576.

Low-energy scattering defined by the Helmholtz equation in one dimension and permittivity profiles with balanced gain and loss

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Abstract: The Helmholtz equation in one dimension, which describes the propagation of electromagnetic waves in effectively one-dimensional systems, is equivalent to the time-independent Schrödinger equation. The fact that the potential term entering the latter is energy-dependent obstructs the application of the results on low-energy quantum scattering in the study of the low-frequency waves satisfying the Helmholtz equation. We use a recently developed dynamical formulation of stationary scattering to offer a comprehensive treatment of the low-frequency scattering of these waves for a general finite-range scatterer. In particular we give explicit formulas for the coefficients of the low-frequency series expansion of the transfer matrix of the system which in turn allow for determining the low-frequency expansions of its reflection, transmission, and absorption coefficients. Our general results reveal a number of interesting physical aspects of low-frequency scattering particularly in relation to permittivity profiles having balanced gain and loss.

Reference:

F. Loran and A. Mostafazadeh, Low-frequency scattering defined by the Helmholtz equation in one dimension, J. Phys. A: Math. Theor., to appear; arXiv: 2105.07895.

Supersymmetric transformations, ladder operators and coherent states for non-rational extensions of the harmonic oscillator

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Abstract: In this work, we generate a family of quantum potentials that are non rational extensions of the harmonic oscillator one. Such family can be obtained via two different but equivalent supersymmetric transformations. We construct ladder operators for these extensions as the product of the intertwining operators related to the supersymmetric transformations. Then, we generate families of coherent states as eigenstates of the annihilation operator and study their properties. We find that they possess temporal stability, continuity on the label, and a resolution of the identity. Moreover, we investigate mean energy values, time-dependent probability densities, Wigner functions, and the Mandel Q parameter to elucidate a general non-classical behavior.

Work in collaboration with Alonso Contreras-Astorga and David Fernández.

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Quantization of Rationally Deformed Morse Potentials by Wronskians of Generalized Bessel Polynomials With Common Index.

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Abstract: The presentation underscores certain advantages of quantizing the Morse potential by Romanovski/Bessel (R/Bessel) polynomials with degree-independent indexes (in contrast with the conventional representation of its eigenfunctions in terms of classical Laguerre polynomials with varied positive indexes). The preposition that the Morse potential can be quantized by one of three finite orthogonal sequences discovered by Romanovsky places this potential on an equal foot with hypergeometric Pöschl-Teller (h-PT) and Gendenshtein (Scarf II) potentials quantized by two other finite orthogonal sequences of Romanovski polynomials. The common feature of the Morse and Gendenshtein potentials is that all their rational Darboux-Crum (RDC) transforms are specified by a single series of Maya diagrams and as result any eigenfunction can be represented as a weighted ratio of Wronskians of generalized Bessel and Routh polynomials accordingly. We term the exceptional orthogonal polynomial (EOP) sequences formed by these polynomial Wronskians as X-R/Bessel and X-R/Routh polynomials respectively. In particular if the given set of seed solutions is composed of juxtaposed pairs of eigenfunctions these Wronskians are formed by R/Bessel and correspondingly R/Routh polynomials. On other hand, if the RDC transforms of the Morse potential are constructed using generalized Bessel polynomials of nonzero degree with no positive roots then we come to the RDC subnet of isospectral SUSY partners. The Darboux transforms of the Morse potential recently re-examined by Quesne [Europ. Phys. J. Plus 136 (2021) 128] constitute the simplest representatives of this isospectral subnet of the RDC net of potentials solvable by X-R/Bessel polynomials.

Renormalization Group in Six-derivative Quantum Gravity

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Abstract: The exact one-loop beta functions for the four-derivative terms (Weyl tensor squared, Ricci scalar squared, and the Gauss-Bonnet) are derived for the minimal six-derivative quantum gravity (QG) theory in four spacetime dimensions. The calculation is performed by means of the Barvinsky and Vilkovisky generalized analytic Schwinger-DeWitt technique. With this result we gain, for the first time, the full set of the relevant beta functions in a super-renormalizable model of QG. The complete set of renormalization group (RG) equations, including also these for the Newton and the cosmological constant, is solved explicitly in the general case and for the six-derivative Lee-Wick (LW) quantum gravity. In the ultraviolet regime, the minimal theory is shown to be asymptotically free and describes free gravitons in Minkowski or (anti-) de Sitter backgrounds. We argue that an extension of the theory that involves operators cubic in Riemann tensor may change the beta functions and be useful for constructing UV-finite theory of quantum gravity.

Revisiting the Swanson Model: non-PT symmetry phase

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Abstract: The Swanson Model has been introduced some time ago [1] as an example of a non hermitian hamiltonian that obey Parity-Time Reversal (PT) symmetry [2]. It is well known that it admits real eigenvalues for a well defined region of the parameter space of the model. The similarity between the Swanson Hamiltonian and the Harmonic Oscillator in the PT-symmetry region as well as the dynamic of physical observables have been extensively analyzed [3]. However, to our knowledge, much less has been investigated in the region of PT-broken symmetry. We study the spectrum and the eigenfunctions in the non-PT symmetry phase. Our interest is both from the physical and the mathematical point of view. We show that, depending on the region on the parameter model-space, the Swanson model is similar to different physical systems, i.e. parabolic barrier, or harmonic oscillator with negative mass. From the mathematical point of view, we deal with an infinite dimensional system. From a mathematical point of view, in most of the non-PT symmetry phase, their eigenfunctions do not belong to the Hilbert space.. Because of this fact, though the Swanson hamiltonian is a quadratic model, it is a good example to test different approaches to the problem of providing physical solutions. As a first proposal, we have applied the Complex Scaling Method to the Swanson model, in doing so we have obtained eigenfunctions which are square-integrable function for the different regions. We present the corresponding symmetry and metric operators. Also, we study the time evolution of different initial states under the action of the Swanson Hamiltonian, by computing the corresponding Wigner functions, in the different regions.

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[4] D. P: Musumbu, H. B. Geyer and W. D. Heiss, J. of Phys. A: Math. Theor. 40, (2007)F45

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On Heun equation with applications

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Abstract: We study the analytic solutions of the generalized Heun equation, $(\alpha_0+\alpha_1\,r+\alpha_2\,r^2+\alpha_3\,r^3)\,y''+(\beta_0+\beta_1\,r+\beta_2\,r^2)\,y'+(\varepsilon_0+\varepsilon_1\,r)\,y=0$, where $|\alpha_3|+|\beta_2|\neq 0$, and $\{\alpha_i\}_{i=0}^3$, $\{\beta_i\}_{i=0}^2$, $\{\varepsilon_i\}_{i=0}^1$ are real parameters. The existence conditions for the polynomial solutions are given. A simple procedure based on a recurrence relation is introduced to evaluate these polynomial solutions explicitly. For $\alpha_0=0$, $\alpha_1\neq 0$, we prove that the polynomial solutions of the corresponding differential equation are sources of finite sequences of orthogonal polynomials. Several mathematical properties, such as the recurrence relation, Christoffel-Darboux formulas and the norms of these polynomials are discussed. We shall also show that they exhibit a factorization property that permits the construction of other infinite sequences of orthogonal polynomials.

Drum membrane in contact with a drum stick modeling.

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Abstract: Despite an extensive number of studies and proposed models, little attention has been paid to the interaction of the drum with an object that excites vibrations, and a short point interaction of a membrane with an external force is often used. Although such model is very convenient and greatly simplifies solution of the problem, it can not account for variety of composite gestures performed on the drums. In the present paper, two models describing the interaction of a drumstick with a drum membrane are proposed, based on experimental data. On the basis of the proposed models, a numerical simulation of the drumstick dynamics and analysis of changes in the dynamics depending on such parameters as the type, force, and place of impact are carried out.

Quantum Groups and Polymer Quantum Mechanics

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Abstract: In Polymer Quantum Mechanics, a quantization scheme that naturally emerges from Loop Quantum Gravity, position and momentum operators cannot be both well-defined on the Hilbert space (H_Poly). It is henceforth deemed impossible to define standard creation and annihilation operators. we show that a q-oscillator structure, and hence q-deformed creation/annihilation operators, can be naturally defined on H_Poly , which is then mapped into the sum of many copies of the q-oscillator Hilbert space. This shows that the q-calculus is a natural calculus for Polymer Quantum Mechanics. Moreover, we show that the inequivalence of different superselected sectors of H_Poly is of topological nature.

DYNAMIC SYSTEMS WITH BENIGN GHOSTS

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Abstract: A system with ghosts is a quantum system where the spectrum of the Hamiltonian has no bottom: there are states with arbitrary low and arbitrary high energies. In many such systems, the ghosts are "malignant", bringing about the blow up of the classical trajectories and quantum collapse associated with violation of unitarity. But there are also many systems where the ghosts are there, but they are benign: no blow up in the classical dynamics and no collapse.

We discuss three large classes of such benign systems:

1. The systems obtained by a variation of any ordinary Hamiltonian system with a double set of dynamic variables: $(q_i, p_i; Q_i = \delta q_i, P_i = \delta p_i)$.

2. The systems describing the motion over a Lorentzian manifold whose metric is not positive definite .

3. Exactly solvable systems. In particular, we discuss the 2-dimensional KdV system with reversed time and spatial coordinates. The Lagrangian of such a system involves higher time derivatives.

Non-Hermitian gauge field theory and BPS solutions.

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Abstract: We present an overview of some key results in a recent series devoted to non-Hermitian gauge field theories with SU(N) continuous symmetries and modified CPT symmetries. We demonstrate that Goldstones theorem and Higgs mechanism work conventionally in the CPT symmetric regime. However, it breaks down when the theory is in CPT broken regime. When the fields are in the adjoint representation of SU(N), we identify the t'Hooft-Polyakov monopoles using a fourfold Bogomol'nyi-Prasad-Sommerfield (BPS) limit. We investigate this limit further for other types of non-Hermitian field theories in 1+1 dimensions and 3+1 dimensional Skyrme models for which we find new types of complex solutions. We will present the mechanism for which the energy of the complex soliton is real due to the CPT symmetry of the theory.

New exact and approximation methods for time-dependent non-Hermitian quantum systems

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Abstract: We present several new methods for determining the time-dependent metric for time-dependent non-Hermitian quantum systems. These methods include identifying a complex point transformation as a map from a solvable time-independent system to an explicitly time-dependent non-Hermitian system. This map can then be used to construct the time-dependent non-Hermitian invariant for the latter system, which in turn may be utilized in the construction of Dyson maps, hence metric operators, due to being pseudo-Hermitian.

We will also discuss a recently developed perturbative approach for determining the metric. To demonstrate the effectiveness of this approach we shall apply it to the two dimensional time-dependent harmonic oscillators with weak non-Hermitian coupling and the strongly coupled time-dependent negative quartic anharmonic oscillator.

We conclude by discussing a new general method which allows for the construction of an infinite series of time-dependent Dyson maps for the same non-Hermitian Hamiltonian.

Anharmonic oscillator: almost analytic solution

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Abstract: Closed analytic expressions for eigenfunctions of an anharmonic oscillator are proposed. In particular cases of quartic and sextic oscillators they lead to unprecedented accuracies for both eigenfunctions and eigenvalues.

Exact solutions for time-dependent Hermitian and Non-Hermitian Oscillators through point transformations

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Abstract: In this talk, I discuss the construction of the appropriate form-preserving point transformation so that a given stationary Schrödinger equation deforms into one with time-dependent potential. This allows obtaining a set of orthogonal solutions inherited from the stationary system, where the orthogonality of the set is guaranteed from the preservation of the inner product. The corresponding constants of motion (invariant operators) are extracted straightforwardly by simply performing the appropriate mappings and exploiting the conservation of first integrals available in the initial system. To illustrate the method, I discuss the parametric oscillator as an example in the Hermitian regime. In contrast, a particular realization of the time-dependent Swanson oscillator is discussed for the non-Hermitian case.

Joint collaboration with O. Rosas-Ortiz.