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Analytic and algebraic methods in physics

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→ **THE BOOK OF ABSTRACTS** ←
(the version of September 10, 2018)

(alphabetical ordering)

Petr Ambrož

Palindromic length and morphisms in class \mathbf{P}

Palindromic length of a finite word w – defined in 2013 by Frid, Puzynina and Zamboni – is the smallest integer k such that w can be written as concatenation of k palindromes. Let $P_u(n)$ denote the maximal palindromic length of factors of length n in u . Frid et al. conjectured that $P_u(n)$ is bounded if and only if u is periodic.

In this contribution we study $P_u(n)$ for fixed points of morphisms in class \mathbf{P} ; this class was defined by Hof, Knill and Simon in 1995 in their study of palindromic Schroedinger operators. Languages of fixed points of morphisms in class \mathbf{P} are conjectured to coincide with languages of fixed points of primitive morphisms containing infinite number of palindromes.

We prove that there exists a positive constant K such $P_u(n) < K \cdot \ln(n)$ for all n .

Fabio Bagarello

Recent results on deformed commutation rules

I will review some old and new results on deformed commutation rules of several kinds, and on the ladder operators related to them.

Diana Barseghyan

A magnetic version of the Smilansky-Solomyak model

We discuss a modification of Smilansky model in which a singular potential is replaced by a regular, below unbounded potential and in the presence of a homogeneous magnetic field perpendicular to the plane. We show that, similarly to the original model, such operator exhibits a spectral transition with respect to the coupling constant.

Jussi Behrndt

Spectral shift function for Schrödinger operators with delta-interactions on hypersurfaces

For a pair of self-adjoint operators we recall the notion of the spectral shift function and we provide a new representation formula for it in terms of an abstract Titchmarsh-Weyl function. As an application we discuss Schrödinger operators with delta-interactions supported on compact hypersurfaces.

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This talk is based on joint work with Fritz Gesztesy and Shu Nakamura.

Carl M. Bender

PT-symmetric quantum field theory in $D < 2$

The field of PT-symmetric quantum mechanics began with a study of the Hamiltonian $H = p^2 + x^2(ix)^\varepsilon$. A surprising feature of this non-Hermitian Hamiltonian is that its eigenvalues are discrete, real, and positive when $\varepsilon \geq 0$. This talk examines the corresponding quantum-field-theoretic Hamiltonian $H = \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}\phi^2(i\phi)^\varepsilon$ in D-dimensional spacetime, where ϕ is a pseudoscalar field. For $0 \leq D < 2$ it is shown how to calculate the Green's functions as series in powers ε directly from the Euclidean-space representation of the partition function. Exact expressions for the first few coefficients in this series for the vacuum energy density, the one-point and two-point Green's functions, and the renormalized mass are derived. The remarkable spectral properties of PT-symmetric quantum mechanics appear to persist in PT-symmetric quantum field theory.

Anton Boitsev

Operator extensions theory model for quantum dot radiation

We propose a model describing electronic transport through a boson cavity. We use the JaynesCummings model dealing with a two-level quantum dot coupled to a quantized electromagnetic field and two semi-infinite wires. The mathematical background of our model is given by the theory of self-adjoint extensions of symmetric operators. Using the boundary triplets approach, the gamma-field and the Weyl function were calculated.

Lyonell Boulton

Perturbations of Gibbs semigroups and non-selfadjoint Schrödinger operators

Let $-T$ be the generator of a C_0 one-parameter semigroup e^{-Tt} which is of finite trace for all $t > 0$ (a Gibbs semigroup). Let A be a T -compact operator. In general $-(T + A)$ might or might not be the generator of a Gibbs semigroup. In the first half of this talk we give sufficient conditions on A so that $-(T + A)$ is the generator of a Gibbs semigroup. We determine these conditions in terms of the convergence of the Dyson-Phillips expansion corresponding to the perturbed semigroup in suitable Schatten-von Neumann norms. They are optimal in a sense to be made precise during the talk.

In the second half of the talk we consider $T = H_\vartheta$ and $A = V$ where H_ϑ is the non-selfadjoint harmonic oscillator

$$H_\vartheta = -e^{-i\vartheta} \partial_x^2 + e^{i\vartheta} x^2$$

acting on $L^2(\mathbb{R})$, $0 < \vartheta < \frac{\pi}{2}$ and $V : \mathbb{R} \rightarrow \mathbb{C}$ is a locally integrable potential such that

$$|V(x)| \leq a|x|^\alpha + b \quad \forall x \in \mathbb{R}$$

for some $0 \leq \alpha < 2$, $a > 0$ and $b \in \mathbb{R}$. We show that $-(H_\vartheta + V)$ is the generator of a Gibbs semigroup $e^{-(H_\vartheta + V)\tau}$ for $|\arg \tau| \leq \frac{\pi}{2} - |\vartheta|$. By virtue of a non-selfadjoint version of an inequality due to Ginibre and Gruber found in 2001 by Cachia and Zagrebnov, it is possible to deduce this on an open sector. But what is remarkable and far from obvious here is the fact that the finite trace property extends all the way to the edges of the sector above. To achieve the latter, we invoke the first part of the talk and show that the Dyson-Phillips expansion converges in this case in r Schatten-von Neumann norm for $r > \frac{4}{2-\alpha}$.

Biagio Cassano

Self-adjointness for the Dirac operator with Coulomb-type spherically symmetric perturbations via boundary conditions

We determine all the self-adjoint realizations of the Dirac operator

$$-i\alpha \cdot \nabla + m\beta + \mathbb{V}(x)$$

in \mathbb{R}^3 for $m \in \mathbb{R}$ and \mathbb{V} a Coulomb-type spherically symmetric potential. We characterize the self-adjointness of the operator in terms of the behaviour of the functions of its domain in the origin, and we relate our results with the theory of the *boundary triples*. Moreover, in some cases we select a *distinguished* extension requiring that its domain is included in the domain of the appropriate quadratic form.

This is a joint work with Fabio Pizzichillo (Basque Center for Applied Mathematics)

Alonso Contreras-Astorga

Photonic systems with two-dimensional landscapes of complex refractive index via time-dependent supersymmetry

We present a framework for the construction of solvable models of optical settings with genuinely two-dimensional landscapes of refractive index. The solutions of the associated non-separable Maxwell equations in paraxial approximation are found with the use of the time-dependent supersymmetry. We discuss peculiar theoretical aspects of the construction. Sufficient conditions for existence of localized states are discussed. Localized solutions vanishing for large $|\vec{x}|$, that we call light dots, as well as the guided modes that vanish exponentially outside the wave guides are constructed. We consider different definitions of the parity operator and analyze general properties of the \mathcal{PT} -symmetric systems, e.g. presence of localized states or existence of symmetry operators. Despite the models with parity-time symmetry are of the main concern, the proposed framework can serve for construction of non- \mathcal{PT} -symmetric systems as well. We explicitly illustrate the general results on a number of physically interesting examples, e.g. wave guides with periodic fluctuation of refractive index or with a localized defect, curved wave guides, two coupled wave guides or a uniform refractive index system with a localized defect.

Erik Díaz-Bautista

Coherent states in graphene

Graphene is a material that exhibits interesting electronic properties and that is why it is considered a good material for the development of electronic devices. In this sense, the interaction between its electrons and magnetic fields has become an interesting research subject to achieve the confinement or control of such particles. In this work, we use a symmetric gauge to describe this system and to construct the coherent states for the interaction of the electrons in graphene with a constant magnetic field that is perpendicular to the surface of the layer in order to describe the electron dynamics in a semiclassical approach.

Manuel Gadella

Point potentials in one and two dimensions

This is a contribution to the study of quantum solvable models. We study the behaviour of bound states of a one dimensional Hamiltonian with a potential proportional to $|x|$ plus a point perturbation. We consider three different families of point perturbation depending on one or two real parameters and analyze the dependence of bound states with respect to these parameters. Some levels are invariant after the point perturbation and some others change, a phenomenon which depends on the parity of the energy levels. Level crossings occur. We also consider three types of two dimensional potentials, isotropic harmonic oscillator, pyramidal and mixed, perturbed by a Dirac delta at the origin. In order to obtain the energy levels, we use the Green function for the perturbed Hamiltonian, that in many cases requires a regularization procedure. Level crossings also occur.

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Prepared in collaboration with S. Fassari, M. L. Glasser, L. M. Nieto and F. Rinaldi.

Uwe Günther

**The IR-truncated \mathcal{PT} -symmetric $V = ix^3$ model
and its asymptotic spectral scaling graph**

The \mathcal{PT} -symmetric quantum mechanical $V = ix^3$ model over the real line, $x \in \mathbb{R}$, is IR truncated and considered as Sturm-Liouville problem over a finite interval $x \in [-L, L] \subset \mathbb{R}$. Via WKB and Stokes graph analysis, the location of the complex spectral branches of the $V = ix^3$ model as well as those of more general $V = -(ix)^{2n+1}$ models over $x \in [-L, L] \subset \mathbb{R}$ are obtained. Underlying asymptotic spectral scaling graphs $\mathcal{R} \subset \mathbb{C}$ are extracted which are L -invariant so that the IR limit $L \rightarrow \infty$ can be formally taken. Implications for the $V = ix^3$ model over $x \in \mathbb{R}$ are discussed.

This talk is based on joint work with Frank Stefani.

W. Dieter Heiss

It looks like an Exceptional Point but it isnt

It is demonstrated that, in single particle potential scattering, the meeting point of resonance poles at zero energy under variation of the potential strength has some characteristics of an exceptional point, but it is not an EP under some more careful scrutiny. While this appears to be a mathematical curiosity it is of physical interest, e.g. in low energy neutron scattering in nuclear physics where the cross section shows a very special behaviour around zero energy.

Markus Holzmann

Self-adjoint realizations of Dirac operators with boundary conditions on domains in \mathbb{R}^3

Let $\Omega \subset \mathbb{R}^3$ be a domain with compact C^2 -smooth boundary. In this talk Dirac operators acting on functions which satisfy suitable boundary conditions that yield self-adjoint operators in $L^2(\Omega; \mathbb{C}^4)$ are discussed. Such operators are the relativistic counterparts of Laplacians on Ω with Robin-type boundary conditions. In the study of the self-adjointness of the operators it turns out that there exist critical boundary values for which the corresponding operators have significantly different properties. In the case of non-critical boundary values the self-adjointness and several basic spectral properties of the operators are obtained. Eventually, it is shown that in the critical case the spectral properties can be of a completely different nature.

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This talk is based on a joint work with J. Behrndt and A. Mas.

Artur Ishkhanyan

The second Exton potential for the Schrödinger equation

Analytical solutions of the Schrödinger equation with a singular, fractional power potential, referred to as the second Exton potential, are derived and analyzed. The potential is defined on the positive half-axis and supports an infinite number of bound states. It is conditionally integrable and belongs to a bi-confluent Heun family. The fundamental solutions are expressed as irreducible linear combinations of two Hermite functions of a scaled and shifted argument. The energy quantization condition results from the boundary condition imposed at the origin. For the exact eigenvalues, which are solutions of a transcendental equation involving two Hermite functions, highly accurate approximation by simple closed-form expressions is derived.

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Work done together with J. Karwowski (Toruń).

Jacek Karwowski

Semi-exact solutions of radial Schrödinger equations

One-particle potentials expressed as polynomials of the radial variable are referred to as the power potentials. Many years ago M. Znojil demonstrated that the Schrödinger equations with power potentials are semi-exactly-solvable (or nearly-exactly solvable) and their solutions may be expressed in terms of the Hessenberg determinants [1]. Several numerical experiments illustrating properties of these solutions are presented. Specific properties of the polynomial solutions as well as some consequences of hidden symmetries of the equations are briefly discussed.

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[1] M. Znojil, J. Phys. A: Math. Gen. 27 (1994) 4945-68; Int. J. Mod. Phys. A 12 (1997) 299-304.

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Work done together with Henryk A. Witek from Hsinchu, Taiwan.

Andrii Khrabustovskyi

Singular Schrödinger operators with predefined spectrum

We construct a singular Schrödinger operator on a compact interval with predefined essential spectrum and predefined finite part of the discrete spectrum. The required structure of the spectrum is realized by a special choice of a sequence of point interactions of δ and δ' types. Our construction is inspired by a celebrated paper [1] and its sequel [2], where similar problem was treated for Neumann Laplacians on bounded domains.

This is a joint work with Jussi Behrndt (TU Graz).

References

- [1] R. Hempel, L. Seco, B. Simon, The essential spectrum of Neumann Laplacians on some bounded singular domains, *J. Funct. Anal.* 102 (1991), no. 2, 448–483.
- [2] R. Hempel, T. Kriecherbauer, P. Plankensteiner, Discrete and Cantor spectrum for Neumann Laplacians of combs, *Math. Nachr.* 188 (1997), 141–168.

Daya Shankar Kulshreshta

Strings and Superstrings

Quantization of bosonic and superstrings would be considered.

Usha Kulshreshta

Warmholes

Wormholes are considered in a theory of complex scalar phantom field in general relativity.

Tereza Kurimaiová (poster)

What is the optimal geometry for vibrating membranes?

We introduce the self-adjoint Robin Laplacian with negative boundary parameter and some of the properties of its spectrum. Of our main interest is the spectral isoperimetric inequality for the first eigenvalue which is the only known example where the ball is not an optimiser. This was proved using the asymptotic expansion of the first eigenvalue of the ball and annulus. Afterwards, we present an attempt to prove that the annulus is the optimiser using the so called parallel coordinates. Finally, an upper bound for particular not simply connected domains is obtained.

Sergiusz Kuzhel

Mathematical aspects of C-symmetries

The aim of this talk is to study properties of C-symmetries in the Krein space setting. For this purpose, extensions of dual definite subspaces to dual maximal definite ones are described. The obtained results are applied to the classification of C-symmetries. The concepts of dual quasi maximal subspaces and quasi bases are introduced and studied. It is shown that complex shift $g(\cdot) \rightarrow g(\cdot + ia)$ of Hermite functions g_n is an example of quasi bases in $L_2(\mathbb{R})$. The talk is based on results of [1].

Reference

[1] A. Kamuda, S. Kuzhel, V. Sudilovskaya, On dual definite subspaces in Krein space, arXiv:1802.08647, math.FA

Mustapha Maamache

Mending the broken PT-symmetry via a non unitary time-dependent transformation

Parity-time (PT) symmetry, the invariance under parity time reflection, is an important concept in physics recently developed in application to optical systems. Of particular interest for a non-Hermitian Hamiltonian is the situation when all quasi-energies are imaginary and constant. We demonstrate that non-Hermitian Hamiltonian systems with spontaneously broken PT-symmetry and imaginary eigenvalue spectrum can be made meaningful in a quantum mechanical sense when one introduces an explicit time-dependent non unitary transformation into the Shrodinger equation for the non Hermitian Hamiltonian H . For a meaningful physical picture one only needs to guarantee now that the expectation values of the obtained new Hamiltonian $H'(t)$ are real and instead identify a new symmetry (pseudo-hermiticity) to be responsible for this property.

Jan Mazáč

Linear mappings on model sets

Projections of high-dimensional lattices onto two orthogonal subspaces form the discrete sets that can be used as a mathematical model of quasicrystals and of their diffraction patterns. These patterns can exhibit, for example, the 5-, 9-, 10-fold symmetry. For a pattern with 5-fold symmetry one has to use at least 4-dimensional lattice. This lattice together with the projections is called the cut-and-project scheme. We build a matrix formalism for the description of the cut-and-project schemes. Using it we can answer the following question: For a given linear mapping A does there exist a cut-and-project scheme such that the first projection of a lattice is invariant under this mapping? This question is closely connected to the determination of the minimal dimension of the lattice in the cut-and-project scheme. We give a few estimates of this dimension, thus following up the results of Baake, Lagarias, Moody and others who only considered scalings, rotations and scaled rotations.

Prepared in collaboration with Zuzana Masáková and Edita Pelantová (FNSPE).

Recommended references:

- J. C. Lagarias. Geometric Models for Quasicrystals I. Delone Sets of Finite Type. *Discrete Comput Geom*, 21:161–191, 1999.
- Michael Baake and Uwe Grimm, *Aperiodic order. Vol. 1: A mathematical invitation*, *Encyclopedia of Mathematics and its Applications*, vol. 149, Cambridge University Press, Cambridge, 2013.
- Michael Baake and Uwe Grimm, *Aperiodic Order. Vol. 2: Crystallography and Almost Periodicity*, *Encyclopedia of Mathematics and its Applications*, vol. 166, Cambridge University Press, Cambridge, 2017.

Alexander Moroz

Constraint polynomial approach - an alternative to the functional Bethe Ansatz method?

Inspired by the Kus constraint polynomials in the quantum Rabi model, a general constraint polynomial approach has been recently developed. It (i) reproduced known constraint polynomials for the usual and driven Rabi models and (ii) immediately generated hitherto unknown constraint polynomials for the two-mode, two-photon, and generalized Rabi models, implying that the eigenvalues of corresponding polynomial eigenfunctions can be determined algebraically. The constraint polynomial approach is shown to replace a set of algebraic equations of the functional Bethe Ansatz method by a single polynomial constraint. As the proof of principle, the usefulness of the method is demonstrated for a number of quasi-exactly solvable potentials of the Schrödinger equation, such as two different sets of modified Manning potentials with three parameters, an electron in Coulomb and magnetic fields and relative motion of two electrons in an external oscillator potential, a perturbed double sinh-Gordon system, and the hyperbolic Razavy potential. For the quasi-exactly solvable examples considered here, constraint polynomials terminate a finite chain of orthogonal polynomials in an independent variable that need not to be necessarily energy. The finite chain of orthogonal polynomials is characterized by a positive-definite moment functional \mathcal{L} , implying that a corresponding constraint polynomial has only real and simple zeros. Constraint polynomials are shown to be different from the weak orthogonal Bender-Dunne polynomials.

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Work done in collaboration with Andrey E. Miroshnichenko (Canberra)

Ali Mostafazadeh

Nonlinear Transfer Matrix and Blowing up Light

We introduce a nonlinear generalization of the notion of transfer matrix and use it to devise a method for obtaining a physical application of blow-up solutions of the time-independent nonlinear Schrödinger equation. In particular we construct a scattering set-up consisting of a Kerr slab with a defocusing Kerr constant and a unidirectionally invisible linear slab that serves as a nonlinear amplifier for electromagnetic waves with definite frequency and amplitude.

J. Gonzalo Muga

Symmetries of non-Hermitian Hamiltonians

Hamiltonian symmetries are usually considered to be formulated as the commutation of H with some unitary or anti unitary operator. For non-Hermitian Hamiltonians this has to be generalized. Several groups of possible symmetries emerge. A super operator formulation is presented. Physical consequences are drawn, in particular with respect to possible scattering asymmetries.

Gregory Natanson

On Quantization of the Bagchi-Quesne-Roychoudhury Potential by a Finite X-Orthogonal Sequence of Heun Eigenpolynomials

As recently reported by the author [researchgate.net/publication/322851312 (2018)] there are three distinct rationally deformed hyperbolic Pöschl-Teller (h-PT) potentials quantized by finite sequences of orthogonal eigenpolynomials. Each polynomial sequence starts from an eigenpolynomial of nonzero degree and therefore belongs to a class of exceptional Bochner (X-Bochner) eigenpolynomials. Since the eigenpolynomials in question can be obtained by a rational Darboux transformation of Romanovsky-Jacobi (R-Jacobi) polynomials the cited study refers to these three polynomial sequences as XR-Jacobi eigenpolynomials distinguishing them by prefixes J1, J2, and D.

The Bagchi-Quesne-Roychoudhury (BQR) potential [Pramana J. Phys. 73, 337 (2009)] represents the very special case when all three rational SUSY partners constructed using the first-order seed polynomials can be formally described by a single rational expression with three poles on the real axis. We thus come to the RC-SLE with four regular singular points (including infinity) so quasi-rational solutions of this equation take form of Lambe-Ward kernels [Quat. J. Math. 5, 81 (1934)] whereas the corresponding XR-Jacobi eigenpolynomials turn into X-orthogonal Heun polynomials. It is shown that the X-orthogonal sequences of J1-Heun and J2-Heun polynomials identically coincide constituting a subset of generally non-orthogonal X1-Jacobi polynomials (with both indexes allowed to take arbitrary real values). It is also proven that the monic X-orthogonal Heun polynomial of order $m+1$ coincides with the second polynomial in the X_m -Jacobi orthogonal polynomial system after the latter is re-written in its monic form. As for the branch of the BQR potential conditionally exactly quantized by XR-Jacobi D-eigenpolynomials it has only one bound energy level and thereby is of little interest in this context.

Satoshi Ohya

Supersymmetry and Non-Abelian Geometric Phase

Supersymmetric quantum mechanics provides a natural playground for studying non-Abelian geometric phase, because supersymmetry always guarantees degeneracies in energy levels. In this talk I will present simple supersymmetric quantum mechanical models where Berry's connections become non-Abelian magnetic monopoles and are given by classical solutions of SU(2) Yang-Mills gauge theory.

Axel Pérez-Obiol

Stationary real solutions of the nonlinear Schrödinger equation on a ring with a defect

We analyze the 1D cubic nonlinear stationary Schrödinger equation on a ring with a defect for both focusing and defocusing nonlinearity. All possible d and $d = 0$ boundary conditions are considered at the defect, computing for each of them the real eigenfunctions, written as Jacobi elliptic functions, and eigenvalues for the ground state and first few excited energy levels. All six independent Jacobi elliptic functions are found to be solutions of some boundary condition. We also provide a way to map all eigenfunctions satisfying $d/d = 0$ conditions to any other general boundary condition or point-like potential.

Alfredo Raya

Supersymmetric electronic states in graphene under uniaxial strain

We study uniaxially strained graphene under the influence of non-uniform magnetic fields perpendicular to the material sample with a coordinate independent strain tensor. For that purpose, we solve the Dirac equation with anisotropic Fermi velocity and explore the conditions upon which such an equation possesses a supersymmetric structure in the quantum mechanical sense through examples. Working in a Landau-like gauge, wave functions and energy eigenvalues are found analytically in terms of the magnetic field intensity, the anisotropy scales and other relevant parameters that shape the magnetic field profiles.

Peter Schlosser

Propagators of the Schrödinger equation: Time evolution of superoscillations

Superoscillation is the phenomenon that functions can (locally) oscillate faster than any of their Fourier components. This strange and paradoxical behaviour was first found by Y. Aharonov and M. Berry who constructed and investigated specific functions having this property. Since the origin of such functions is physics, with the motivation e.g. to get higher resolutions of optical measurements, a strict mathematical investigation is not yet existing. Our work now concentrates on superoscillating functions as initial condition of the time dependent Schrödinger equation. Therefore, we derive propagators for specific potentials and treat them as operators between function spaces which are suitable to describe superoscillations. The main goal is then to prove the persistence of superoscillations over time.

Iana Van-Iun-Sian

On the scattering on star-like metric graph with fourth-order operator at the edges

Star-like metric graph consisting of a segment attached to a line is considered. We assume the fourth-order differential operator at the graph edges. The stationary scattering problem and resonances are investigated. As for the analogous problem for a half-line. The problem reduces to the following differential equation

$$p \frac{d^4 y_j}{dx^4} - q \frac{d^2 y_j}{dx^2} - k^2 y_j = 0 \quad j = 1, 2, 3,$$

with the conditions at the graph vertices:

$$y_2(L_2) = y_2^0(L_2) = 0, \quad y_1(0) = y_2(0) = y_3(0),$$
$$y_1^0(0) = y_2^0(0) = y_3^0(0) = 0, \quad -y_1^{000}(0) + y_2^{000}(0) + y_3^{000}(0) = \alpha y_1(0).$$

Here L_2 is the length of the segment (point L_2 corresponds to the boundary vertex), p is the coefficient characterizing the response to bending in the corresponding mechanical problem, q is the coefficient describing the tension of the rod, k is the wave number, α is related to the properties of rods connection at the internal vertex. The transmission and the reflection coefficients dependencies on the wave number and the model parameters are studied.

Oktaý Veliev

On the finite-zone potentials

I am going to give a talk about the Schrodinger operator $L(q)$ with a periodic potential q . First I find the necessary and sufficient conditions in terms of periodic and antiperiodic eigenvalues as well as in terms of the Fourier coefficients of the potential for which the number of the gaps in the real part of the spectrum of $L(q)$ with a PT-symmetric periodic complex-valued potential q is finite. Then using it I construct the large and easily checkable classes of the finite-zone PT-symmetric periodic potentials. Moreover, we discuss the diversity of the results for the PT-symmetric finite-zone potentials and the real finite-zone potentials.

Günter Wunner¹

Computing eigenfunctions and eigenvalues of boundary-value problems with the orthogonal spectral renormalization method

The spectral renormalization method was introduced in 2005 as an effective way to compute ground states of nonlinear Schrödinger and Gross-Pitaevskii type equations. In this paper, we introduce an orthogonal spectral renormalization (OSR) method to compute ground and excited states (and their respective eigenvalues) of linear and nonlinear eigenvalue problems. The implementation of the algorithm follows four simple steps: (i) reformulate the underlying eigenvalue problem as a fixed-point equation, (ii) introduce a renormalization factor that controls the convergence properties of the iteration, (iii) perform a Gram-Schmidt orthogonalization process in order to prevent the iteration from converging to an unwanted mode, and (iv) compute the solution sought using a fixed-point iteration. The advantages of the OSR scheme over other known methods (such as Newton's and self-consistency) are (i) it allows the flexibility to choose large varieties of initial guesses without diverging, (ii) it is easy to implement especially at higher dimensions, and (iii) it can easily handle problems with complex and random potentials. The OSR method is implemented on benchmark Hermitian linear and nonlinear eigenvalue problems as well as linear and nonlinear non-Hermitian PT -symmetric models.

Work done in collaboration with Holger Cartarius¹, Ziad H. Musslimani^{1,2} and Lukas Schwarz¹

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