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→ THE BOOK OF ABSTRACTS ←

(the version of September 8, 2017)

(alphabetical ordering)

Anna Allilueva

Waves on geometric graphs

We describe certain properties of wave equations on geometric graphs. In particular, we study equations corresponding to different types of Laplacians and corresponding reflections on the vertices. We discuss the distribution of wave energy; in particular we show that in certain cases this distribution is governed by the location of eigenspaces of the corresponding unitary operator. For strongly localized solutions, this operator reduced to reflection from a subspace.

Petr Ambrož

Voronoi Tilings of Quasicrystals

Quasicrystals are materials whose diffraction pattern reveals rotational symmetries forbidden in periodic patterns. Experimentally observed were symmetries of order 8, 10 and 12. One of possible models of quasicrystals are point sets constructed using the well-known cut-and-project method. Local configurations of such point sets can be characterized via corresponding Voronoi tiling. As quasicrystals with decagonal symmetry were already treated, we consider quasicrystals with 12-fold rotational symmetry and we describe their distinct Voronoi tiles based on the size of the acceptance window of said cut-and-project method. For one particular size of the dodecagonal acceptance window we obtain a Voronoi tiling composed of only two prototiles.

Fabio Bagarello

kq-representation for non-Hermitian position and momentum operators, and bi-coherent states

We show how the Zak kq-representation, originally used in many-body theory, can be adapted to deal with pseudo-bosons, and under which conditions this can be done. By means of this representation we prove completeness of a discrete set of bi-coherent states constructed out of pseudo-bosonic operators. The case of Riesz bi-coherent states is analyzed in detail.

Holger Cartarius

Open many-body quantum systems with topologically non-trivial phases

Non-Hermitian systems with PT symmetry can possess purely real eigenvalue spectra if the eigenstates are PT symmetric as well. At the edges of multi-site many-body systems topologically protected states can appear, which lead to complex relations with PT-symmetric potentials. We study one-dimensional bosonic and fermionic systems, in which topologically nontrivial edge states appear, and investigate their behaviour in relation to balanced gain and loss of the probability amplitude modelled by complex PT-symmetric potentials. Typically the edge states are not PT symmetric since their probability amplitude is localised only on one side of the system, however, a particle-hole symmetry can ensure their PT symmetry. For a many-particle system a dynamical approach with a master equation provides a more direct modelling of the in- and outcoupling of particles. Thus, we investigate our models with a master equation in Lindblad form. It is shown that the dynamics of the density matrix follows the predictions of stationary calculations using PT-symmetric potentials. In particular it is found that there is a clear distinction in the dynamics between the topologically different cases known form the stationary eigenstates.

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Work done together with Marcel Klett, Felix Dangel, Marcel Wagner, Dennis Dast and Guenter Wunner

Francisco M. Fernández and Javier Garcia

Highly accurate calculation of the real and complex eigenvalues of one-dimensional anharmonic oscillators

We draw attention on the fact that the Riccati-Padé method developed some time ago enables the accurate calculation of bound-state eigenvalues as well as of resonances embedded either in the continuum or in the discrete spectrum. We apply the approach to several one-dimensional models that exhibit different kind of spectra. In particular we test a WKB formula for the imaginary part of the resonance in the discrete spectrum of a three-well potential.

Daniel Hook

Behavior of eigenvalues in a region of broken PT symmetry

PT-symmetric quantum mechanics began with a study of the Hamiltonian $H = p^2 + x^2(ix)^\epsilon$. When $\epsilon \geq 0$, the eigenvalues of this non-Hermitian Hamiltonian are discrete, real, and positive. This portion of parameter space is known as the region of unbroken PT symmetry. In the region of broken PT symmetry, $\epsilon < 0$, only a finite number of eigenvalues are real and the remaining eigenvalues appear as complex-conjugate pairs. The region of unbroken PT symmetry has been studied but the region of broken PT symmetry has thus far been unexplored. This paper presents a detailed numerical and analytical examination of the behavior of the eigenvalues for $-4 < \epsilon < 0$. In particular, it reports the discovery of an infinite-order exceptional point at $\epsilon = -1$, a transition from a discrete spectrum to a partially continuous spectrum at $\epsilon = -2$, a transition at the Coulomb value $\epsilon = -3$, and the behavior of the eigenvalues as ? approaches the conformal limit $\epsilon = -4$.

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Results of collaboration with

Carl M. Bender, Nima Hassanpour, S. P. Klevansky, Christoph Snderhauf and Zichao Wen, published, on May 15th, 2017, in Phys. Rev. A 95, 052113.

Jiří Hrivnák

Discrete Cosine and Sine transforms on Honeycomb Lattice

The discrete Fourier-like analysis of generalized cosine and sine functions on the two-dimensional honeycomb lattice is presented. The theoretical background stems from the concept of Weyl-orbit functions, discretized simultaneously on the weight and root lattices of the Weyl group A_2. The introduced class of extended Weyl-orbit functions generalizes periodicity and boundary properties of the one-dimensional cosine and sine functions. Three types of discrete complex Fourier-Weyl transforms and three types of real-valued Hartley-Weyl transforms are detailed. Examples of unitary transform matrices and interpolation behavior of the discrete transform is demonstrated. Consequences of the developed discrete transforms for transversal eigenvibrations of the mechanical graphene model are discussed.

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This is a joint work with Lenka Motlochová.

A.M. Ishkhanyan

Bi-confluent Heun solutions of the Schrödinger equation

We present the five six-parametric Lamieux-Bose potentials for which the general solution of the one-dimensional Schrdinger equation is written in terms of the bi-confluent Heun functions [1,2]. To derive the confluent hypergeometric reductions of this family of potentials, we construct an expansion of the solutions of the bi-confluent Heun equation in terms of the Hermite functions. The series is governed by a three-term recurrence relation between successive coefficients of the expansion. We examine the restrictions that are imposed on the involved parameters in order that the series terminates thus resulting in closed-form finite-sum solutions of the bi-confluent Heun equation. We further identify a particular conditionally integrable potential for which the involved bi-confluent Heun function admits a four-term finite-sum expansion in terms of the Hermite functions [3]. This is an infinite well defined on a half-axis. We present the explicit solution of the one-dimensional Schrdinger equation for this potential and discuss the bound states supported by the potential. We derive the exact equation for the energy spectrum and construct a highly accurate approximation for the bound-state energy levels.

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Results of collaboration with T. A. Ishkhanyan

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[1] A. Lamieux and A.K. Bose, "Construction de potentiels pour lesquels l'quation de Schrödinger est soluble", Ann. Inst. Henri Poincaré A 10, 259-270 (1969).

[2] A.M. Ishkhanyan and V.P. Krainov, "Discretization of Natanzon potentials".

[3] T.A. Ishkhanyan and A.M. Ishkhanyan, "Solutions of the bi-confluent Heun equation in terms of the Hermite functions", Ann. of Phys. (2017),

DOI: https://doi.org/10.1016/j.aop.2017.04.015, arXiv:1608.02245 [quant-ph].

Yogesh N. Joglekar

Parity-time symmetric systems with time-delayed gain and loss

In recent years, open classical and quantum systems, described by an effective, non-hermitian Hamiltonian, have emerged as a research frontier. Such Hamiltonians are instead invariant under combined parity and time-reversal operations, and as such are known as PT-symmetric Hamiltonians. All of these studies are confined to either static or time-periodic (Floquet) Hamiltonians that are experimentally realized in coupled optical waveguides, resonators, ultra cold atoms, and mechanical systems. I will define the notion of a PT-symmetric system with time-delay, present analytical and numerical results for dynamics of such systems, and show their experimental realization in coupled semiconductor lasers.

Vladimir V. Konotop

CPT-symmetric coupled nonlinear Schrdinger equations

There will be considered two models of the coupled nonlinear Schrödinger equation which are symmetric with respect to simultaneous time-reversal (T), spatial-inversion (P), and field exchange (C). The first model describes the spin-orbit coupled Bose-Einstein condensate with loading of atoms in one hyperfine state and removing from the other one. The second model describes two optical waveguides, one with gain and another with loss, whose coupling is highly dispersive. For both models there will be discussed features of the (CPT)-symmetry breaking, the existence of nonlinear modes (the first model) and solitons (the second model), as well as their stability. Both works are done in collaboration with Y. V. Kartashov and D. A. Zezyulin.

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[1] Kartashov, Y. V, Konotop, V. V, & Zezyulin, D. A. (2014). CPT -symmetric spin-orbit - coupled condensate. EPL (Europhysics Letters), 107, 50002.

[2] Zezyulin, D. A., Kartashov, Y. V., & Konotop, V. V. (2017). CPT-symmetric coupler with intermodal dispersion, 42(7), 1273-1276.

Jan Kotrbatý

Lie fields and representations of the Poincaré groups

Irreducible unitary representations of the Poincaré group $\mathcal{P}_4$ are of fundamental importance in quantum physics. The representations were classified by E. P. Wigner in 1939 and later on his result was broadened to much wider class of Lie groups by G. W. Mackey.

Within the talk, an alternative method for construction of irreducible unitary representations is suggested and illustrated on the Poincaré groups $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$. The technique is motivated by the famous Gelfand-Kirillov conjecture, namely we make use of the relationship between the fields of fractions corresponding to Weyl algebras and universal enveloping algebras, respectively. Connection to Mackey theory is discussed in each case in order to show that both methods lead to the same results.

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The talk is based on joint work with Miloslav Havlíček, Severin Pošta (both CTU Prague) and Patrick Moylan (Penn State).

Albert T. Kotvytskiy

The Analysis of a Set of Images of an N-point Gravitational Lens by the Algebraic Geometry Methods

The N-point gravitational lens is studied. Let X^2 be the image of the transformation $L : Y^2 \to X^2$ from the plane of the source into the plane of the lens. The inverse transformation has a rather simple form: and we study it by the methods of algebraic geometry. To this end, we work in coordinates and reduce the map to a polynomial relation $F_1(x_1, x_2, y_1) = 0$, $F_2(x_1, x_2, y_2) = 0$. In the case of the identically zero resultant the solution set is infinite, and its equations have a common component. In the case of a single-point lens, the physical meaning of this phenomenon is the Einstein ring. If the solution set is finite, then by the Bezout theorem, the number of solutions is $p = degF_1 \cdot degF_2$, and the number of the real solutions p_r is such that $p_r \leq p$ and $p_r \equiv p(mod\ 2)$. It is known that the total number of real solutions is odd. However, the set of images is one-to-one with respect to the set of not all real solutions, but only different ones. From this, to determine the images number of an N-point gravitational lens, additional information is needed, for example, the values of some parameters. We studied cases: N=1, N=2 and N=3.

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Talk prepared together with S. D. Bronza (Ukrainian State University of Railway Transport, bronza.sem @ gmail.com) and V. Yu. Shablenko (Karazin Kharkiv National University, shablenkov @ gmail.com)

Géza Lévai

The finite $\mathcal{PT}$-symmetric square well potential

Exactly solvable potentials are typically applied to approximate realistic physical problems. Here we reverse this usual scenario and construct a potential that mimics an exactly solvable one in order to decide whether its unusual features are specific to its mathematical construction, or to the physical content behind it. This study is motivated by the finding [1] that $V(x) = v\,\mathrm{sech}^2(x) + \mathrm{i}w\tanh(x)$, the $\mathcal{PT}$-symmetric Rosen–Morse II potential possesses purely real discrete energy eigenvalues while the spectrum of the $\mathcal{PT}$-symmetric Scarf II potential that differs from it only in its imaginary component ($w\tanh(x)\mathrm{sech}(x)$), is either purely real or purely complex, depending on w.

We approximate the real and imaginary components of the Rosen–Morse potential with the finite real square well and the imaginary step potential, respectively. We determine the solutions in terms of hyperbolic and exponential functions, and construct the transmission and reflection amplitudes. We search for the poles of T_R to identify the bound-state solutions. It turns out that only solutions with real energy eigenvalues can be found, which is in agreement with the results concerning the $\mathcal{PT}$-symmetric Rosen–Morse II potential. We attribute this finding to the asymptotically non-vanishing imaginary potential components.

[1] G. Lévai and E. Magyari, J. Phys. A:Math. Gen. **42** (2009) 195302.

Ali Mostafazadeh

Exact solution of the two-dimensional scattering problem for the delta-function potentials supported on subsets of a line

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The recently developed dynamical formulation of scattering theory offers means for solving a number of open problems in scattering theory. In this talk we outline its application in obtaining exact solution of the scattering problem for a class of delta-function potentials in two dimensions whose support is a subset of a line. These include finite linear arrays of delta-function potentials in two dimensions, as well as delta-function potentials supported on a line and having an arbitrary periodic spatial dependence along this line. A particular example of the latter is a periodic infinite linear array of two-dimensional delta-functions, i.e., a Dirac comb embedded in a plane. We also prove a theorem on potentials having identical scattering properties in specific spectral intervals.

Satoshi Ohya

Exactly Solvable Bound-State Problems and Lie Algebras so(2,1), so(3), and iso(2)

I shall revisit very old bound-state problems in quantum mechanics. I shall focus on five specific examples: Coulomb, trigonometric Rosen-Morse, Manning-Rosen, hyperbolic Rosen-Morse, and attractive Liouville potentials, all of whose bound state problems are known to be exactly solvable. I show that in these problems the energy eigenfunctions as well as energy eigenvalues are all obtained by means of the unitary representations of the Lie algebras so(2,1), so(3), or iso(2).

Axel Pérez-Obiol

Spontaneous twisting and shrinking of carbon nanotubes

Deformations of single-wall carbon nanotubes are investigated within the tight-binding model with deformation-dependent hopping energies. We show that the nanotubes tend to twist and shrink spontaneously at zero temperature. The explicit values of the deformation parameters are computed for a wide range of nanotubes with varying diameter and chirality. The changes of the spectral gap associated with the spontaneous deformation are shown to depend on the chirality of the nanotubes

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Talk prepared together with Vít Jakubský

Daniel Reitzner

Navigating a maze using quantum-walk searches

We show that it is possible to use a quantum walk to find a path from one marked vertex to another. In the specific case of M stars connected in a chain, one can find the path from the first star to the last one in $O(M\sqrt{N})$ steps, where N is the number of spokes of each star. First we provide analytical result showing this speedup by starting in a phase-modulated highly superposed initial state. Next, we show, that the search can also be performed by a series of successive searches when we start at the last known position and search for the next connection in $O(\sqrt{N})$ steps. For this result we use the analytical solution that can be obtained for a ring of stars of double the length of the chain.

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Reference: arxiv.org/abs/1707.01581.

Fabio Rinaldi

Gaussian potential is essentially solvable

In this presentation we expand the preliminary findings showing that the one-dimensional Schrdinger equation with an attractive Gaussian potential is essentially solvable in the sense that its eigenvalues, whose number depends on the magnitude of the coupling parameter but is always greater than or equal to one, can be determined by diagonalising a trace class operator, whose integral kernel can be suitably expressed in p-space, and solving the corresponding number of transcendental equations to get the solutions as functions of the coupling constant.

Etsuo Segawa

Discrete-time quantum walks induced by quantum graphs

I introduce discrete-time quantum walks induced by quantum graphs with some boundary conditions. In this talk, we discuss on a resonance of the quantum walks.

Andrei Shafarevich

Laplacians and wave equations on polyhedral surfaces

We study properties of Laplacialns and wave equations on polyhedral surfaces. These equations appear, particularly in the problem of long waves' scattering on point obstacles. We describe different types of Laplacians, their kernels (spaces of harmonic functions), trace formulas and wave fronts for localized solutions of wave equations.

Teoman Turgut

Uniqueness of the ground state for Lee model

We define an oversimplified version of the Lee model, in which a two level system is interacting with bosonic particles on a three dimensional manifold. We show that this system can be understood by an associated operator, so called the principal operator, after removing an infinity (mass renormalization). We indicate that this operator contains all the information about the dynamics and the spectrum is bounded from below. We explain a method to show that there is a well defined self-adjoint Hamiltonian defined by this operator. We sketch the proof that on a compact manifold the lowest eigen-state of this operator is unique, using the theory of positivity improving semigroups.

Oktay Veliev

Schrödinger operator with the periodic PT-symmetric potentials

I am going to give a talk about the one-dimensional Schrödinger operator $L(q)$ with a complex-valued PT-symmetric periodic potential q. First we consider the general spectral property of the spectrum of $L(q)$ and prove that the main part of its spectrum is real and contains the large part of $[0, \infty)$. Using this we find necessary and sufficient condition on the potential for finiteness of the number of the nonreal arcs in the spectrum of $L(q)$. Moreover, we find necessary and sufficient conditions for the equality of the spectrum of $L(q)$ to the half line and consider the connections between spectrality of $L(q)$ and the reality of its spectrum for some class of PT-symmetric periodic potentials. Besides, we give a complete description of the shape of the spectrum of the Hill operator with optical potential $4\cos^2 x + 4iV \sin 2x$.

Finally I will speak about the spectrum and Bloch functions of the multidimensional Schrodinger operator $L(q)$ in $L_2\left(\mathbb{R}^d\right)$ $(d \geq 2)$ with complex-valued periodic, with respect to a lattice Ω, potential q when the Fourier coefficients q_γ of q with respect to the orthogonal system $\{e^{i\langle\gamma x\rangle} : \gamma \in \Gamma\}$ vanish if γ belong to a half-space, where Γ is the lattice dual to Ω. We prove that the Bloch eigenvalues are $\mid \gamma + t \mid^2$ for $\gamma \in \Gamma$, where t is a quasimomentum and find explicit formulas for the Bloch functions. It implies that the Fermi surfaces of $L(q)$ and $L(0)$ are the same. The considered set of operators includes a large class of PT-symmetric operators used in the PT-symmetric quantum theory.

Václav Zatloukal

Hamiltonian constraint formulation of field theories

The Hamiltonian constraint is a concept useful for Hamiltonian formulation not only of general relativity, but, in fact, of a generic field theory, as pointed out in Ref. [1, Ch. 3], and exploited in [2]. Characteristic features of this formulation are: finite-dimensional configuration space, and multivector-valued momentum variable (cf., the De Donder-Weyl theory). The Hamiltonian is a function $H(q, P)$ of spacetime and field-space coordinates (combined in points q), and the momentum P, and it has to vanish on classical field configurations.

I show how the canonical equations of motion, which generalize the Hamilton's equations of analytic mechanics, can be derived, as well as a field-theoretic generalization of the Hamilton-Jacobi equation. Symmetries of the Hamiltonian lead to conservation laws via a Hamiltonian version of the Noether theorem. Arguments based on local symmetries then lead to the introduction of static gauge fields.

The Hamiltonian constraint formulation of classical field theories offers an attractive ground for an unorthodox quantization of field theories, which deserves further investigation.

In this work I use the mathematical language of geometric algebra and calculus [3], which proves to be very efficient when it comes to handling higher-dimensional geometric objects such as the field configurations (viewed as surfaces with oriented surface elements), the momentum multivector, etc.

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P. T. O.

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